



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE
*NASIONALE
SENIOR SERTIFIKAAT***

GRADE/*GRAAD* 11

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2016

MEMORANDUM

MARKS/*PUNTE*: 150

**This memorandum consists of 20 pages.
*Hierdie memorandum bestaan uit 20 bladsye.***

NOTE:

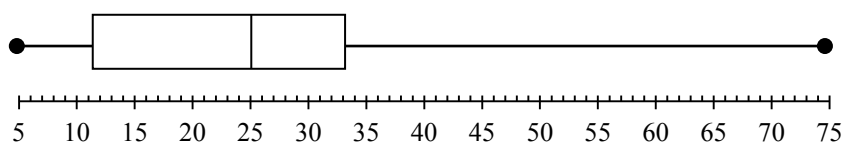
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.
- Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing.
- Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.

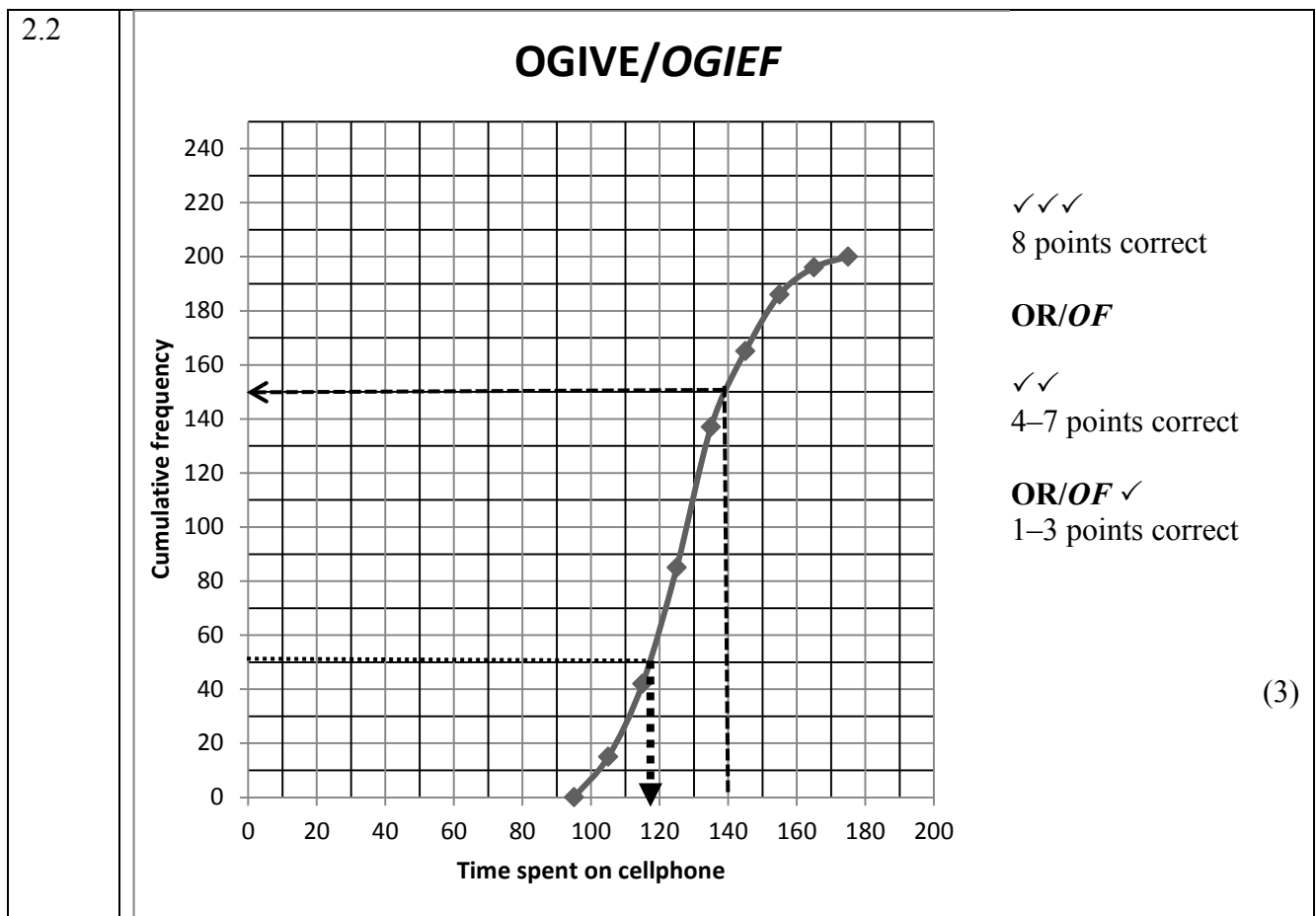
QUESTION/VRAAG 1

5	8	15	20	25	27	31	36	75
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1.1	Range/Omvang = $75 - 5$ = 70	✓ answer/antw (1)
1.2	Std dev/Std afwyking = 19,56	✓ ✓ answer/antw (2)
1.3	Median/Mediaan = 25	✓ answer/antw (1)
1.4	$Q_1 = \frac{8+15}{2} = 11,5$ $Q_2 = \frac{31+36}{2} = 33,5$ IQR = $Q_3 - Q_1$ = $33,5 - 11,5$ = 22	✓ $Q_1 = 11,5$ ✓ $Q_3 = 33,5$ ✓ answer/antw (3)
1.5		✓ box/mond ✓ whiskers/snor ✓ min and max (maks) (3)
1.6	Skewed to the right/skeef na regs Positively skewed/positief skeef	✓ answer/antw (1)
1.7	Outlier/uitskieter = 75 OR/OF $33.5 + 1.5(22) = 66.5$ Outlier/uitskieter = 75	✓ answer/antw ✓ answer/antw (1) [12]

QUESTION/VRAAG 2

2.1	TIME SPENT/ TYD SPANDEER (IN MINUTES/ MINUTE)	FREQUENCY FREKWENSIE f	CUMULATIVE FREQUENCY/ KUMULATIEWE FREKWENSIE f	✓✓ correct CF values/ korrekte KF-waardes (2)
	$95 < x \leq 105$	15	15	
	$105 < x \leq 115$	27	42	
	$115 < x \leq 125$	43	85	
	$125 < x \leq 135$	52	137	
	$135 < x \leq 145$	28	165	
	$145 < x \leq 155$	21	186	
	$155 < x \leq 165$	10	196	
	$165 < x \leq 175$	4	200	



2.3	$Q_1 = 118$ Accept any answer between (115 and 120)	✓ ✓ answer/antw (2)
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2.4	$Number\ of\ learners / Getal\ leerders = 200 - 150$ $= 50$ Accept 150 or any other reading between (145 and 155)	✓ 150 ✓ 50 (2) [9]
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QUESTION/VRAAG 3

3.1	$M = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$ $= \left(\frac{6+2}{2} ; \frac{-2+15}{2} \right)$ $= \left(4 ; \frac{13}{2} \right)$	✓ subst into/in midpt form/ midpnt vorm. ✓ answer/antw (2)
3.2	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{15 - 3}{2 - (-4)}$ $= 2$ $m_{MN} = m_{BC} = 2 \quad [BC \parallel MN]$	✓ subst into gradient form./subst in gradiëntvorm ✓ answer/antw ✓ gradients equal/ gradiënte gelyk (3)
3.3	$y - y_1 = m(x - x_1) \qquad y = mx + c$ $y - \frac{13}{2} = 2(x - 4) \qquad \frac{13}{2} = 2(4) + c$ $y = 2x - \frac{3}{2} \quad \text{OR/OF} \qquad -\frac{3}{2} = c$ $\qquad \qquad \qquad y = 2x - \frac{3}{2}$	✓ subst $\left(4 ; \frac{13}{2} \right)$ and $m = 2$ into str line eq. ✓ answer/antw (2)

3.4	<p>N is a midpoint of AC [Line through midpoint of one side parallel to second side]</p> $N\left(\frac{-4+6}{2}; \frac{3+(-2)}{2}\right)$ $= N\left(1; \frac{1}{2}\right)$ <p>OR/OF</p> $m_{AC} = \frac{3-(-2)}{(-4)-6}$ $= -\frac{1}{2}$ <p>Equation of AC $y - y_1 = m(x - x_1)$ $y - 3 = -\frac{1}{2}(x - (-4))$ $y = -\frac{1}{2}x + 1$ $-\frac{1}{2}x + 1 = 2x - \frac{3}{2}$ $-x + 2 = 4x - 3$ $x = 1$ $y = 2(1) - \frac{3}{2}$ $= \frac{1}{2}$ $N\left(1; \frac{1}{2}\right)$</p>	<p>✓ S ✓ R ✓ x-value/waarde ✓ y-value/waarde (4)</p> <p>✓ gradient of AC</p> <p>✓ equation of AC ✓ equating/gelykstelling</p> <p>✓ $N\left(1; \frac{1}{2}\right)$ (4)</p>
3.5	<p>N is the midpoint of BD and the midpoint of AC [diagonals of parm bisect] <i>N is die midpt v BD en midpt v AC [hoeklyne van parm halveer]</i></p> $\left(\frac{2+x}{2}; \frac{y+15}{2}\right) = \left(1; \frac{1}{2}\right)$ $\frac{2+x}{2} = 1 \qquad \frac{y+15}{2} = \frac{1}{2}$ $x = 0 \qquad y = -14$ <p>D(0 ; -14)</p>	<p>✓ $\frac{2+x}{2} = 1$ ✓ $\frac{y+15}{2} = \frac{1}{2}$ ✓ $x = 0$ ✓ $y = -14$ (4)</p>

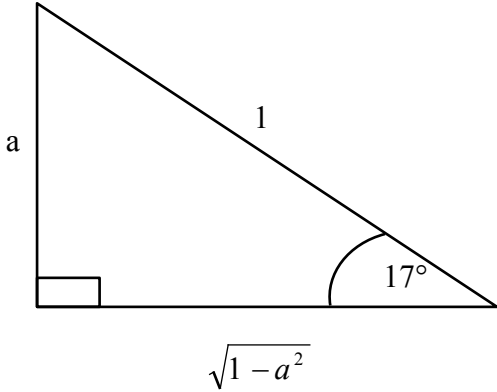
	<p>OR/OF From B to A</p> <p>$(x; y) \rightarrow (x+4; y-17)$ $D(-4+4; 3-17)$ $D(0; -14)$</p>	<p>✓ $x+4$ ✓ $y-17$ ✓ subst ✓ $D(0; -14)$</p> <p>(4) [15]</p>
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QUESTION/VRAAG 4

4.1	$m_{MP} = m_{PN}$ $\frac{2-0}{0-k} = \frac{4-2}{3-0}$ $\frac{2}{-k} = \frac{2}{3}$ $k = -3$	✓ $m_{MP} = m_{PN}$ ✓ subst. into gradient form. ✓ answer/antw (3)
4.2	$\tan \alpha = m_{PN}$ $\tan \alpha = \frac{2}{3}$ $\alpha = 33,69^\circ$ $\tan \beta = m_{AB}$ $\tan \beta = -\frac{1}{2}$ $\beta = -26,57^\circ + 180^\circ$ $= 153,43^\circ$ $\theta = 153,43^\circ - 33,69^\circ$ $= 119,74^\circ$	✓ $\alpha = 33,69^\circ$ ✓ $\tan \beta = -\frac{1}{2}$ ✓ $153,43^\circ$ ✓ $119,74^\circ$ (4)
4.3	$-\frac{1}{2}x + 4 = 0$ $x = 8$ $R(8;0)$ $MR = 8 - (-3)$ $= 11 \text{ units}$	✓ $y = 0$ ✓ $R(8;0)$ ✓ answer/antw (3)
4.4	$\text{Area of } \Delta MNR = \frac{1}{2}(MR) \cdot \perp \text{ height}$ $= \frac{1}{2}(11)(y - \text{value of } N)$ $= \frac{1}{2}(11)(4)$ $= 22 \text{ sq units/vk eenh}$ OR/OF	✓ area formula ✓ subst y-value of N ✓ answer/antw

	$MN = \sqrt{(3 - (-3))^2 + (4 - 0)^2}$ $= \sqrt{36 + 16}$ $= \sqrt{52} \text{ units/eenh}$ $\text{Area of/Opp van } \triangle MNR = \frac{1}{2} \times \sqrt{52} \times 11 \times \sin 33,69^\circ$ $= 21,999$ $\approx 22 \text{ sq units/vk eenh}$	<p>✓ $\sqrt{52}$</p> <p>✓ subst in area form</p> <p>✓ answer/antw</p> <p>(3) [13]</p>
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QUESTION/VRAAG 5

5.1.1	$x^2 + y^2 = r^2$ $(-8)^2 + (t)^2 = 17^2$ $t^2 = 225$ $t = -15$	✓ subst in pyth ✓ answer/antw (2)
5.1.2(a)	$\cos(-\theta)$ $= \cos \theta$ $= \frac{-8}{17}$	✓ $\cos \theta$ ✓ answer/antw (2)
5.1.2(b)	$1 - \sin \theta = 1 - \frac{-15}{17}$ $= \frac{17}{17} + \frac{15}{17}$ $= \frac{32}{17}$	✓ subst ✓ answer/antw (2)
5.2.1	$\tan 17^\circ = \frac{a}{\sqrt{1-a^2}}$ 	✓ sketch ✓ $\sqrt{1-a^2}$ ✓ answer/antw (3)
5.2.2	$\sin 107^\circ$ $= \sin(90^\circ + 17^\circ)$ $= \cos 17^\circ$ $= \sqrt{1-a^2}$ <p>OR/OF</p> $\sin 107^\circ$ $= \sin(180^\circ - 73^\circ)$ $= \sin 73^\circ$ $= \sqrt{1-a^2}$	✓ $\cos 17^\circ$ ✓ $\sqrt{1-a^2}$ (2) ✓ $\sin 73^\circ$ ✓ $\sqrt{1-a^2}$ (2)

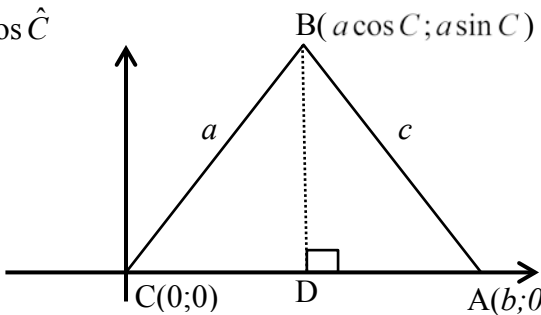
<p>5.2.3</p>	$\begin{aligned} & \cos^2 253^\circ + \sin^2 557^\circ \\ & = (-\cos 73^\circ)^2 + (-\sin 17^\circ)^2 \\ & = (-a)^2 + (-a)^2 \\ & = 2a^2 \end{aligned}$	<p>✓ $\cos^2 73^\circ$ ✓ $\sin^2 17^\circ$ ✓ subst of ratios ✓ answer/antw (4)</p>
<p>5.3</p>	$\begin{aligned} & \frac{\cos(180^\circ + 45^\circ)\sin(180^\circ - 45^\circ) + \sin(360^\circ - 30^\circ)}{\tan(180^\circ + 45^\circ)} \\ & = \frac{(-\cos 45^\circ) \cdot (\sin 45^\circ) - \sin 30^\circ}{\tan 45^\circ} \\ & = \frac{\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}}{1} \\ & = -1 \end{aligned}$	<p>✓ $-\cos 45^\circ$ ✓ $\sin 45^\circ$ ✓ $-\sin 30^\circ$ ✓ $\tan 45^\circ$ ✓ numerator/teller ✓ answer/antw Answer only: 1 mark out of 6 (6)</p>
<p>5.4</p>	$\begin{aligned} RHS &= \frac{-1}{\tan^2 x \cdot \cos^2 x} \\ &= \frac{-1}{\frac{\sin^2 x}{\cos^2 x} \times \cos^2 x} \\ &= \frac{-1}{\sin^2 x} \\ &= \frac{-1}{1 - \cos^2 x} \\ &= \frac{1}{\cos^2 x - 1} \\ &= \frac{1}{(\cos x + 1)(\cos x - 1)} \\ &= LHS \end{aligned}$ <p>OR/OF</p>	<p>✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ simplification/vereenv. ✓ identity ✓ factors (4)</p>

	$LHS = \frac{1}{(\cos x + 1)(\cos x - 1)}$ $= \frac{1}{\cos^2 x - 1}$ $= \frac{1}{-\sin^2 x}$ $= \frac{-1}{\sin^2 x}$ $= \frac{-1}{\frac{\sin^2 x}{\cos^2 x} \times \frac{\cos^2 x}{1}}$ $= \frac{-1}{\tan^2 x \cdot \cos^2 x}$ <p>OR/OF</p> $RHS = \frac{-\cos^2 x}{\sin^2 x \cdot \cos^2 x}$ $= \frac{-1}{\sin^2 x}$ $LHS = \frac{1}{\cos^2 x - 1}$ $= \frac{1}{-\sin^2 x}$ <p>$RHS = LHS$</p>	$\checkmark \cos^2 x - 1$ $\checkmark -\sin^2 x$ $\checkmark \frac{-1}{\sin^2 x}$ $\checkmark \frac{\sin^2 x}{\cos^2 x} \times \frac{\cos^2 x}{1}$ (4) $\checkmark \frac{1}{\tan^2 x} = \frac{\cos^2 x}{\sin^2 x}$ $\checkmark \frac{-1}{\sin^2 x}$ $\checkmark \cos^2 x - 1$ $\checkmark -\sin^2 x$ (4)
<p>5.5</p>	$2 \sin x \cos x - \cos x = 0$ $\cos x(2 \sin x - 1) = 0$ $\cos x = 0 \qquad \text{or} \qquad \sin x = \frac{1}{2}$ $x = 90^\circ + 360^\circ \cdot k, \quad k \in Z \qquad \qquad x = 30^\circ + 360^\circ \cdot k, \quad k \in Z$ or $x = 270^\circ + 360^\circ \cdot k, \quad k \in Z \qquad \qquad x = 150^\circ + 360^\circ \cdot k, \quad k \in Z$	\checkmark factors \checkmark both equations/ <i>beide verg.</i> \checkmark both general solutions for $\cos x = 0$ $\checkmark \checkmark$ general solutions for $\sin x = \frac{1}{2}$ $\checkmark k \in Z$ (6) [31]

QUESTION/VRAAG 6

6.1	$b = 30^\circ$	✓ answer/antw (1)
6.2	360°	✓ answer/antw (1)
6.3	$f(x) = g(x)$ $x = -150^\circ$ $x = 30^\circ$	✓ $x = -150^\circ$ ✓ $x = 30^\circ$ (2)
6.4	$\sin(90^\circ - x) > g(x)$ $\cos x > g(x)$ $f(x) > g(x)$ $x \in (-150^\circ; 30^\circ)$ or $-150^\circ < x < 30^\circ$	✓ $\cos x$ ✓ end points/eindpnte ✓ notation/notasie (3)
6.5	Range: $y \in [2; 4]$ or $2 \leq y \leq 4$	✓ end points/eindpnte ✓ notation/notasie (2)
		[9]

QUESTION/VRAAG 7

<p>7.1</p>	<p>$AB^2 = AD^2 + BD^2$ [pythagoras]</p> <p>$c^2 = (b - a \cos \hat{C})^2 + (a \sin \hat{C})^2$</p> <p>$= b^2 - 2ab \cos \hat{C} + a^2 \cos^2 \hat{C} + a^2 \sin^2 \hat{C}$</p> <p>$= b^2 - 2ab \cos \hat{C} + a^2 (\cos^2 \hat{C} + \sin^2 \hat{C})$</p> <p>$= a^2 + b^2 - 2ab \cos \hat{C}$</p> 	<ul style="list-style-type: none"> ✓ $B(a \cos C; a \sin C)$ ✓ $A(b; 0)$ ✓ distance formula/ <i>afstbdfor.</i> ✓ expansion/<i>ontwikk.</i> ✓ common factor ✓ square identity <p style="text-align: right;">(6)</p>
<p>7.1.2</p>	<p>$c^2 = a^2 + b^2 - 2ab \cos \hat{C}$</p> <p>$\therefore \cos \hat{C} = \frac{a^2 + b^2 - c^2}{2ab}$</p> <p>$RHS = \frac{(a + b)^2 - c^2}{2ab}$</p> <p>$= \frac{a^2 + 2ab + b^2 - c^2}{2ab}$</p> <p>$= \frac{a^2 + b^2 - c^2}{2ab} + \frac{2ab}{2ab}$</p> <p>$= \cos \hat{C} + 1$</p> <p>$= LHS$</p> <p>OR/OF</p> <p>$c^2 = a^2 + b^2 - 2ab \cos C$</p> <p>$2ab \cos C = a^2 + b^2 - c^2$</p> <p>$2ab + 2ab \cos C = a^2 + 2ab + b^2 - c^2$</p> <p>$2ab(1 + \cos C) = (a + b)^2 - c^2$</p> <p>$1 + \cos C = \frac{(a + b)^2 - c^2}{2ab}$</p>	<ul style="list-style-type: none"> ✓ making cos C subject of form ✓ simplifying ✓ expansion/<i>ontwikk.</i> ✓ subst cos C <p style="text-align: right;">(4)</p> <ul style="list-style-type: none"> ✓ making $2ab \cos C$ the subject of the formula ✓ adding 2ab on both sides of equation ✓ common factor ✓ factorise the trinomial/<i>drieterm</i> <p style="text-align: right;">(4)</p>

7.2.1	<p style="text-align: center;">In $\triangle ABD$</p> $\frac{BD}{\sin 109,6^\circ} = \frac{90,52}{\sin 31,23^\circ}$ $BD = \frac{90,52 \times \sin 109,16^\circ}{\sin 31,23^\circ}$ $= 164,92 \text{ m}$	<p>✓ sine rule/<i>sinusreël</i></p> <p>✓ subst</p> <p>✓ answer/<i>antw.</i></p> <p style="text-align: right;">(3)</p>
7.2.2	$CD^2 = 164,92^2 + 235^2 - 2 \times 164,92 \times 235 \times \cos 48,88^\circ$ $CD^2 = 31448,4874$ $CD = 177,34 \text{ m}$	<p>✓ cos rule/<i>cosinusreël</i></p> <p>✓ subst</p> <p>✓ answer/<i>antw.</i></p> <p style="text-align: right;">(3)</p>
		[16]

QUESTION/VRAAG 8

8.1	$\tan 35,5^\circ = \frac{0,5}{AB}$ $AB = \frac{0,5}{\tan 35,5^\circ}$ $= 0,7 \text{ m}$	✓ subst ✓ answer/antw (2)
8.2	$\text{Volume of cone} = \frac{1}{3} \times \pi (0,5)^2 \times 0,7$ $= 0,18 \text{ m}^3$ $\text{Volume of a cylinder} = \pi (0,5)^2 \times 1,1$ $= 0,86 \text{ m}^3$ $\frac{3}{4} \text{ of volume} = \frac{3}{4} \times (0,18 + 0,86)$ $= \frac{3}{4} \times (1,04) \text{ m}^3$ $= 0,78 \text{ m}^3$ $\text{Time taken by pump} = \frac{0,78 \text{ m}^3}{0,52 \text{ m}^3/\text{h}}$ $= 1,5 \text{ hours}$	✓ V of cone/keël ✓ V of cylinder ✓ 0,78 m ³ ✓ answer/antw (4)

[6]

QUESTION/VRAAG 9

9.1	Equal to twice the angle subtended by the arc at the circumference	✓ ✓ answer/antw (2)
9.2.1	$\hat{R} = 30$ [\angle at centre = $2 \times \angle$ at circumference]	✓ S ✓ R (2)
9.2.2	$\hat{NST} = 30^\circ$ [equal chords subtends equal angles]	✓ S ✓ R (2)
		[6]

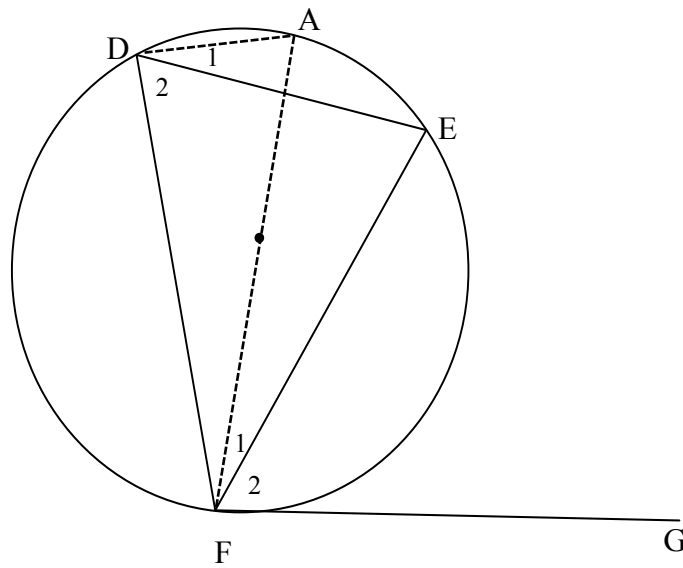
QUESTION/VRAAG 10

<p>10.1</p>	<p>$D\hat{E}G = x + 20^\circ$ [alt \angle's, ED FG]</p> <p>OR/OF</p> <p>$D\hat{E}G = 170^\circ - 2x$ [opp angles of cyclic quad]</p>	<p>✓ S ✓R (2)</p> <p>✓ S ✓R (2)</p>
<p>10.2</p>	<p>$x + 20^\circ + 2x + 10^\circ = 180^\circ$ [opp \angle of cyclic quad] $3x = 150^\circ$ $x = 50^\circ$ $D\hat{H}G = 2(50^\circ) + 10^\circ$ $= 110^\circ$</p> <p>OR/OF</p> <p>$x + 20^\circ = 170^\circ - 2x$ [alt 's, ED FG] $3x = 150^\circ$ $x = 50$ $D\hat{H}G = 2(50^\circ) + 10^\circ$ $= 110^\circ$</p>	<p>✓ S ✓R ✓ answer/antw ✓ 110° (4)</p> <p>✓ S ✓R ✓ answer/antw ✓ 110° (4)</p> <p>[6]</p>

QUESTION/VRAAG 11

11.1	$SP = SR$ [tangents from the same point] $\hat{P}RS = 42,83^\circ$ [\sphericalangle 's between equal sides] $\hat{O}RS = 90^\circ$ [tan \perp rad] $\hat{O}RN = 90^\circ - 42,83^\circ$ $= 47,17^\circ$ $\hat{N}OR = 90^\circ - 47,17^\circ$ [sum \sphericalangle 's of Δ] $= 42,83^\circ$	✓ S ✓ S ✓ R ✓ $\hat{O}RN$ ✓ answer/antw (5)
11.2	$Let\ OR = x$ $OS = x + 9$ $\hat{O}RS = 90^\circ$ [tan \perp rad] $x^2 + 15^2 = (x + 9)^2$ [Pythagoras] $x^2 + 225 = x^2 + 18x + 81$ $18x = 144$ $x = 8units$ $radius = 8units$	✓ S ✓ S ✓ Using Pythagoras ✓ answer/antw (4)
		[9]

QUESTION/VRAAG 12



<p>12.1</p>	<p>Construction: Draw diameter AOF. Join A to D. $\hat{F}_1 + \hat{F}_2 = 90^\circ$ [tan \perp diameter] $\hat{D}_1 = \hat{F}_1$ [\angle's in the same segment] $\hat{D}_1 + \hat{D}_2 = 90^\circ$ [\angle in a semi circle] $\therefore \hat{F}_2 = \hat{D}_2$ $E\hat{F}G = F\hat{D}E$</p>	<p>✓ Constr ✓ S ✓ R ✓ S ✓ R (5)</p>
<p>12.2.1</p>	<p>$B\hat{A}C = 90^\circ$ [\angle in a semi circle] $\hat{E}_2 = 90^\circ$ [line drawn from centre to midpont of chord] $\therefore B\hat{A}C = \hat{E}_2$ $BA \parallel OD$ [corresp. \angle's are equal] OR/OF $B\hat{A}C = 90^\circ$ [\angle in a semi circle] $\hat{E}_4 = 90^\circ$ [Line from centre to midpo int of chord] $B\hat{A}C = \hat{E}_4$ $\Rightarrow BA \parallel OD$ [Alt \angle's are equal]</p>	<p>✓ S / R ✓ S ✓ R ✓ R (4) ✓ S / R ✓ S ✓ R ✓ R</p>

