



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NASIONALE
SENIOR SERTIFIKAAT**

GRAAD 12

WISKUNDE V2

FEBRUARIE/MAART 2012

MEMORANDUM

PUNTE: 150

Hierdie memorandum bestaan uit 18 bladsye.

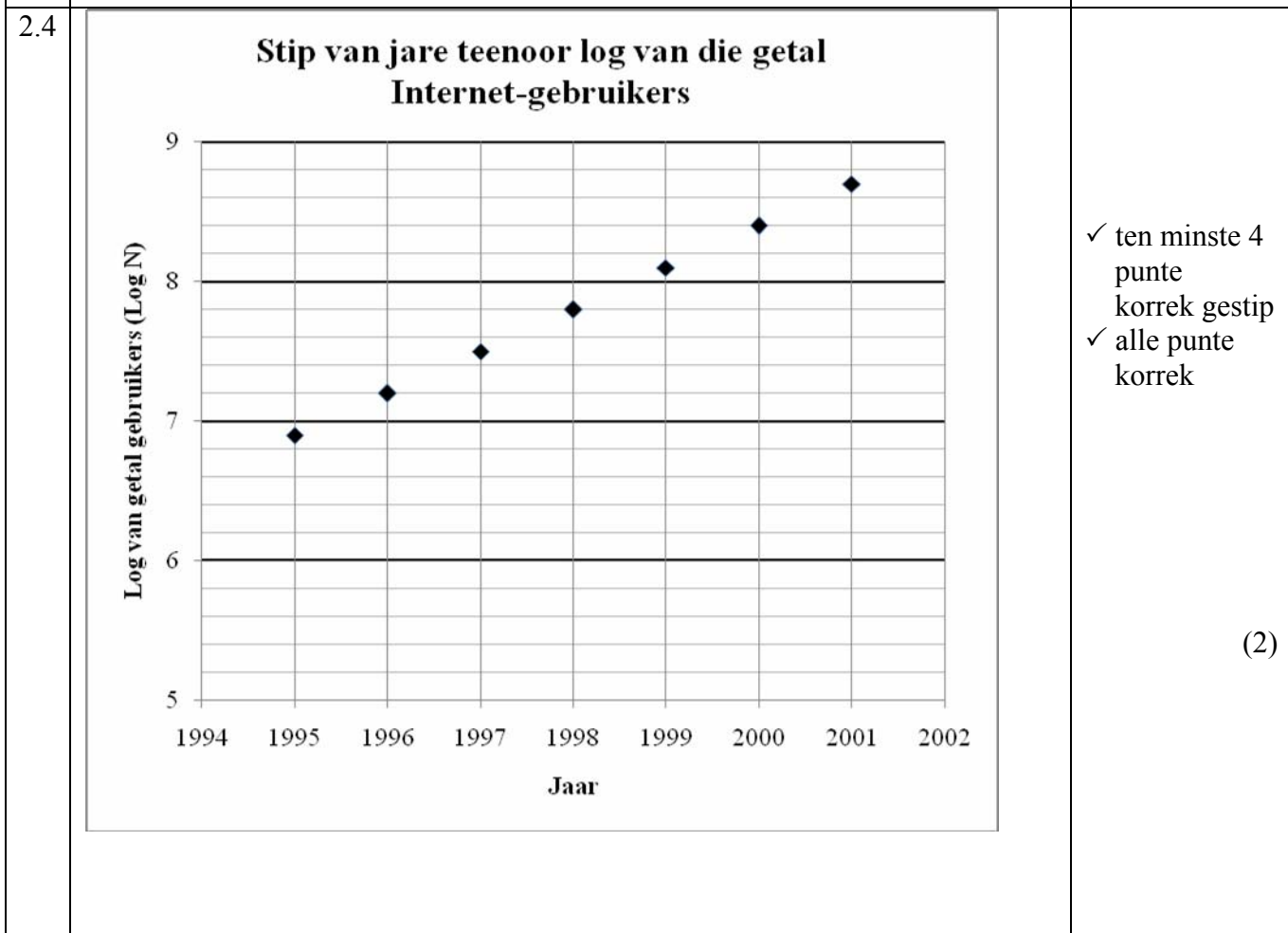
VRAAG 1

1.1	<p>Gemiddeld</p> $\frac{\sum_1^n x_1}{n} = \frac{102100}{9}$ $= R11\ 344,44$	<p>✓ 102100</p> <p>✓ antwoord (2)</p>
1.2	<p>Standaardafwyking</p> $\sqrt{\frac{\sum_1^n (x_1 - \bar{x})^2}{n}} = R4\ 460,97$	<p>✓✓ antwoord (2)</p>
1.3	<p>Waarde van een standaardafwyking bo gemiddeld = R11 344,44 + R4 460,97 = R15 805,41 Slegs een persoon het kommissie bo R 15 805,41 verdien. Dus het slegs 1 persoon 'n gradering van 'goed' ontvang.</p>	<p>✓ tel gemiddeld en std. af. by</p> <p>✓ afleiding (2)</p> <p>[6]</p>

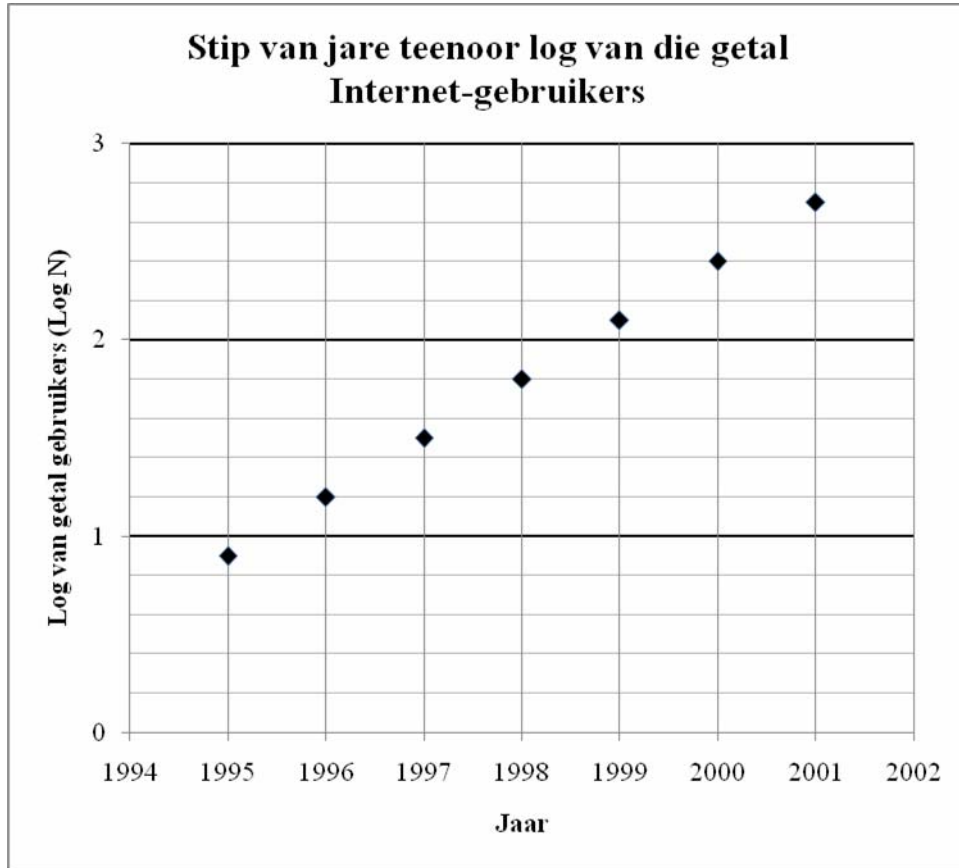
VRAAG 2

2.1	<p style="text-align: center;">Stip van jare teenoor log van die getal Internet-gebruikers</p> <table border="1" style="margin-top: 10px;"> <caption>Data points from the scatter plot</caption> <thead> <tr> <th>Jaar</th> <th>Getal (N) gebruikers (in Miljoene)</th> </tr> </thead> <tbody> <tr><td>1995</td><td>~10</td></tr> <tr><td>1996</td><td>~20</td></tr> <tr><td>1997</td><td>~35</td></tr> <tr><td>1998</td><td>~70</td></tr> <tr><td>1999</td><td>~140</td></tr> <tr><td>2000</td><td>~280</td></tr> <tr><td>2001</td><td>~560</td></tr> </tbody> </table>	Jaar	Getal (N) gebruikers (in Miljoene)	1995	~10	1996	~20	1997	~35	1998	~70	1999	~140	2000	~280	2001	~560	<p>✓ ten minste vier punte korrek</p> <p>✓ al die punte korrek (2)</p>
Jaar	Getal (N) gebruikers (in Miljoene)																	
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2001	~560																	
2.2	<p>Eksponensieel, want die toename in groei wys amper 'n verdubbeling elke jaar.</p>	<p>✓ eksponensieel</p> <p>✓ rede (2)</p>																

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OF (indien slegs logwaardes in table in berekening gebring is)



✓ ten minste 4 punte korrek gestip
 ✓ alle punte korrek

(2)

2.5 Lineêr, want die punte lê baie naby aan 'n reguitlyn

✓ lineêr
 ✓ rede (2)
[9]

VRAAG 3

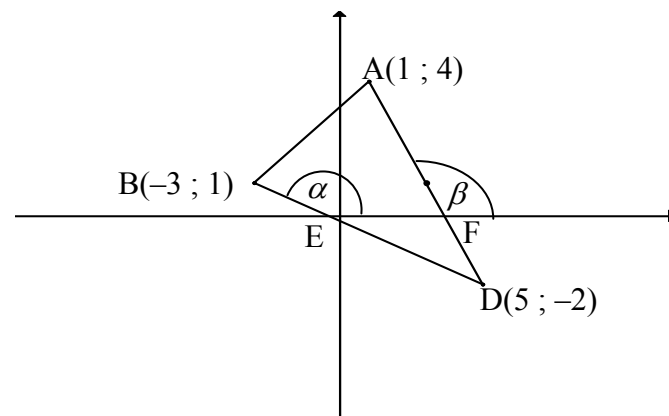
3.1	40	✓ 40 (1)												
3.2	<table border="1"> <thead> <tr> <th>Tyd, t, in minute</th> <th>Frekwensie</th> </tr> </thead> <tbody> <tr> <td>$0 \leq t < 5$</td> <td>3</td> </tr> <tr> <td>$5 \leq t < 10$</td> <td>5</td> </tr> <tr> <td>$10 \leq t < 15$</td> <td>10</td> </tr> <tr> <td>$15 \leq t < 20$</td> <td>15</td> </tr> <tr> <td>$20 \leq t < 25$</td> <td>7</td> </tr> </tbody> </table>	Tyd, t, in minute	Frekwensie	$0 \leq t < 5$	3	$5 \leq t < 10$	5	$10 \leq t < 15$	10	$15 \leq t < 20$	15	$20 \leq t < 25$	7	✓ vir intervalle in tabel ✓ vir eerste drie korrekte frekwensies ✓ vir laaste twee korrekte frekwensies (3)
Tyd, t, in minute	Frekwensie													
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$15 \leq t < 20$	15													
$20 \leq t < 25$	7													
3.3		✓ korrekte frekwensies ✓ middelpuntwaardes ✓ geen spasies tussen stawe (3) [7]												

VRAAG 4

$a = 7$	$b = 15$	$c = 17$	$d = 23$	$e = 34$	$f = 37$	$g = 42$	✓ elke korrekte antwoord (7)
OF							
$g = 42 ; a = 7 ; d = 23 ; f = 37 ; b = 15$ $\frac{42 + 7 + 23 + 37 + 15 + 3c}{7} = 25$ $3c = 51$ $c = 17$ $e = 34$							✓ g ✓ a ✓ d ✓ f ✓ b ✓ c ✓ e (7) [7]

VRAAG 5

5.1	$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-2 - 4}{5 - 1}$ $= -\frac{6}{4} = -\frac{3}{2}$	✓ vir substitusie ✓ vir antwoord (2)
5.2	$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5 - 1)^2 + (-2 - 4)^2}$ $= \sqrt{16 + 36}$ $= \sqrt{52}$	✓ vir substitusie ✓ $\sqrt{52}$ (2)
5.3	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ $M = \left(\frac{1 + 5}{2}, \frac{4 - 2}{2} \right)$ $M = (3; 1)$	✓ x-waarde ✓ y-waarde (2)
5.4	$m_{BC} = m_{AD}$ $= -\frac{3}{2}$ $y - y_1 = m(x - x_1)$ $y - 1 = -\frac{3}{2}(x + 3)$ $2y - 2 = -3x - 9$ $3x + 2y + 7 = 0$ <p style="text-align: center;">OF</p> $y = -\frac{3}{2}x + c$ $1 = -\frac{3}{2}(-3) + c$ $c = -\frac{7}{2}$ $y = -\frac{3}{2}x - \frac{7}{2}$ $3x + 2y + 7 = 0$	lyne is parallel ✓ waarde m_{BC} ✓ subst $(-3; 1)$ ✓ vergelyking (3) ✓ waarde m_{BC} ✓ subst $(-3; 1)$ ✓ vergelyking (3)

<p>5.5.1</p>	$m_{AD} = -\frac{3}{2}$ $\tan \beta = -\frac{3}{2}$ $\beta = 180^\circ - 56,31^\circ$ $\beta = 123,69$ 	<p>✓ $\tan \beta = m_{AD}$</p> <p>✓ $123,69^\circ$</p> <p>(2)</p>
<p>5.5.2</p>	$m_{BD} = \frac{-2-1}{5-(-3)} = \frac{-3}{8}$ $\tan \alpha = -\frac{3}{8}$ $\alpha = 180^\circ - 20,56^\circ$ $\alpha = 159,44^\circ$ $\widehat{FED} = 180^\circ - 159,44^\circ = 20,56^\circ$ $\widehat{EFD} = 123,69^\circ$ $\widehat{FDE} = 180^\circ - (20,56^\circ + 123,69^\circ) = 35,75^\circ$	<p>✓ $m_{BD} = \frac{-3}{8}$</p> <p>✓ $159,44^\circ$</p> <p>✓ $20,56^\circ$</p> <p>✓ $123,69^\circ$</p> <p>✓ $35,75^\circ$</p> <p>(5)</p>
<p>5.6</p>	<p>Koördinate van middelpunt M (3 ; 1) Radius van sirkel: $\frac{1}{2}$ of AD = $\frac{1}{2} (2\sqrt{13}) = \sqrt{13} = \frac{1}{2}\sqrt{52}$ Vergelyking van die sirkel is: $(x-3)^2 + (y-1)^2 = 13$</p> <p style="text-align: center;">OF</p> <p>$r^2 = (3-1)^2 + (1-4)^2 = 13$ Vergelyking van die sirkel is: $(x-3)^2 + (y-1)^2 = 13$</p>	<p>✓ waarde van radius</p> <p>✓ substitusie in vergelyking vir sirkelmiddelpunt-vorm (2)</p> <p>✓ waarde van r^2</p> <p>✓ substitusie in vergelyking in sirkelmiddelpunt vorm (2)</p>
<p>5.7</p>	<p>M(3 ; 1) B(-3 ; 1) $MB = \sqrt{(3+3)^2 + (1-1)^2}$ $MB = 6$ Punt B lê buite die sirkel want $MB >$ radius</p> <p style="text-align: center;">OF</p> <p>M(3 ; 1) B(-3 ; 1) $MB = 3 + 3 = 6$ Radius van die sirkel = $\sqrt{13} < 6$ Punt B lê buite die sirkel want $MB >$ radius</p>	<p>✓ substitusie</p> <p>✓ buite (2)</p> <p>✓ substitusie</p> <p>✓ buite (2)</p> <p>[20]</p>

VRAAG 6

6.1	Koördinate van middelpunt M (-2 ; 1) $(1+2)^2 + (-2-1)^2 = 18 = r^2$ Radius = $\sqrt{18}$ of $3\sqrt{2}$	✓✓ koördinate van middelpunt ✓ berekening ✓ waarde (4)
6.2	$m_{MS} = \frac{-3}{3} = -1$ $m_{MS} \times m_{RS} = -1$ OF raaklyn \perp radius $m_{RS} = 1$ $y - y_1 = m(x - x_1)$ $y + 2 = 1(x - 1)$ $y = x - 3$ <p style="text-align: center;">OF</p> $m_{MS} = \frac{-3}{3} = -1$ $m_{MS} \times m_{RS} = -1$ $m_{RS} = 1$ $y = x + c$ $-2 = 1 + c$ $c = -3$ $y = x - 3$	✓ gradiënt MS ✓ gradiënt RS ✓ subst (1 ; -2) ✓ vergelyking (4) ✓ gradiënt MS ✓ gradiënt RS ✓ subst (1 ; -2) ✓ vergelyking (4)
6.3	$\frac{MS}{MP} = \frac{1}{3}$ $\therefore MP = 3MS$ $MP^2 = 9MS^2$ $(a+2)^2 + (b-1)^2 = 9(3^2 + 3^2) = 162$ (1) $MS \perp SR$ en $PS \perp SR$ $\therefore m_{PS} = m_{MS}$ $\frac{b+2}{a-1} = \frac{3}{-3} = -1$ $b+2 = -a+1$ $b = -a-1$ (2) Vervang (2) in (1)	✓ MP = 3MS ✓ vergelyking ✓ dieselfde gradiënte ✓ gradiënt ✓ $b = -a - 1$

	$(a+2)^2 + (-a-1-1)^2 = 162$ $(a+2)^2 + (a+2)^2 = 162$ $2(a+2)^2 = 162$ $(a+2)^2 = 81$ $a+2 = 9 \text{ of } -9$ $a = 7 \text{ of } -11$ $b = -a - 1 = -8$ $P(7; -8)$ <p>OF</p> $\frac{MS}{MP} = \frac{1}{3}$ $\therefore MP = 3MS$ $MP^2 = 9MS^2$ $(a+2)^2 + (b-1)^2 = 9(3^2 + 3^2) = 162 \quad (1)$ $MS \perp SR \text{ en } PS \perp SR \quad \therefore m_{PS} = m_{MS}$ $\frac{b+2}{a-1} = \frac{3}{-3} = -1$ $b+2 = -a+1$ $b = -a-1 \quad (2)$ <p>Vervangt (2) in (1)</p> $a^2 + 4a + 4 + a^2 + 4a + 4 = 162$ $2a^2 + 8a - 154 = 0$ $a^2 + 4a - 77 = 0$ $(a+11)(a-7) = 0$ $a = 7 \text{ of } -11$ <p>Maar $a > 0$</p> $\therefore a = 7$ $b = -a - 1 = -8$ $P(7; -8)$ <p>OF</p>	<p>✓ substitusie</p> <p>✓ $a = 7$ ✓ $b = -8$</p> <p>(8)</p> <p>✓ $MP = 3MS$</p> <p>✓ vergelyking</p> <p>✓ dieselfde gradiënte</p> <p>✓ gradiënt</p> <p>✓ $b = -a - 1$</p> <p>✓ substitusie</p> <p>✓ $a = 7$ ✓ $b = -8$</p> <p>(8)</p>
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P(a ; b)
MSP is 'n reguitlyn (MS ⊥ SR)

$$m_{PM} = -1$$

$$\frac{b-1}{a+2} = -1$$

$$b-1 = -a-2$$

$$b = -a-1 \dots\dots(1)$$

$$PS = 2MS = 2\sqrt{9+9} = 2\sqrt{18}$$

$$PS^2 = 4(18) = 72$$

$$(a-1)^2 + (b+2)^2 = 72 \dots\dots(2)$$

$$(a-1)^2 + (-a-1+2)^2 = 72$$

$$2a^2 - 4a - 70 = 0$$

$$a^2 - 2a - 35 = 0$$

$$(a-7)(a+5) = 0$$

$$a = 7 \text{ of } a = -5$$

$$b = -7-1 = -8$$

$$P(7 ; -8)$$

OF

$$2(a-1)^2 = 72$$

$$(a-1)^2 = 36$$

$$a-1 = 6 \text{ of } -6$$

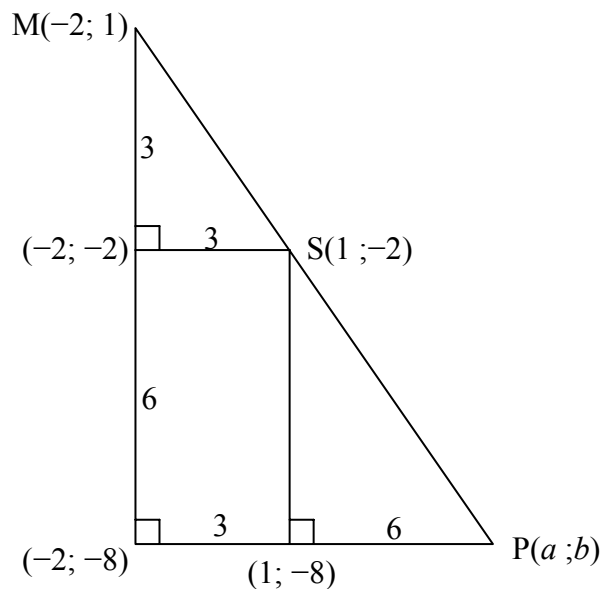
$$a = 7 \text{ of } -5$$

$$a = 7$$

$$b = -8$$

$$P(7 ; -8)$$

OF



✓ MSP is 'n reguitlyn

✓ $m_{PM} = -1$

✓ $\frac{b-1}{a+2}$

✓ vergelyking 1

✓ vergelyking 2

✓ substitusie van vergelyking 1 in vergelyking 2

✓✓ koördinate

(8)

✓✓ diagram

✓✓ (-2; -8)

✓ (-2; -2)

✓ (1; -8)

✓✓ P(7 ; -8)

(8)

	<p>P(a ; b)</p> $\frac{x_S - x_M}{x_P - x_M} = \frac{y_S - y_M}{y_P - y_M} = \frac{1}{3}$ $\frac{-3}{b-1} = \frac{3}{a+2} = \frac{1}{3}$ $-9 = b-1$ $b = -8$ $9 = a+2$ $a = 7$ <p>P(7 ; -8)</p>	<p>✓✓ verdeling van 'n lynstuk in 'n gegewe verhouding</p> <p>✓✓ substitusie</p> <p>✓ vergelyking</p> <p>✓ vergelyking</p> <p>✓ koördinate</p> <p style="text-align: right;">(8) [16]</p>
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VRAAG 7

7.1		<p>Vir korrekte koördinate en benoeming van elke beeld:</p> <p>✓ K'</p> <p>✓ L'</p> <p>✓ M'</p> <p>✓ N'</p> <p style="text-align: right;">(4)</p>
7.2.1	Transformasie is nie rigied (star) nie, want die area het verander as gevolg van die vergroting.	<p>✓ nie rigied nie</p> <p>✓ grootte nie behou nie</p> <p style="text-align: right;">(2)</p>
7.2.2	$N''(-2 ; -2)$	<p>✓✓ koördinate van N''</p> <p style="text-align: right;">(2)</p>
7.3	$(x ; y) \rightarrow (-y ; x) \rightarrow (-2y ; 2x)$	<p>✓ $-y$</p> <p>✓ x</p> <p>✓ $-2y$</p> <p>✓ $2x$</p> <p style="text-align: right;">(4)</p>
7.4	Area van KLMN : area van $K''L''M''N'' = 1 : 4$	<p>✓✓ antwoord</p> <p style="text-align: right;">(2)</p>
7.5	<p>Indien die verste punt vanaf die oorsprong in die sirkel gedruk/gestuur word, dan sal die hele vierhoek in die sirkel gedruk wees. K is die verste weg.</p> $KO = \sqrt{3^2 + 3^2} = \sqrt{18}$	<p>✓ K – verste</p> <p>✓ $KO = \sqrt{18}$</p> <p>✓ antwoord</p> <p style="text-align: right;">(3)</p>

$p.KO = 1, p = \frac{1}{\sqrt{18}}$	[17]
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VRAAG 8

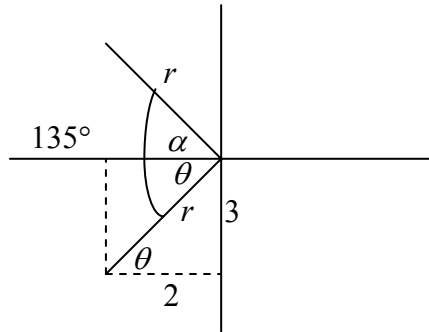
8.	$x_Q = x \cos \theta + y \sin \theta$ $x_Q = -2 \cos 135^\circ + (-3) \sin 135^\circ$ $x_Q = \frac{2}{\sqrt{2}} - \frac{3}{\sqrt{2}} = \frac{-1}{\sqrt{2}} \text{ of } \frac{-\sqrt{2}}{2} \text{ or } -0,71$ $y_Q = y \cos \theta - x \sin \theta$ $y_Q = -3 \cos 135^\circ - (-2) \sin 135^\circ$ $y_Q = \frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} = 3,54$ $Q\left(\frac{-1}{\sqrt{2}}; \frac{5}{\sqrt{2}}\right)$ <p style="text-align: center;">OF</p> $x_Q = x \cos \theta - y \sin \theta$ $x_Q = -2 \cos(-135^\circ) - (-3) \sin(-135^\circ)$ $x_Q = \frac{2}{\sqrt{2}} - \frac{3}{\sqrt{2}} = \frac{-1}{\sqrt{2}} \text{ of } \frac{-\sqrt{2}}{2} \text{ or } -0,71$ $y_Q = y \cos \theta + x \sin \theta$ $y_Q = -3 \cos(-135^\circ) + (-2) \sin(-135^\circ)$ $y_Q = \frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} = 3,54$ $Q\left(\frac{-1}{\sqrt{2}}; \frac{5}{\sqrt{2}}\right)$ <p style="text-align: center;">OF</p> $x' = x \cos \theta - y \sin \theta$ $-2 = x \cos 135^\circ - y \sin 135^\circ$ $-2 = \frac{-x}{\sqrt{2}} - \frac{y}{\sqrt{2}}$ $-2\sqrt{2} = -x - y \tag{1}$ $y' = y \cos \theta + x \sin \theta$ $-3 = y \cos 135^\circ + x \sin 135^\circ$ $-3 = \frac{-y}{\sqrt{2}} + \frac{x}{\sqrt{2}}$ $-3\sqrt{2} = x - y \tag{2}$ <p>Los (1) en (2) gelyktydig op:</p>	<ul style="list-style-type: none"> ✓ subst -2 en -3 in korrekte formule vir x_Q ✓ gebruik 135° ✓ x-koördinate (in enige formaat) ✓ subst -2 en -3 in korrekte formule vir y_Q ✓ vir y-koördinate (in enige formaat) (5) ✓ subst -2 en -3 in korrekte formule vir x_Q ✓ gebruik -135° ✓ x-koördinate (in enige formaat) ✓ subst -2 en -3 in korrekte formule vir y_Q ✓ vir y-koördinate (in enige formaat) (5) ✓ subst -2 en 135° in korrekte formule vir x' ✓ vereenvoudiging ✓ subst -2 en 135° in korrekte formule vir y' ✓ y-koördinaat ✓ x-koördinaat (5)
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$$-5\sqrt{2} = -2y$$

$$y = \frac{5}{\sqrt{2}} \quad \text{en} \quad x = \frac{-1}{\sqrt{2}}$$

OF

Gebruik eerste beginsels: $Q = (-r \cos \alpha; r \sin \alpha)$



$$Q' = (-2; -3)$$

$$\tan \theta = \frac{3}{2}$$

$$r = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$\theta = 56,31^\circ$$

$$\therefore \alpha = 135^\circ - 56,31^\circ = 78,69^\circ$$

$$Q = (-r \cos \alpha; r \sin \alpha)$$

$$= (-0,71; 3,54)$$

$$\checkmark \tan \theta = \frac{3}{2}$$

$$\checkmark r = \sqrt{13}$$

$$\checkmark \theta = 56,31^\circ$$

✓

$$Q = (-r \cos \alpha; r \sin \alpha)$$

✓ antwoord

(5)

[5]

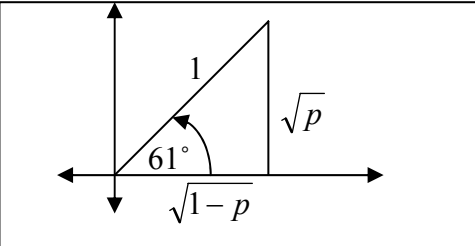
VRAAG 9

9.1.1	$r = 13$ $\cos \alpha = \frac{12}{13}$	$\checkmark 13$ $\checkmark \frac{12}{13}$ (2)
9.1.2	$\widehat{QOR} = 180^\circ - (90^\circ + \alpha)$ $= 90^\circ - \alpha$	$\checkmark 180^\circ - (90^\circ + \alpha)$ $\checkmark 90^\circ - \alpha$ (2)
9.1.3	$\cos \widehat{QOR} = \frac{QR}{OQ}$ $\cos(90^\circ - \alpha) = \frac{7,5}{OQ}$ $OQ = \frac{7,5}{\cos(90^\circ - \alpha)}$ $OQ = \frac{7,5}{\sin \alpha}$ $OQ = \frac{7,5}{\frac{5}{13}}$ $OQ = 19,5$ <p style="text-align: center;">OF</p> $\sin(\widehat{RQO}) = \frac{7,5}{OQ}$ $\therefore OQ = \frac{7,5}{\sin \alpha}$ $OQ = \frac{7,5}{\frac{5}{13}}$ $OQ = 19,5$	\checkmark $\cos(90^\circ - \alpha) = \frac{7,5}{OQ}$ $\checkmark \frac{7,5}{\sin \alpha}$ $\checkmark \frac{5}{13}$ $\checkmark 19,5$ (4)
9.2	$LK = \frac{\cos x \cdot \cos x(-\tan x)}{-\cos x}$ $= \cos x \cdot \frac{\sin x}{\cos x}$ $= \sin x$ $= RK$	$\checkmark \cos x$ $\checkmark -\tan x$ $\checkmark \frac{\sin x}{\cos x}$ $\checkmark \text{antwoord}$ (4) [12]

VRAAG 10

10.1	Periode = 120°	✓ 120° (1)
10.2	$\sin 3x = -1$ $x = -30^\circ$ of $x = 90^\circ$	✓ -30° ✓ 90° (2)
10.3	Maksimum waarde van $f(x)$ is 1 \therefore Maksimum waarde van $h(x)$ is 0	✓ maks van $f(x)$ ✓ antwoord (2)
10.4		✓ -90° ; 90° ✓ $(0^\circ; 3)$ ✓ $(180^\circ; -3)$ (3)
10.5	$\frac{\sin 3x}{3} - \cos x = 0$ $\sin 3x - 3 \cos x = 0$ $\therefore \sin 3x = 3 \cos x$ <p>Daar is 2 oplossings waar die grafieke van f en g gelyk is</p>	✓ $\sin 3x = 3 \cos x$ ✓ antwoord (2)
10.6	$f(x) \cdot g(x) < 0$ $x \in (-60^\circ; 0^\circ)$ of $(60^\circ; 90^\circ)$ of $(120^\circ; 180^\circ)$ <p style="text-align: center;">OF</p> $-60^\circ < x < 0^\circ$ of $60^\circ < x < 90^\circ$ of $120^\circ < x < 180^\circ$	✓✓✓ vir elke interval ✓ korrekte hakies of korrekte simbole (4) [14]

VRAAG 11

11.1.1	$\sin 61^\circ = \sqrt{p}$ $\sin 241^\circ = \sin (180^\circ + 61^\circ)$ $= -\sin 61^\circ$ $= -\sqrt{p}$		<p>✓ $-\sin 61^\circ$ ✓ antwoord</p> <p style="text-align: right;">(2)</p>
11.1.2	$\cos 61^\circ = \sqrt{1 - \sin^2 61^\circ}$ $= \sqrt{1 - p}$		<p>✓ identiteit ✓ antwoord</p> <p style="text-align: right;">(2)</p>
11.1.3	$\cos 122^\circ = \cos 2(61^\circ)$ $= 2\cos^2 61^\circ - 1$ $= 2(\sqrt{1-p})^2 - 1$ $= 2(1-p) - 1$ $= 2 - 2p - 1$ $= 1 - 2p$		<p>✓ dubbelhoek ✓ uitbreiding</p> <p>✓ antwoord</p> <p style="text-align: right;">(3)</p>
11.1.4	$\cos 73^\circ \cos 15^\circ + \sin 73^\circ \sin 15^\circ$ $= \cos(73^\circ - 15^\circ)$ $= \cos 58^\circ = (\cos 180^\circ - 122^\circ)$ $= -(\cos 122^\circ)$ $= -(1 - 2p)$ $= 2p - 1$		<p>✓ $\cos(73^\circ - 15^\circ)$ ✓ $-(\cos 122^\circ)$ ✓ antwoord</p> <p style="text-align: right;">(3)</p>
11.2.1	$LK = \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \frac{\cos^2 x + 2\cos x \sin x + \sin^2 x - (\cos^2 x - 2\sin x \cos x + \sin^2 x)}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \frac{4\cos x \sin x}{\cos^2 x - \sin^2 x}$ $= \frac{2\sin 2x}{\cos 2x}$ $= 2\tan x$ $= RK$		<p>✓</p> $\frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ <p>✓ teller ✓ $4\cos x \sin x$ ✓ $\cos^2 x - \sin^2 x$ ✓ $2\sin 2x$ ✓ $\cos 2x$</p> <p style="text-align: right;">(6)</p>
11.2.2	$x = 45^\circ; 135^\circ$		<p>✓ 45° ✓ 135°</p> <p style="text-align: right;">(2)</p>
11.3.1	$\sin x = \cos 2x - 1$ $\sin x = 1 - 2\sin^2 x - 1$ $\sin x = -2\sin^2 x$ $2\sin^2 x + \sin x = 0$		<p>✓ $1 - 2\sin^2 x$</p> <p style="text-align: right;">(1)</p>

<p>11.3.2</p>	$\sin x = \cos 2x - 1$ $2 \sin^2 x + \sin x = 0$ $\sin x (2 \sin x + 1) = 0$ $\sin x = 0 \text{ of } \sin x = -\frac{1}{2}$ $\therefore x = 0^\circ + 180^\circ k; k \in \mathbb{Z} \text{ of } x = \{210^\circ \text{ or } 330^\circ\} + 360^\circ k; k \in \mathbb{Z}$ <p style="text-align: center;">OF</p> $x = n \cdot 180^\circ$ $x = n \cdot 360^\circ - 30^\circ$ $x = (2n + 1) \cdot 180^\circ + 30^\circ, n \in \mathbb{Z}$	$\checkmark \sin x (2 \sin x + 1) = 0$ $\checkmark \sin x = 0 \text{ of } \sin x = -\frac{1}{2}$ $\checkmark 0^\circ + 180^\circ k$ $\checkmark 210^\circ$ $\checkmark 330^\circ$ $\checkmark + 360^\circ k; k \in \mathbb{Z}$ <p style="text-align: right;">(6)</p>
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<p>11.4</p>	$\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \times \tan 4^\circ \times \dots \times \tan 87^\circ \times \tan 88^\circ \times \tan 89^\circ$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ}\right) \left(\frac{\sin 2^\circ}{\cos 2^\circ}\right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ}\right) \dots \left(\frac{\sin 88^\circ}{\cos 88^\circ}\right) \left(\frac{\sin 89^\circ}{\cos 89^\circ}\right)$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ}\right) \left(\frac{\sin 2^\circ}{\cos 2^\circ}\right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ}\right) \dots \left(\frac{\sin(90^\circ - 2^\circ)}{\cos(90^\circ - 2^\circ)}\right) \left(\frac{\sin(90^\circ - 1^\circ)}{\cos(90^\circ - 1^\circ)}\right)$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ}\right) \left(\frac{\sin 2^\circ}{\cos 2^\circ}\right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ}\right) \dots \left(\frac{\cos 2^\circ}{\sin 2^\circ}\right) \left(\frac{\cos 1^\circ}{\sin 1^\circ}\right)$ $= \tan 45^\circ$ $= 1$ <p style="text-align: center;">OF</p> $\tan 89^\circ = \cot 1^\circ \quad \tan 88^\circ = \cot 2^\circ \dots$ $\therefore \text{produk is } (\tan 1^\circ \cdot \cot 1^\circ)(\tan 2^\circ \cdot \cot 2^\circ) \dots (\tan 44^\circ \cdot \cot 44^\circ) \cdot \tan 45^\circ$ $= 1 \times 1 \times 1 \times \dots \times 1 = 1$	$\checkmark \text{ identiteit}$ $\checkmark \text{ ko-verhoudings}$ $\checkmark \text{ vereenvoudiging}$ $\checkmark \text{ vir antwoord}$ <p style="text-align: right;">(4)</p> $\checkmark \text{ identiteit}$ $\checkmark \text{ ko-verhoudings}$ $\checkmark \text{ vereenvoudiging}$ $\checkmark \text{ vir antwoord}$ <p style="text-align: right;">(4) [29]</p>
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VRAAG 12

12	<p>By $\triangle CBG$ en $\triangle CDH$:</p> <p>$CG^2 = x^2 + y^2$ Pythagoras</p> <p>$CH^2 = x^2 + y^2$ Pythagoras</p> <p>By $\triangle FAE$</p> <p>$AE^2 = x^2 + x^2$</p> <p style="padding-left: 20px;">$= 2x^2$</p> <p style="padding-left: 20px;">$= GH^2$</p> <p>By $\triangle CGH$</p> <p>$GH^2 = CG^2 + CH^2 - 2 CG \cdot CH \cdot \cos GCH$</p> <p>$\cos \hat{GCH} = \frac{CG^2 + CH^2 - GH^2}{2CG \cdot CH}$</p> <p>$\cos \hat{GCH} = \frac{x^2 + y^2 + x^2 + y^2 - 2x^2}{2\sqrt{x^2 + y^2} \cdot \sqrt{x^2 + y^2}}$</p> <p>$\cos \hat{GCH} = \frac{2y^2}{2(x^2 + y^2)}$</p> <p>$\cos \hat{GCH} = \frac{y^2}{x^2 + y^2}$</p>	<p>✓ CG^2</p> <p>✓ CH^2</p> <p>✓ AE^2</p> <p>✓ $AE^2 = GH^2$</p> <p>✓ gebruik van cos-reël</p> <p>✓ manipulatie van formule</p> <p>✓ substitusie</p> <p>✓ $\cos \hat{GCH} = \frac{2y^2}{2(x^2 + y^2)}$</p> <p style="text-align: right;">(8)</p> <p style="text-align: right;">[8]</p>
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