basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P1

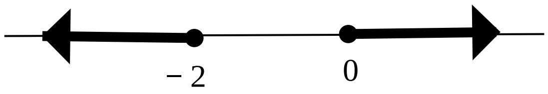
MARKING GUIDELINES

EXEMPLAR 2018

MARKS: 150

These marking guidelines consist of 12 pages.

QUESTION 1

1.1.1	$x(x+2)=0$ $\therefore x=0$ or $x=-2$	✓ $x=0$ ✓ $x=-2$ (2)
1.1.2	$x(x+2)\geq 0$ $\therefore x\leq -2$ OR $x\geq 0$  	✓ $x\leq -2$ ✓ $x\geq 0$ ✓ **OR** ✓Graphical representation (4)
1.2	$5x^2=2+x$ $5x^2-x-2=0$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\dfrac{-(-1)\pm\sqrt{(-1)^2-4(5)(-2)}}{2(5)}=\dfrac{1\pm\sqrt{41}}{10}$ $\therefore x\approx 0{,}74$ or $x\approx -0{,}54$	✓Standard form ✓Substitution into the quadratic formula ✓ $x\approx 0{,}74$ ✓ $x\approx -0{,}54$ (4
1.3	$m-t-1=0$ $m=t+1$ $m^2+t^2=5$ $(t+1)^2+t^2=5$ $t^2+2t+1+t^2-5=0$ $2t^2+2t-4=0$ $t^2+t-2=0$ $(t+2)(t-1)=0$ $\therefore t=-2$ or $t=1$ $m=-2+1=-1$ or $m=1+1=2$ **OR** $m-t-1=0$ $t=m-1$ $m^2+t^2=9$ $m^2+(m-1)^2=5$ $m^2+m^2-2m+1-5=0$ $2m^2-2m-4=0$ $m^2-m-2=0$ $(m-2)(m+1)=0$ $\therefore m=2$ or $m=-1$ $t=2-1=1$ or $t=-1-1=-2$	✓Making m the subject ✓Substitution ✓Simplification ✓Factors ✓Both values of t ✓Both values of m **OR** ✓Making t the subject ✓Substitution ✓Simplification ✓Factors ✓Both values of m ✓Both values of t (6)

1.4.1	$\varepsilon=\frac{L_2-L_1}{L_1}$ $\varepsilon L_1=L_2-L_1$ $\varepsilon L_1+L_1=L_2$ $L_1(\varepsilon+1)=L_2$ $L_1=\frac{L_2}{(\varepsilon+1)}$ OR $\varepsilon=\frac{L_2}{L_1}-1$ $\varepsilon+1=\frac{L_2}{L_1}$ $L_1(\varepsilon+1)=L_2$ $\therefore L_1=\frac{L_2}{(\varepsilon+1)}$	✓multiply with LCD ✓common factor ✓divide by factor (3)
1.4.2	$L_1=\frac{L_2}{\varepsilon+1}$ $=\frac{18}{1+0,8}$ cm $=10$ cm	✓Substitution ✓Simplification (2)
1.4.3	$10=8+2\ =2^3+2$ $=1010_2$	✓ 2^3+2 ✓ 1010_2, (2)
1.5	$12\times0,00361$ $=0,04332$ $=4,332\times10^{-2}$	✓ $0,04332$ ✓ $4,332\times10^{-2}$ **[25]**

QUESTION 2

2.1.1	$p=-1$	✓ $p=-1$ (1)
2.1.2	$9-3p<0$ $9<3p$ $\therefore p>3$	✓ $9-3p<0$ ✓ $p>3$ (2)
2.1.3	0 **OR** 3	✓0 **OR** 3 (1)
2.2	$x^2-4x+(k-1)=0$ For equal roots, $\Delta=b^2-4ac=0$ $(-4)^2-4(1)(k-1)=0$ $16-4k+4=0$ $-4k=-20$ $\therefore k=5$	✓For equal roots, $\Delta=0$ ✓Substitution ✓Simplification ✓Value of k (4) **[8]**

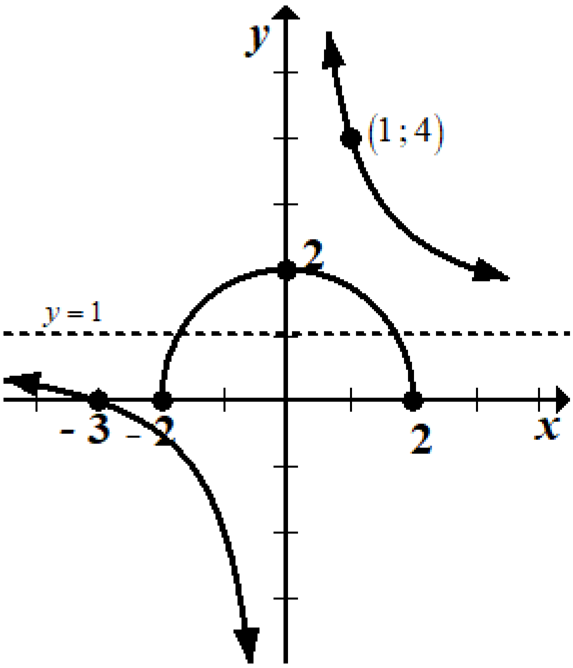
QUESTION 3

3.1.1	$\frac{5\times 2^{n-1}-2^n}{2^n}$ $=\frac{2^n\left(5\times 2^{-1}-1\right)}{2^n}$ $=5\times\frac{1}{2}-1=\frac{3}{2}$ **OR** $\frac{5\times 2^{n-1}-2^n}{2^n}$ $=\frac{5\times 2^{n-1}}{2^n}-\frac{2^n}{2^n}\ =5\times 2^{-1}-1$ $=2\frac{1}{2}-1=\frac{3}{2}$	✓✓Common factor ✓Simplification ✓✓Dividing each term by the denominator ✓Simplification (3)
3.1.2	$\sqrt{64+16}-\sqrt{20}$ $=\sqrt{80}-\sqrt{4\times 5}$ $=4\sqrt{5}-2\sqrt{5}$ $=2\sqrt{5}$	✓Addition ✓Simplified surd ✓Simplified surd ✓Simplification (4)
3.1.3	$\log_6 216\times\log 0{,}001$ $=\log_6 6^3\times\log\frac{1}{1000}$ $=\log_6 6^3\times\log 10^{-3}$ $=3\log_6 6\times\left(-3\log 10\right)$ $=3(1)\times(-3)(1)$ $=-9$	✓ $\log_6 6^3$ ✓ $\log 10^{-3}$ ✓ $3\log_6 6-3\log 10$ ✓Simplification (4)
3.2.2	$\log(x+18)-\log x=1$ $\log\frac{(x+18)}{x}=1$ $\frac{(x+18)}{x}=10$ $10x=x+18$ $9x=18$ $\therefore x=2$	✓Apply log property ✓Change from log form to exp. Form ✓Simplification ✓Value of x (4)

3.3	$z=3+\sqrt{3}i$ $\lvert z\rvert=r=\sqrt{x^2+y^2}$ $=\sqrt{(3)^2+\left(\sqrt{3}\right)^2}\quad=\sqrt{12}$ $\tan\theta=\frac{\sqrt{3}}{3}$ $\theta=30°$ $z=\sqrt{12}\,\text{cis}(30°)$ OR $z=\sqrt{12}\left[\cos 30°+i\sin 30°\right]$	✓Calculating the modulus ✓Simplification ✓ $\tan\theta=\frac{\sqrt{3}}{3}$ ✓Argument ✓Correct polar form (5)
3.4	$x+yi=(3+5i)(2-7i)$ $x+yi=6-11i-35i^2$ $x+yi=6-11i-35(-1)$ $x+yi=6-11i+35$ $x+yi=41-11i$ $\therefore x=41$ and $y=-11$	✓ $6-11i-35i^2$ ✓ $i^2=-1$ ✓ $x=41$ ✓ $y=-11$ (4) **[24]**

QUESTION 4

4.1.1	$x-$intercepts, $f(x)=0$ $2x^2+4x-6=0$ $2(x+3)(x-1)=0$ OR $(x+3)(2x-21)=0$ $\therefore x=-3$ or $x=1$ $\therefore \text{B}(1\ ;\ 0)$	✓Finding other factor ✓Coordinates of B. (2)
4.1.2	$f(x)=2x^2+4x-6$ $\left(\frac{-b}{2a};\frac{4ac-b^2}{4a}\right)=\left(\frac{-4}{2(2)};\frac{4(2)(-6)-(4)^2}{4(2)}\right)$ $\therefore \text{D}(-1\ ;-8)$ **OR** $x=\frac{-b}{2a}=\frac{-4}{2(2)}$ $\therefore x=-1$ $f(-1)=2(-1)^2+4(-1)-6=-8$ $\therefore \text{D}(-1\ ;-8)$ **OR** $x_D=\frac{-3+1}{2}=-1$ $f(-1)=2(-1)^2+4(-1)-6=-8$ $\therefore \text{D}(-1\ ;-8)$ **OR** $f(x)=2x^2+4x-6$ $f'(x)=4x+4=0$ $\therefore x=-1$ $f(-1)=2(-1)^2+4(-1)-6=-8$ $\therefore \text{D}(-1\ ;-8)$	✓✓Substitution in formula ✓Coordinates of D **OR** ✓Substitution in formula ✓Substitution to find *y* ✓Coordinates of D **OR** ✓Using *x*-intercepts ✓Substitution to find *y* ✓Coordinates of D **OR** ✓Using the derivative ✓Substitution to find *y* ✓Coordinates of D (3)
4.1.3	$g(x)=k^x+q$ $10=k^2+6$ $k^2=4$ $\therefore k=2$	✓Substituting coordinates of *Q* ✓Simplified equation ✓Correct value of *k*. (3)
4.1.4	$y=6$	✓ $y=6$ (1)
4.1.5	$-3<x<1$	✓Correct critical values ✓Correct notation (2)

4.2.1	$x=0$ and $y=1$	✓ $x=0$ ✓ $y=1$ (2)
4.2.2	$h(x)=\frac{3}{x}+1$ $0=\frac{3}{x}+1$ $-1=\frac{3}{x}$ $\therefore x=-3$	✓Substituting coordinates of Q ✓Value of x (2)
4.2.3	$r=2$	✓ $r=2$ (1)
4.2.4		✓Shape of *h* ✓Asymptote ✓*x*-intercept ✓Any other point on the graph of *h* ✓Shape of *g* ✓*x*-intercepts of *g* ✓*y*-intercept of *g* (7)
4.2.5	$0\le y\le 2$	✓ $0\le y$ ✓ $y\le 2$ (2) **[25]**

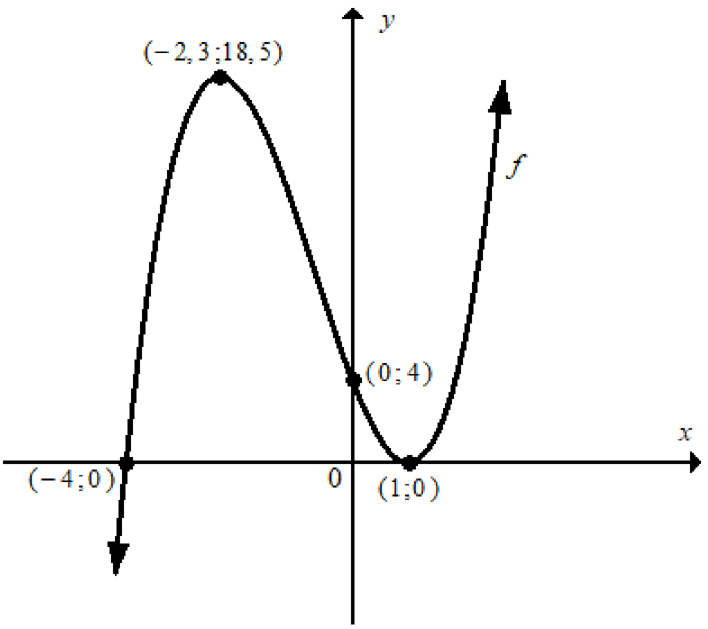
QUESTION 5

5.1	$i_{eff} = \left(1+\frac{i}{m}\right)^m - 1$ $i_{eff} = \left(1+\frac{0,072}{2}\right)^2 - 1$ $\approx 0,073296$ $\therefore$ annual effective interest rate is $7,33\%$	✓Correct substitution ✓Simplification ✓Effective rate as % (3)
5.2	$\text{A} = \text{P}(1-i)^n$ $70 = 220(1-0,08)^n$ $\frac{7}{22} = (0,92)^n$ $n = \log_{0,92}\frac{7}{22}$ $\therefore n \approx 13,73363166$ It will take approximately 14 minutes.	✓Correct formula ✓Correct substitution ✓Simplified power form ✓Using logarithms ✓Nearest minute (5)
5.2.2	Value of A after 3 years: $\text{A} = \text{P}(1+i)^n$ $\text{A} = \text{R}150\,000\left(1+\frac{10,5\%}{4}\right)^{3\times4}$ $= \text{R}\,204\,705,40$ Value of P after withdrawal: $\text{P} = \text{R}\,204\ 705,40 - \text{R}\,30\ 000 = \text{R}174\,705,40$ Amount received at the end of the investment period: $\text{A} = \text{R}174\,705,40\left(1+\frac{10,5\%}{4}\right)^{2\times4}$ $\therefore \text{A} = \text{R}\,214\ 947,15$	✓Correct formula ✓Correct substitution ✓ R204 705,40 ✓ P = R174 705,40 ✓Correct substitution ✓Final amount (6) **[14]**

QUESTION 6

6.1	$f(x)=2x^2-3$ Average gradient $=\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$ $=\dfrac{[2(1)^2-3]-[2(-2)^2-3]}{1-(-2)}$ $=\dfrac{-1-5}{3}$ $=-2$	✓Corresponding y-value ✓Corresponding y-value ✓Substitution in formula ✓Simplification (4)
6.2	$f(x)=4-3x$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{[4-3(x+h)]-(4-3x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{4-3x-3h-4+3x}{h}$ $=\lim\limits_{h\to 0}\dfrac{-3h}{h}$ $=\lim\limits_{h\to 0}(-3)$ $=-3$	✓Definition ✓Substitution in the definition ✓Simplification (removing brackets) ✓Simplification (division) ✓Simplification (5)
6.3	$y=\dfrac{2}{x^3}+\sqrt{x}$ $y=2x^{-3}+x^{\frac{1}{2}}$ $\dfrac{dy}{dx}=-6x^{-4}+\dfrac{1}{2}x^{-\frac{1}{2}}$	✓ $2x^{-3}$ ✓ $x^{\frac{1}{2}}$ ✓ $-6x^{-4}$ ✓ $\dfrac{1}{2}x^{-\frac{1}{2}}$ (4)
6.4	$g(x)=-x^2-x$ $g(2)=-(2)^2-2=-6$ The point of contact is $(2;-6)$ $g'(x)=-2x-1$ $\therefore m_{\tan}=g'(2)=-2(2)-1=-5$ $y=mx+c$ OR $y-y_1=m(x-x_1)$ $-6=-5(2)+c$ OR $y-(-6)=-5(x-2)$ $c=4$ OR $y+6=-5x+10$ $\therefore y=-5x+4$	✓ value of y ✓ $m_{\tan}=-5$ ✓Correct substitution ✓Value of c (simplification) ✓Equation (any form) (5) **[18]**

QUESTION 7

7.1	$f(x)=x^3+2x^2-7x+4$ $f(1)=(1)^3+2(1)^2-7(1)+4$ $\therefore f(1)=0$ $\therefore x-1$ is a factor of f	✓Substitution ✓0 (2)
7.2	x-intercepts: $f(x)=0$ $x^3+2x^2-7x+4=0$ $(x-1)(x^2+3x-4)=0$ $(x-1)(x-1)(x+4)=0$ $x=1$ or $x=-4$	✓ (x^2+3x-4) (quadratic) ✓ $(x-1)(x-1)(x+4)$ (linear) ✓x-intercepts (3)
7.3	$f(x)=x^3+2x^2-7x+4$ $f^{/}(x)=3x^2+4x-7$ $f^{/}(x)=0$ $\therefore 3x^2+4x-7=0$ $(3x+7)(x-1)=0$ $\therefore x=-\frac{7}{3}$ or $x=1$ $(-2,3;18{,}5)$ and $(1;0)$	✓Derivative ✓ $f^{/}(x)=0$ ✓Factorisation ✓Both values of x ✓Coordinates of the turnings (5)
7.4	 	✓Shape ✓Intercepts with x-axis ✓y-intercept ✓Turning points (4) **[14]**

QUESTION 8

8.1.1	After 2 hrs $D(2) = 4 + 0,5(2)^2 - 0,25(2)^3$ m $= 4\text{m}$	✓Substituting 2 ✓Simplification (2)
8.1.2	$D = 4 + 0,5t^2 - 0,25t^3$ $D^{/}(t) = t - 0,75t^2$ At 12:00 (3 hours later): $D^{/}(3) = (3) - 0,75(3)^2$ $= -3,75\,\text{m.h}^{-1}$ $\therefore$	✓Derivative ✓Substitution of 3 ✓Simplified rate (3)
8.2.1	$P = -3v^2 + 30v$ Neither profit nor loss at $P = 0$ $-3v^2 + 30v = 0$ $-3v(v - 10) = 0$ $\therefore v = 0$ or $v = 10$ $v = 10\,\text{km.h}^{-1}$	✓ $P = 0$ ✓Factors ✓Correct value of v (3)
8.2.2	$P = -3v^2 + 30v$ $\frac{dP}{dv} = -6v + 30 = 0$ $\therefore v = 5\,\text{km.h}^{-1}$	✓Derivative ✓Equating to 0 ✓Value of v (3)
8.2.3	$P_{\max}$ (in R1000) = $-3(5)^2 + 30(5) = 75$ **OR** R75 000	✓Substitution ✓Profit in R1 000 (2) **[13]**

QUESTION 9

9.1	$\int\left(x^{-4}+\frac{7}{x}-1\right)dx$ $=\int x^{-4}\,dx+7\int\frac{1}{x}dx-\int dx$ $=\frac{x^{-3}}{-3}+7\ln x-x+C$	✓ $\frac{x^{-5}}{-5}$ ✓ $7\ln x$ ✓ $-x$ ✓ C (4)
9.2	$h(x)=-2x^2-6x$ $\int_{-3}^{0}(-2x^2-6x)dx$ $=\left[-\frac{2x^3}{3}-3x^2\right]_{-3}^{0}$ $=\left[\left(-\frac{2(0)^3}{3}-3(0)^2\right)-\left(-\frac{2(-3)^3}{3}-3(-3)^2\right)\right]$ $=-18+27$ $=9$ units square	✓ $-\frac{2x^3}{3}$; ✓ $-3x^2$ ✓Substituting 0 ✓Substituting -3 ✓Simplification (5) **[9]**

TOTAL: 150