



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL/NASIONALE
SENIOR
CERTIFICATE/SERTIFIKAAT**

GRADE/GRAAD 12

**MATHEMATICS P2/WISKUNDE V2
FEBRUARY/MARCH/FEBRUARIE/MAART 2017
MEMORANDUM**

MARKS / PUNTE: 150

**This memorandum consists of 21 pages.
*Hierdie memorandum bestaan uit 21 bladsye.***

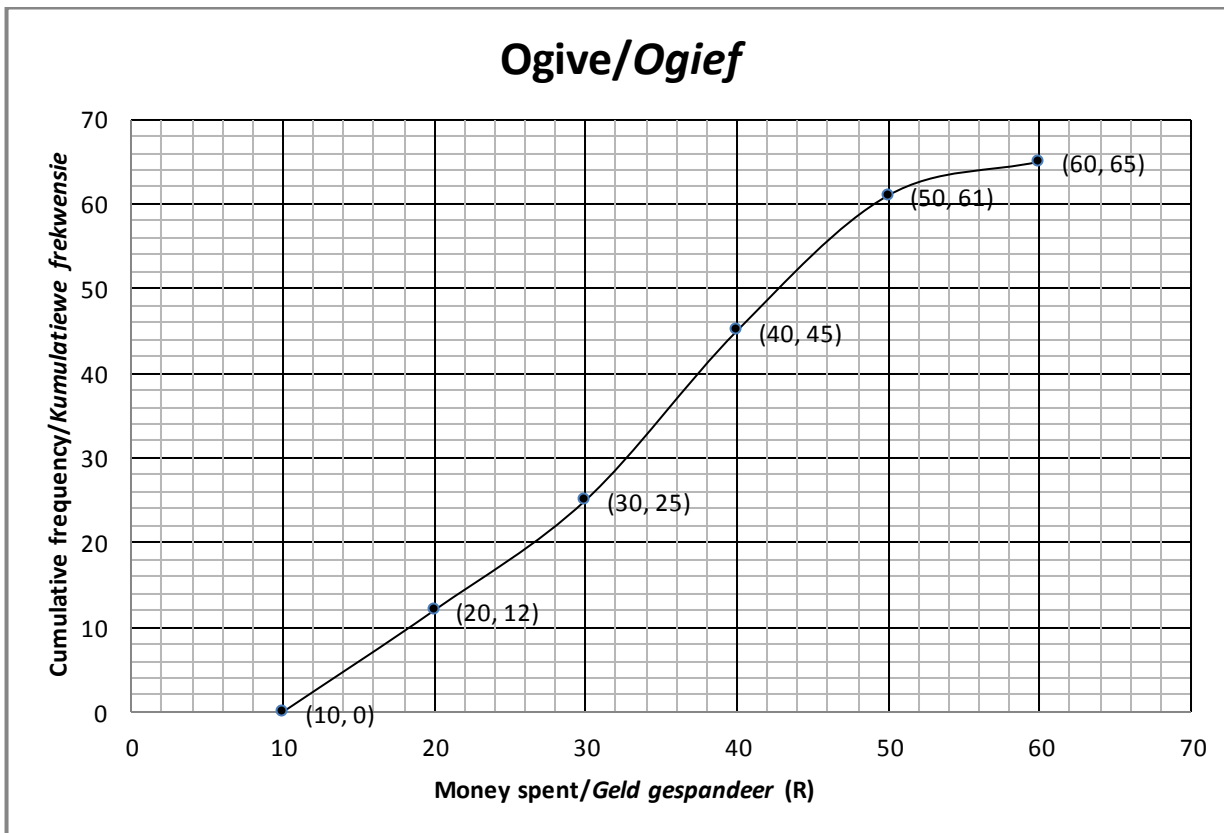
NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

QUESTION/VRAAG 1



Amount of money/ Bedrag geld (in R)	$10 \leq x < 20$	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$
Frequency Frekwensie	a	13	20	b	4

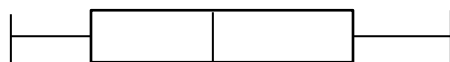
1.1	65 learners/ <i>leerders</i>	✓ answer (1)
1.2	Modal class/ <i>Modale klas</i> : $30 \leq x < 40$	✓ answer (1)
1.3	$a = 12$ $b = 61 - 45$ $= 16$	✓ answer ✓ answer (2)
1.4	No. of learners/ <i>Aantal leerders</i> = $65 - 54$ OR/OF $65 - 55$ $= 11$ $= 10$	✓ 54 or 55 ✓ 11 or 10 (2) [6]

Answer only: full marks

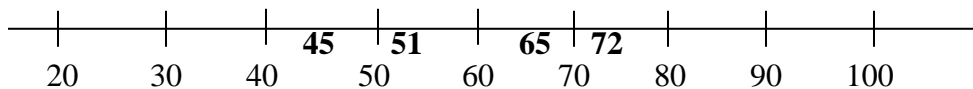
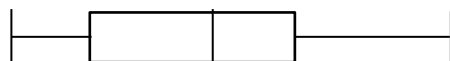
QUESTION/VRAAG 2

2.1

Class/Klas A



Class/Klas B



2.1.1	$\text{IQR of Class B/IKV van Klas B} = Q_3 - Q_1$ $= 72 - 51$ $= 21 \text{ marks/punte}$	✓ 72 and 51 ✓ 21 only (2)
2.1.2	Although the boxes contain the same number of data points, the marks for Class A are more widely spread./Alhoewel die monde dieselfde aantal datapunte bevat, is die punte van Klas A meer verspreid. OR/OF Although the boxes contain the same number of data points, the marks for Class B are more clustered./Alhoewel die monde dieselfde aantal datapunte bevat, is die punte van Klas B nader aan mekaar.	✓ ✓ Class A is more widely spread (2) ✓ ✓ Class B is more clustered (2)
2.1.3	Medians are the same/Mediane is dieselfde Ranges are the same OR Maximum and minimum values are the same/Variasiewydtes is dieselfde OF die maksimum en minimum waarde is dieselfde 75% of both classes obtained 51 and above/75% van albei klasse behaal 51 en meer.	✓ ✓ any TWO of the 3 reasons mentioned (2)

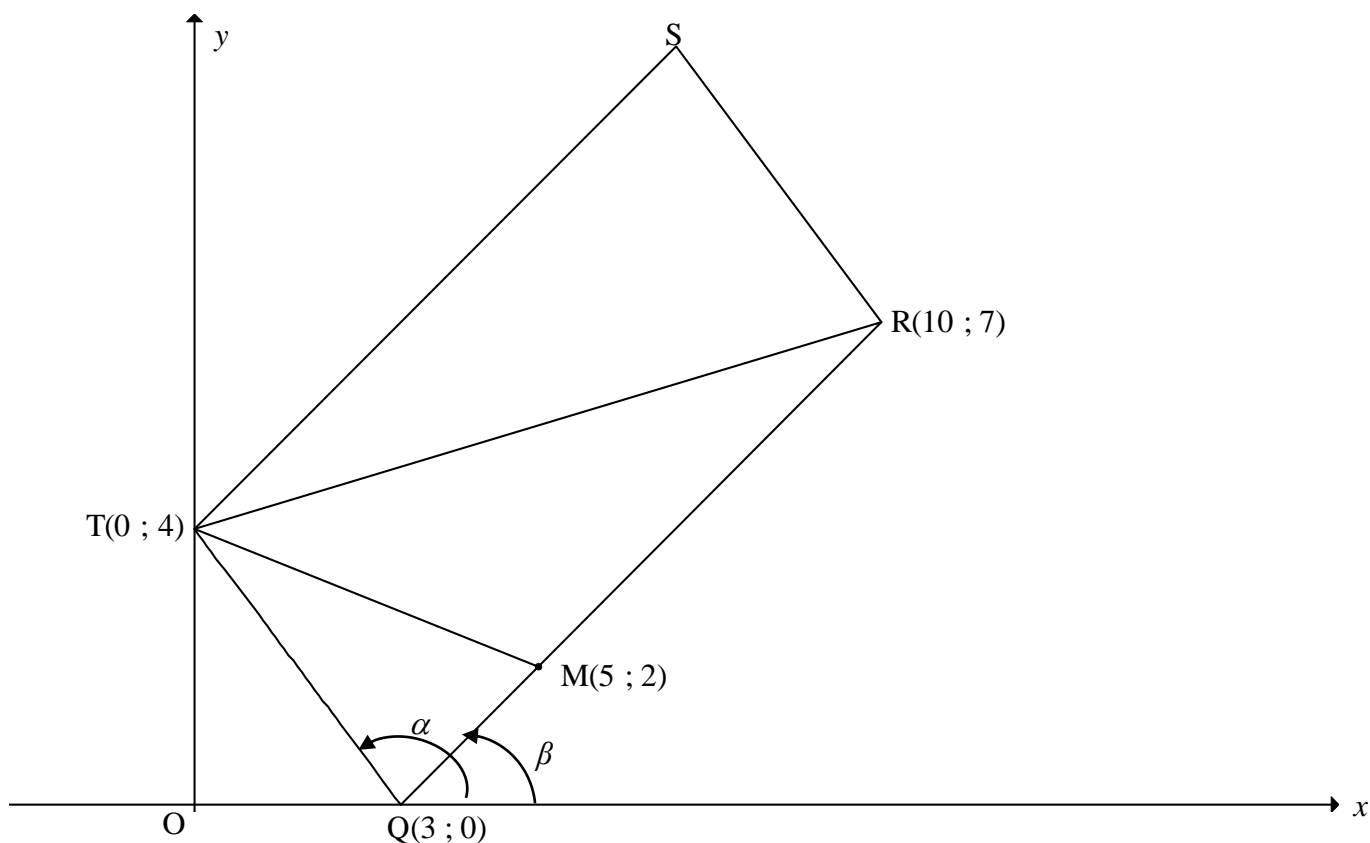
2.2

COUPLE/PAAR	1	2	3	4	5	6	7	8
JUDGE 1/ BEOORDELAAR 1	18	4	6	8	5	12	10	14
JUDGE 2/ BEOORDELAAR 2	15	6	3	5	5	14	8	15

2.2.1	$a = -0,03$ $b = 0,93$ $\hat{y} = -0,03 + 0,93x$	✓ value a ✓ value b ✓ equation (3)
2.2.2	$\hat{y} = -0,03 + 0,93(15)$ $= 13,92$ OR/OF 13,85 ≈ 14	✓ substitution ✓ answer (2)
2.2.3	Yes OR they are consistent, because $r = 0,9$. ($r = 0,89567\dots$)/Ja OF hulle is konsekwent, want $r = 0,9$. ($r = 0,89567\dots$)	✓ statement ✓ $r = 0,9$ (2)

[13]

QUESTION/VRAAG 3



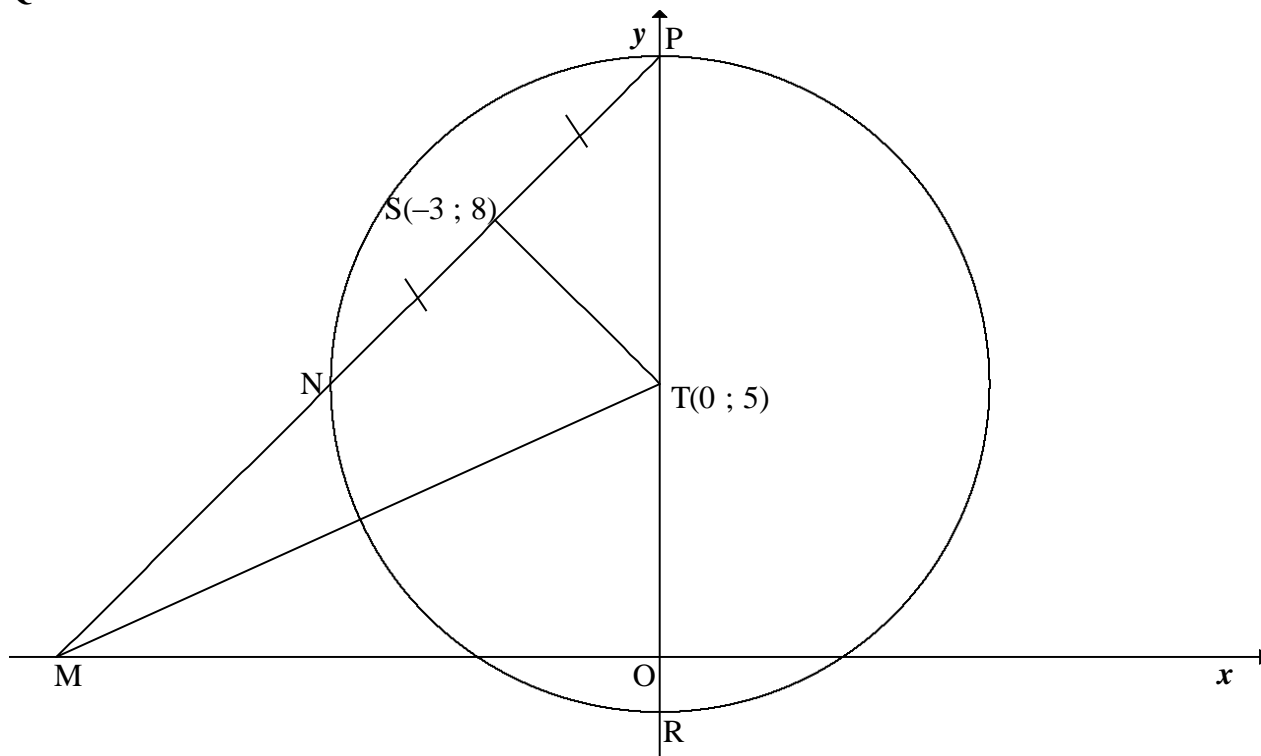
<p>3.1</p>	$m_{TQ} = \frac{4-0}{0-3}$ $= -\frac{4}{3}$	<p>✓ answer (1)</p>
<p>3.2</p>	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $RQ = \sqrt{(10-3)^2 + (7-0)^2}$ $RQ = \sqrt{98} = 7\sqrt{2}$	<p>✓ substitution/substitusie ✓ answer in surd form (2)</p>
<p>3.3</p>	$m_{FQ} = m_{TQ}$ $\frac{-8}{k-3} = -\frac{4}{3} \quad \text{OR/OF}$ $4k - 12 = 24$ $k = 9$ <p>OR/OF</p> <p>Equation of TQ: $y = -\frac{4}{3}x + 4$</p> $-8 = -\frac{4}{3}k + 4$ $k = 9$	<p>✓ equating gradients/stel gradient gelyk ✓ $m_{FQ} = \frac{-8}{k-3}$ ✓ simplification/vereenvoudig ✓ answer (4)</p> <p>✓ gradient ✓ equation of TQ/vgl van TQ ✓ substitution of (k ; - 8) /substitusie van (k ; - 8) ✓ answer (4)</p>

<p>3.4</p>	<p>Using transformation/<i>Gebruik transformasie</i>: $\therefore S(7 ; 11)$</p> <p>OR/OF Midpoint of TR = midpoint of SQ [diag m/hkle m] Midpoint of TR = $(5 ; \frac{11}{2})$ $\frac{x_S + 3}{2} = 5$ and $\frac{y_S + 0}{2} = \frac{11}{2}$ $\therefore x_S = 7$ and $y_S = 11$ $\therefore S(7 ; 11)$</p> <p>OR/OF Equation of TS: $y = \left(\frac{7-2}{10-5}\right)x + 4 = x + 4$ Equation of RS: $y - 7 = -\frac{4}{3}(x - 10)$ $y = -\frac{4}{3}x + \frac{61}{3}$ $x + 4 = -\frac{4}{3}x + \frac{61}{3}$ $7x = 49$ $x = 7$ $\therefore y = 11$ $\therefore S(7 ; 11)$</p>	<p>✓ ✓ <i>x</i>-value/waarde ✓ ✓ <i>y</i>-value/waarde (4)</p> <p>✓ <i>x</i>-value/waarde of/van T ✓ <i>y</i>-value/waarde of/van T</p> <p>✓ <i>x</i>-value/waarde of/van S ✓ <i>y</i>-value/waarde of/van S (4)</p> <p>✓ equations of TS and RS/vgls van TS en RS</p> <p>✓ equating / gelykstel</p> <p>✓ <i>x</i>-value/waarde ✓ <i>y</i>-value/waarde (4)</p>
<p>3.5</p>	<p>$T\hat{S}R = T\hat{Q}R$ [opp \angles of m/teenoorst \anglee m] $T\hat{Q}R = \alpha - \beta$ $\tan \alpha = m_{TQ} = -\frac{4}{3}$ $\therefore \alpha = 180^\circ - 53,13^\circ = 126,87^\circ$ $\tan \beta = m_{RQ} = \frac{7}{7} = 1$ $\therefore \beta = 45^\circ$ $T\hat{Q}R = 126,87^\circ - 45^\circ$ $= 81,87^\circ$ $T\hat{S}R = 81,87^\circ$</p> <p>OR/OF</p>	<p>✓ $T\hat{Q}R = \alpha - \beta$ ✓ $\tan \alpha = m_{TQ}$ ✓ α ✓ $\tan \beta = m_{RQ}$ ✓ β</p> <p>✓ answer (6)</p>

	$TQ = SR = 5$ $TR = \sqrt{100+9} = \sqrt{109}$ $RQ = TS = \sqrt{49+49} = \sqrt{98}$ $\cos \hat{RQT} = \cos \hat{TSR} = \frac{TQ^2 + RQ^2 - TR^2}{2 \cdot TQ \cdot RQ}$ $= \frac{25 + 98 - 109}{2(5)(\sqrt{98})}$ $= 0,141\dots$ $\hat{RQT} = \hat{TSR} = 81,87^\circ$	<ul style="list-style-type: none"> ✓ length of TQ OR SR ✓ length of TR ✓ length of RQ OR TS ✓ correct subst into cosine rule ✓ simplification ✓ answer <p>(6)</p>
<p>3.6.1</p>	$MQ = \sqrt{(5-3)^2 + (2-0)^2}$ $MQ = \sqrt{8}$ $\frac{MQ}{RQ} = \frac{\sqrt{8}}{\sqrt{98}}$ $= \frac{2}{7} \quad \text{or} \quad 0,29$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> Answer only: full marks </div>	<ul style="list-style-type: none"> ✓ substitution/<i>substitusie</i> ✓ $MQ = \sqrt{8} = 2\sqrt{2}$ ✓ answer <p style="text-align: right;">(3)</p>
<p>3.6.2</p>	$\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{\frac{1}{2} \cdot QM \cdot \perp h}{\frac{1}{2} \cdot QR \cdot \perp h} \quad [\perp h \text{ same/dieselfde}]$ $= \frac{QM}{QR} = \frac{2}{7}$ $\frac{\text{area of } \Delta TQM}{\text{area of parm RQTS}} = \frac{\text{area of } \Delta TQM}{2 \times \text{area of } \Delta TQR}$ $= \frac{1}{2} \left(\frac{2}{7} \right) = \frac{1}{7}$ <p>OR/OF</p> $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{QM}{QR}$ $= \frac{2}{7}$ $\frac{\text{area of } \Delta TQM}{\text{area of parm RQTS}} = \frac{\text{area of } \Delta TQM}{2 \text{area of } \Delta TQR}$ $= \frac{1}{2} \left(\frac{2}{7} \right) = \frac{1}{7}$ <p>OR/OF</p>	<ul style="list-style-type: none"> ✓ $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{2}{7}$ ✓ area parm RQTS = 2area ΔTQR ✓ answer <p style="text-align: right;">(3)</p> <ul style="list-style-type: none"> ✓ $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR} = \frac{2}{7}$ ✓ area parm RQTS = 2area ΔTQR ✓ answer <p style="text-align: right;">(3)</p>

$\frac{\text{area of } \Delta TQM}{\text{area of parm RQTS}} = \frac{\frac{1}{2} QM \cdot \perp h}{RQ \cdot \perp h}$ $= \frac{1}{2} \left(\frac{2}{7} \right)$ $= \frac{1}{7}$ <p>OR/OF</p> $\frac{\text{area of } \Delta TQM}{\text{area of parm RQTS}} = \frac{\frac{1}{2} QT \cdot QM \cdot \sin(\alpha - \beta)}{2 \text{area of } \Delta QTR}$ $= \frac{\frac{1}{2} QT \cdot QM \cdot \sin(\alpha - \beta)}{2 \left[\frac{1}{2} \cdot QT \cdot QR \cdot \sin(\alpha - \beta) \right]}$ $= \frac{1}{2} \left(\frac{2}{7} \right)$ $= \frac{1}{7}$	$\checkmark \frac{\frac{1}{2} QM \cdot \perp h}{RQ \cdot \perp h}$ $\checkmark \frac{1}{2} \left(\frac{2}{7} \right)$ $\checkmark \text{ answer}$ <p style="text-align: right;">(3)</p> \checkmark $\text{area parm RQTS} = 2 \text{area } \Delta TQR$ $\checkmark \frac{\frac{1}{2} QT \cdot QM \cdot \sin(\alpha - \beta)}{2 \left[\frac{1}{2} \cdot QT \cdot QR \cdot \sin(\alpha - \beta) \right]}$ $\checkmark \text{ answer}$ <p style="text-align: right;">(3) [23]</p>
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QUESTION/VRAAG 4



4.1	line from centre to midpt of chord / <i>lyn vanaf midpt na midpt van koord</i>	✓ answer (1)
4.2	$m_{ST} = \frac{8-5}{-3-0}$ $= -1$ $m_{ST} \times m_{NP} = -1 \quad [TS \perp NP]$ $\therefore m_{NP} = 1$ $\therefore y = x + c$ $8 = -3 + c$ $c = 11$ $\therefore y = x + 11$ OR/OF $y - y_1 = 1(x - x_1)$ $y - 8 = 1(x + 3)$ $y = x + 11$	✓ subst (-3 ; 8) and (0 ; 5) into gradient formula ✓ m_{ST} ✓ m_{NP} ✓ subst (-3 ; 8) into equation of a line ✓ equation (5)
4.3	P(0 ; 11) [y-intercept of chord NP] \therefore radius is 6 units R(0 ; -1) Equations of the tangents to the circle parallel to the x-axis/ <i>Vgls van die raaklyne aan die sirkel // aan die x-as:</i> $y = 11$ and $y = -1$	✓ coordinates of P/ koördinate v P ✓ coordinates of R koördinate van R ✓✓ answers (4)
4.4	M(-11 ; 0) [x-intercept of/x-afsnit van NP] $MT = \sqrt{(0-11)^2 + (5-0)^2}$ $MT = \sqrt{146} = 12,08$	✓✓ coordinates of M ✓ substitution ✓ answer (4)

4.5	<p>MT = diameter/middel lyn [conv \angle in $\frac{1}{2}$ circle/omgek \angle in $\frac{1}{2}$ sirkel]</p> <p>radius = $\frac{\sqrt{146}}{2}$ units</p> <p>Centre of circle/Middelpunt v sirkel = Midpoint MT /Middelpunt MT</p> <p>= $\left(\frac{-11}{2}; \frac{5}{2}\right)$</p> <p>Equation of circle through S, T and M: $\left(x + \frac{11}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{146}{4}$</p> <p>OR/OF $\left(x + 5\frac{1}{2}\right)^2 + \left(y - 2\frac{1}{2}\right)^2 = \frac{73}{2} = 6,04$</p>	<p>✓ radius of circle</p> <p>✓ x value of M</p> <p>✓ y value of M</p> <p>✓ LHS of equation</p> <p>✓ RHS of equation</p> <p>(5) [19]</p>
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QUESTION/VRAAG 5

5.1	<p>$a = -1$ $b = 2$</p>	<p>✓ answer</p> <p>✓ answer</p> <p>(2)</p>
5.2	<p>$f(3x) = -\sin 3x$</p> <p>Period of $f(3x) = \frac{360^\circ}{3}$ $= 120^\circ$</p> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 100px;">Answer only: Full marks</div>	<p>✓ $\frac{360^\circ}{3}$</p> <p>✓ answer</p> <p>(2)</p>
5.3	<p>$x \in [90^\circ ; 135^\circ) \cup \{180^\circ\}$</p> <p>OR/OF</p> <p>$90^\circ \leq x < 135^\circ$ or $x = 180^\circ$</p>	<p>✓ 90° and 135° in interval form</p> <p>✓ 180° as single value</p> <p>✓ correct brackets</p> <p>(3)</p> <p>✓ 90° and 135° in interval form</p> <p>✓ 180° as single value</p> <p>✓ correct inequalities</p> <p>(3) [7]</p>

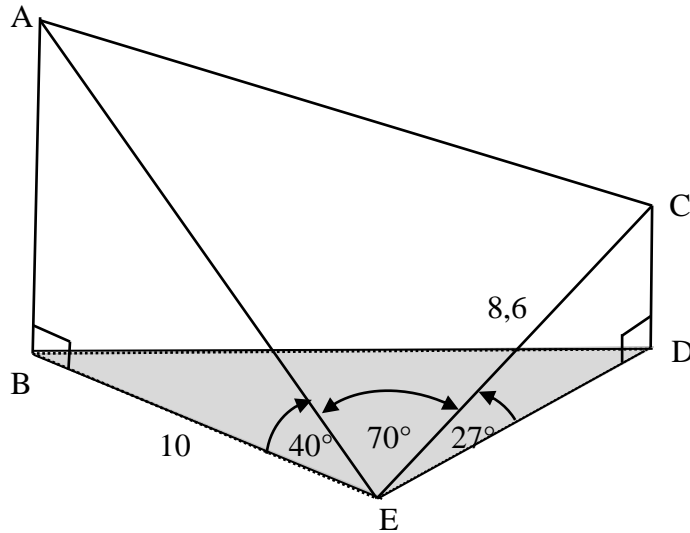
QUESTION/VRAAG 6

6.1.1	$\sin (360^\circ - 36^\circ) = -\sin 36^\circ$	✓ answer (1)
6.1.2	$\cos 72^\circ = \cos(2 \times 36^\circ)$ $= 1 - 2 \sin^2 36^\circ$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">Answer only: Full marks</div>	✓ double angle/dubbelhoek ✓ answer (2)
6.2	<p>R.T.P.: $1 - \frac{\tan^2 \theta}{1 + \tan^2 \theta} = \cos^2 \theta$</p> <p>LHS = $\frac{1 + \tan^2 \theta - \tan^2 \theta}{1 + \tan^2 \theta}$</p> $= \frac{1}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}}$ $= \frac{1}{\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}}$ $= \frac{1}{\frac{1}{\cos^2 \theta}}$ $= \cos^2 \theta$ <p>= RHS</p> <p>OR/OF</p> <p>LHS = $\frac{1 + \tan^2 \theta - \tan^2 \theta}{1 + \tan^2 \theta}$</p> $= \frac{1}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}}$ $= \frac{1}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}} \times \frac{\cos^2 \theta}{\cos^2 \theta}$ $= \frac{\cos^2 \theta}{\cos^2 \theta + \sin^2 \theta}$ $= \frac{\cos^2 \theta}{1}$ $= \cos^2 \theta$ <p>= RHS</p> <p>OR/OF</p>	✓ writing as a single fraction/skryf as enkelbreuk ✓ quotient identity/kwosiëntidentiteit ✓ denominator as a single fraction / Noemer as enkelbreuk ✓ square identity/vierkantidentiteit (4) ✓ writing as a single fraction/skryf as enkelbreuk ✓ quotient identity / kwosiëntidentiteit ✓ $\times \frac{\cos^2 \theta}{\cos^2 \theta}$ ✓ square identity/vierkantidentiteit (4) ✓ quotient identity/

	$\begin{aligned} \text{LHS} &= 1 - \left(\frac{\sin^2 \theta}{\cos^2 \theta} \div \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta} \right) \right) \\ &= 1 - \left(\frac{\sin^2 \theta}{\cos^2 \theta} \times \frac{\cos^2 \theta}{\cos^2 \theta + \sin^2 \theta} \right) \\ &= 1 - \left(\frac{\sin^2 \theta}{\cos^2 \theta} \times \frac{\cos^2 \theta}{1} \right) \\ &= 1 - \sin^2 \theta \\ &= \cos^2 \theta \\ &= \text{RHS} \end{aligned}$	<p><i>kwosiëntidentiteit</i></p> <ul style="list-style-type: none"> ✓ writing as a single fraction/ <i>skryf as enkelbreuk</i> ✓ square identity/<i>vierkantidentiteit</i> ✓ simplification/<i>vereenvoudiging</i> <p style="text-align: right;">(4)</p>
<p>6.3</p>	$\begin{aligned} \cos^2 \frac{1}{2}x &= \frac{1}{4} \\ \cos \frac{1}{2}x &= \frac{1}{2} \text{ or } -\frac{1}{2} \\ \frac{1}{2}x &= 60^\circ + k.360^\circ \text{ or } \frac{1}{2}x = 300^\circ + k.360^\circ \text{ or} \\ \frac{1}{2}x &= 120^\circ + k.360^\circ \text{ or } \frac{1}{2}x = 240^\circ + k.360^\circ \\ x &= 120^\circ + k.720^\circ \text{ or } x = 600^\circ + k.720^\circ \text{ or} \\ x &= 240^\circ + k.720^\circ \text{ or } x = 480^\circ + k.720^\circ; k \in \mathbb{Z} \end{aligned}$ <p>OR/OF</p> $\begin{aligned} \cos^2 \frac{1}{2}x &= \frac{1}{4} \\ \cos \frac{1}{2}x &= \frac{1}{2} \text{ or } -\frac{1}{2} \\ \frac{1}{2}x &= \pm 60^\circ + k.360^\circ \text{ or } \frac{1}{2}x = \pm 120^\circ + k.360^\circ \\ x &= \pm 120^\circ + k.720^\circ \text{ or } x = \pm 240^\circ + k.720^\circ; k \in \mathbb{Z} \end{aligned}$	<ul style="list-style-type: none"> ✓✓ $\cos^2 \frac{1}{2}x = \frac{1}{4}$ ✓ 60° and 300° ✓ 120° and 240° ✓ write at least one general solution as $\frac{1}{2}x = \angle + k.360^\circ$ ✓ write at least one general solution as $x = \angle + k.720^\circ; k \in \mathbb{Z}$ <p style="text-align: right;">(6)</p> <ul style="list-style-type: none"> ✓✓ $\cos^2 \frac{1}{2}x = \frac{1}{4}$ ✓ $\pm 60^\circ$ ✓ $\pm 120^\circ$ ✓ write at least one general solution as $\frac{1}{2}x = \angle + k.360^\circ$ ✓ write at least one general solution as $x = \angle + k.720^\circ k \in \mathbb{Z}$ <p style="text-align: right;">(6)</p>

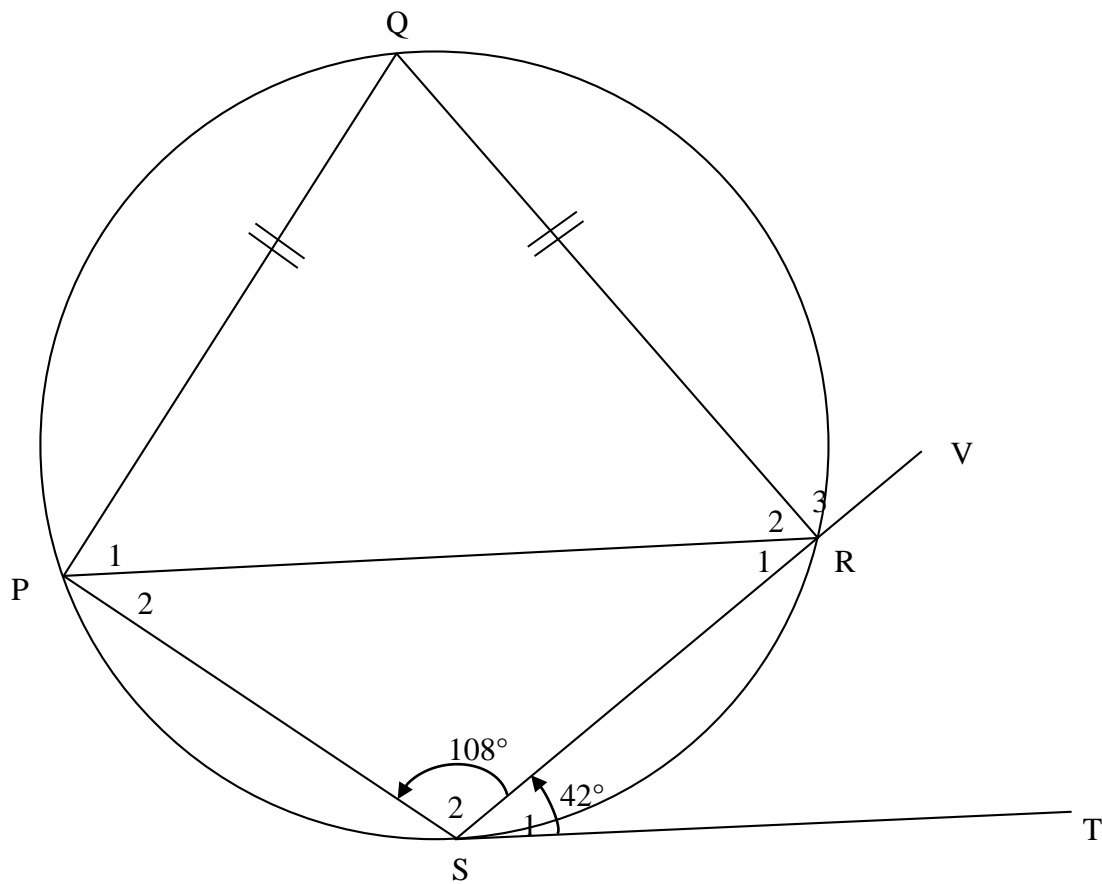
<p>6.4.1</p>	$\begin{aligned} \sin(A - B) &= \cos[90^\circ - (A - B)] \\ &= \cos[(90^\circ - A) - (-B)] \\ &= \cos(90^\circ - A)\cos(-B) + \sin(90^\circ - A)\sin(-B) \\ &= \sin A\cos B + \cos A(-\sin B) \\ &= \sin A\cos B - \cos A\sin B \end{aligned}$ <p>OR/OF</p> $\begin{aligned} \sin(A - B) &= \cos[90^\circ - (A - B)] \\ &= \cos[(90^\circ + B) - A] \\ &= \cos(90^\circ + B)\cos A + \sin(90^\circ + B)\sin A \\ &= -\sin B\cos A + \cos B\sin A \\ &= \sin A\cos B - \cos A\sin B \end{aligned}$	<ul style="list-style-type: none"> ✓ co-ratio/ko-verhouding ✓ writing as a difference of A & B/ <i>skryf as verskil van A & B</i> ✓ expansion/uitbreiding ✓ all reductions/alle reduksies <p style="text-align: right;">(4)</p> <ul style="list-style-type: none"> ✓ co-ratio/ko-verhouding ✓ writing as a difference of A & B/ <i>skryf as verskil van A & B</i> ✓ expansion/uitbreiding ✓ all reductions/alle reduksies <p style="text-align: right;">(4)</p>
<p>6.4.2</p>	$\begin{aligned} &\sin(x + 64^\circ)\cos(x + 379^\circ) + \sin(x + 19^\circ)\cos(x + 244^\circ) \\ &= \sin(x + 64^\circ)\cos(x + 19^\circ) + \sin(x + 19^\circ)[- \cos(x + 64^\circ)] \\ &= \sin(x + 64^\circ)\cos(x + 19^\circ) - \cos(x + 64^\circ)\sin(x + 19^\circ) \\ &= \sin[x + 64^\circ - (x + 19^\circ)] \\ &= \sin 45^\circ \\ &= \frac{1}{\sqrt{2}} \end{aligned}$	<ul style="list-style-type: none"> ✓ $\cos(x + 379^\circ) = \cos(x + 19^\circ)$ ✓✓ $\cos(x + 244^\circ) = -\cos(x + 64^\circ)$ ✓✓ compound formula identity/ <i>saamgestelde identiteit</i> ✓ $\sin 45^\circ$ <p style="text-align: right;">(6) [23]</p>

QUESTION/VRAAG 7



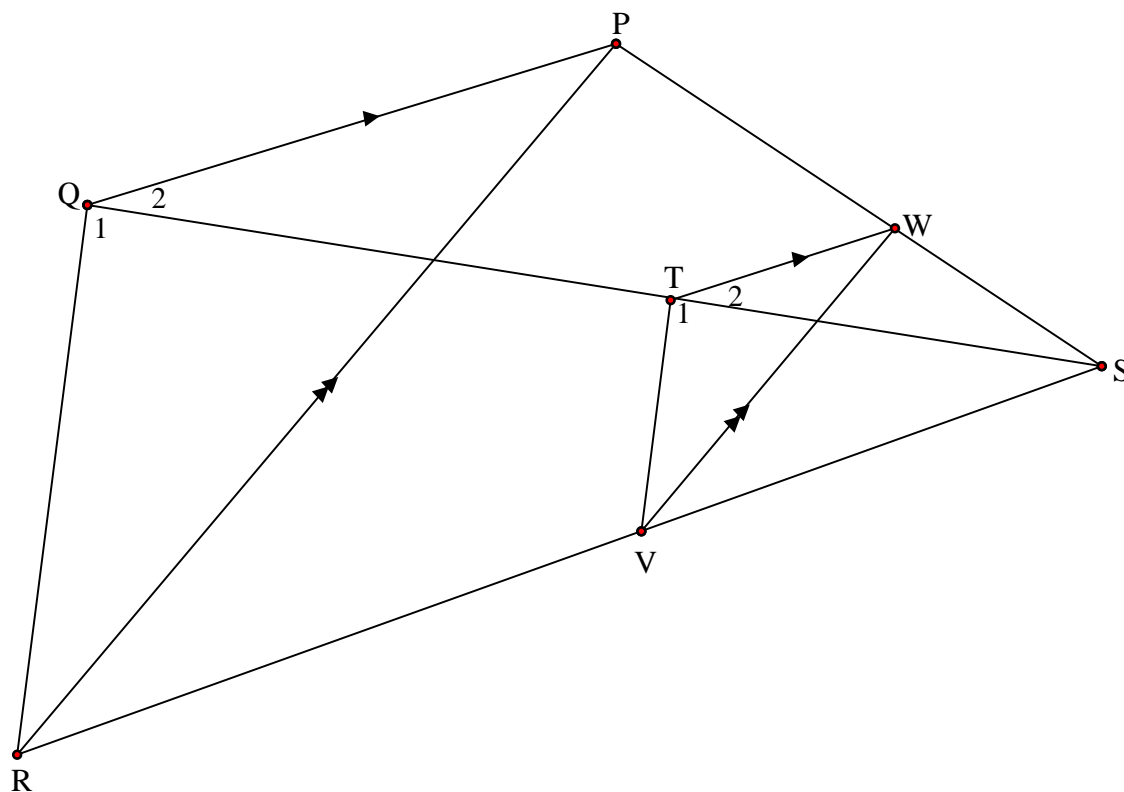
7.1	$\sin 27^\circ = \frac{CD}{8,6}$ $CD = 8,6 \sin 27^\circ$ $CD = 3,90 \text{ m}$	✓ substitution in correct trig ratio / <i>substitusie in korrekte trig verh</i> ✓ answer (2)
7.2	$\cos 40^\circ = \frac{10}{AE}$ $AE = \frac{10}{\cos 40^\circ}$ $AE = 13,05 \text{ m}$	✓ substitution in correct trig ratio / <i>substitusie in korrekte trig verh</i> ✓ answer (2)
7.3	$AC^2 = CE^2 + AE^2 - 2 CE \cdot AE (\cos \hat{AEC})$ $= (8,6)^2 + (13,05)^2 - 2(8,6)(13,05)(\cos 70^\circ)$ $= 167,49$ $AC = 12,94 \text{ m}$	✓ correct use of cosine rule in ΔACE / <i>korrekte gebruik van reel in ΔACE</i> ✓ correct subst into cosine rule ✓ AC^2 ✓ answer (4) [8]

QUESTION/VRAAG 8



8.1	$\hat{Q} = 72^\circ$ [opp \angle s of cyclic quad/teenoorst \angle e koordevh]	\checkmark S \checkmark R (2)
8.2	$\hat{R}_2 = \hat{P}_1$ [\angle s opp equal sides/ \angle e teenoor gelyke sye] $\hat{R}_2 = \frac{180^\circ - 72^\circ}{2}$ [sum of \angle s in Δ /som v \angle e in Δ] $= 54^\circ$	\checkmark S/R \checkmark answer (2)
8.3	$\hat{P}_2 = 42^\circ$ [tan chord theorem/raakl-koordst]	\checkmark S \checkmark R (2)
8.4	$\hat{R}_3 = \hat{P}_1 + \hat{P}_2$ [ext \angle of cyclic quad/buite \angle van koordevh] $= 54^\circ + 42^\circ$ $= 96^\circ$ OR/OF $\hat{R}_1 = 180^\circ - 108^\circ - 42^\circ = 30^\circ$ [sum of/som van \angle s/e in Δ] $\hat{R}_3 = 180^\circ - \hat{R}_1 - \hat{R}_2$ [\angle s on str line/ \angle e op reguitlyn] $= 180^\circ - 30^\circ - 54^\circ$ [sum of/som van \angle s/e in Δ] $= 96^\circ$	\checkmark R \checkmark S \checkmark $\hat{R}_1 = 30^\circ$ \checkmark S (2) [8]

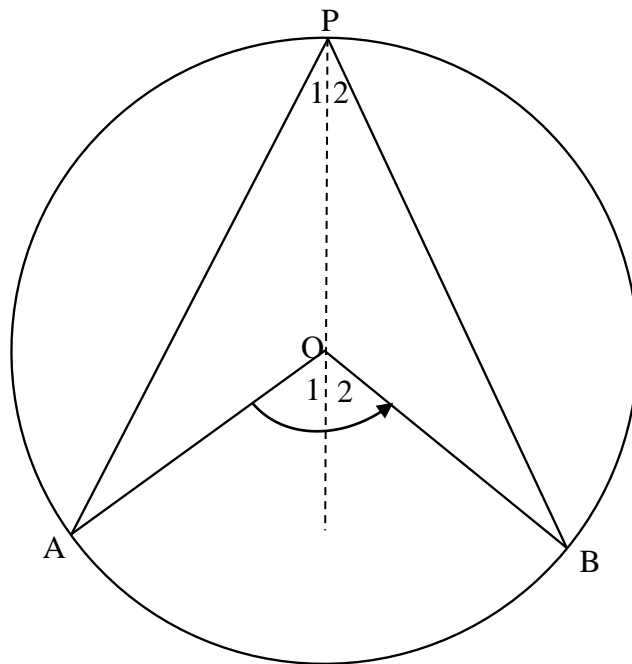
QUESTION/VRAAG 9



9.1.1	$\frac{ST}{TQ} = \frac{SW}{WP}$ $= \frac{2}{3}$	[prop theorem/ <i>eweredighst</i> ; $TW \parallel QP$]	✓ S ✓ S (2)
9.1.2	$\frac{SV}{VR} = \frac{SW}{WP}$ $= \frac{2}{3}$	[prop theorem/ <i>eweredighst</i> ; $VW \parallel RP$]	✓ answer (1)
9.2	$\frac{ST}{TQ} = \frac{SV}{VR}$ [both equal/ <i>beide gelyk</i> $\frac{WS}{PW}$] $\therefore TV \parallel QR$ [line divides 2 sides of Δ in prop/ <i>lyn verdeel 2 sye van Δ in dies verh</i>] $\therefore \hat{T}_1 = \hat{Q}_1$ [corresp/ <i>ooreenkomst</i> \angle s/e; $TV \parallel QR$]		✓ S ✓ S ✓ R ✓ R (4)
9.3	$\Delta VWS \parallel \Delta RPS$		✓ ΔRPS (any order) (1)
9.4	$\frac{WV}{PR} = \frac{SW}{SP}$ $= \frac{2}{5}$	$[\Delta VWS \parallel \Delta RPS]$ OR/OF	$\frac{WV}{PR} = \frac{SV}{SR}$ $= \frac{2}{5}$ $[\Delta VWS \parallel \Delta RPS]$ ✓ ratio ✓ answer (2) [10]

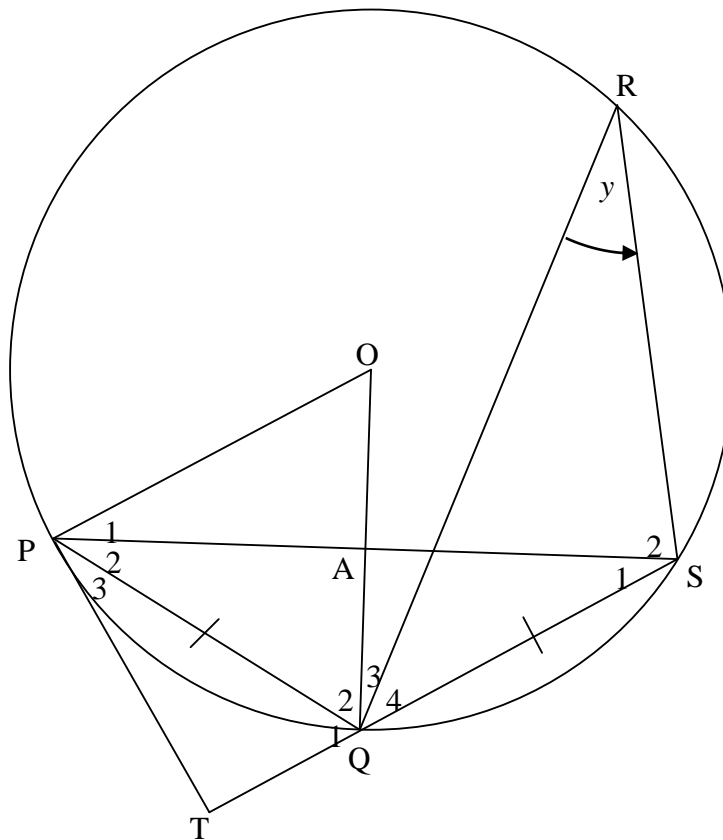
QUESTION/VRAAG 10

10.1



	<p><i>Constr/Konst :</i> Draw line PO and extend /Trek lyn PO en verleng <i>Proof/Bewys :</i> $OP = OA$ [radii] $\therefore \hat{P}_1 = \hat{A}$ [\angles opp/teenoor = sides/sye] but $\hat{O}_1 = \hat{P}_1 + \hat{A}$ [ext \angle of Δ] $\therefore \hat{O}_1 = 2\hat{P}_1$ Similarly/Netso, $\hat{O}_2 = 2\hat{P}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{P}_1 + \hat{P}_2)$ i.e. $\hat{A}OB = 2\hat{A}PB$</p>	<p>✓ construction ✓ S/R ✓ S/R ✓ S ✓ S</p> <p>(5)</p>
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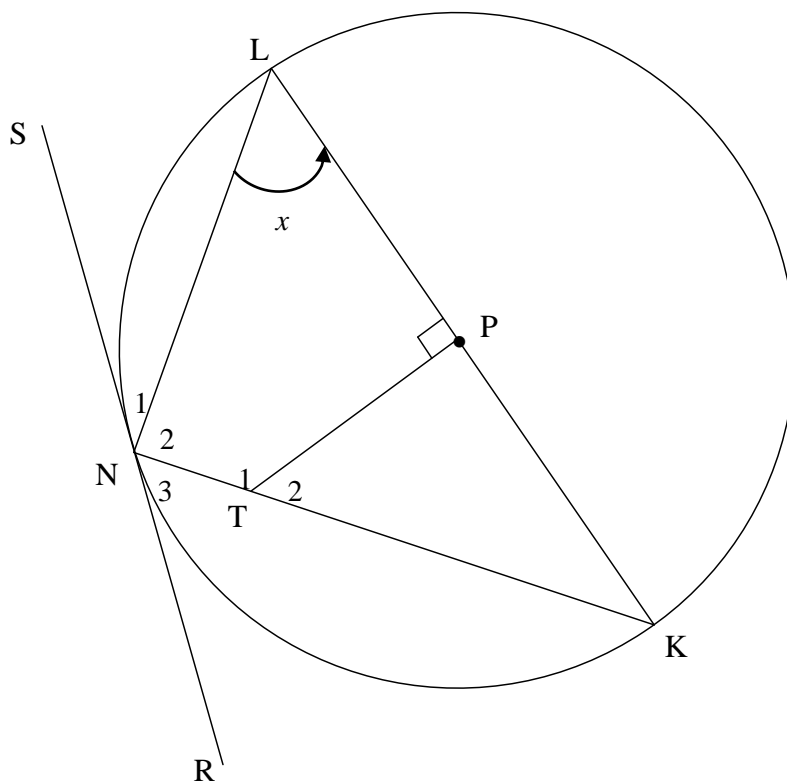
10.2



10.2.1	\angle s in the same segment/ \angle e in dieselfde sirkelsegment	✓ R (1)
10.2.2	$\hat{P}_2 = \hat{S}_1 = y$ [∠s opp equal sides/∠e teenoor = sye] $\hat{S}_1 = \hat{P}_3 = y$ [tan chord theorem/raakl-koordst] $\therefore \hat{P}_2 = \hat{P}_3$ \therefore PQ bisects $\hat{T}PS$	✓ S ✓ R ✓ S ✓ R (4)
10.2.3	$\hat{P}OQ = 2\hat{S}_1 = 2y$ [∠at centre = $2 \times$ ∠at circ/midpts∠ = $2 \times$ omtreks∠]	✓ S ✓ R (2)
10.2.4	$\hat{T}PA = \hat{P}_2 + \hat{P}_3 = 2y$ [proved/bewys in 11.2.2] $\therefore \hat{T}PA = \hat{P}OQ$ [proved/bewys in 11.2.3] \therefore PT = tangent [converse tan chord theorem/omgek raakl-koordst]	✓ $\hat{T}PA = \hat{P}OQ$ ✓ R (2)

10.2.5	$\hat{O}PQ + \hat{O}QP = 180^\circ - 2y$ [sum of/sum v \angle s/e in Δ] $\therefore \hat{O}QP = 90^\circ - y$ [\angle s opp equal sides/ \angle e to = sye; $OP = OQ$] In ΔPAQ : $\hat{O}QP + \hat{P}_2 + \hat{Q}AP = 180^\circ$ $90^\circ - y + y + \hat{Q}AP = 180^\circ$ [sum of/sum v \angle s/e in Δ] $\hat{Q}AP = 90^\circ$ $\therefore \hat{O}AP = 90^\circ$ [\angle s/e on straight line/op reguitlyn]	✓ S ✓ S ✓ R ✓ S ✓ S (5)
	OR/OF	
	$\hat{O}PT = 90^\circ$ [radius \perp tangent/raaklyn] $\therefore \hat{P}_1 = 90^\circ - 2y$ $\hat{P}_1 + \hat{O} + \hat{O}AP = 180^\circ$ [sum of/sum v \angle s/e in Δ] $(90^\circ - 2y) + 2y + \hat{O}AP = 180^\circ$ $\therefore \hat{O}AP = 90^\circ$	✓ S ✓ R ✓ S ✓ S ✓ S (5)
	OR/OF	
	POSQ is a kite/'n vlieër $\therefore OQ \perp PS$ [diag of a kite/hoeklyne v vlieër] $\therefore \hat{O}AP = 90^\circ$	✓✓✓ S ✓✓ R (5)
	OR/OF	
	In ΔOAP and ΔOAS $OP = OS$ (radii) OA is common $\hat{P}OA = 2y$ $= 2\hat{P}_2$ $= \hat{Q}OS$ $\Delta OAP \cong \Delta OAS$ (SAS) $\hat{O}AP = \hat{O}AS$ ($\cong \Delta$ s) $\hat{O}AP = \hat{O}AS = 90^\circ$ (\angle s on str line)	✓ S ✓ S ✓ S ✓ R ✓ S (5)
		(5) [19]

QUESTION/VRAAG 11



11.1	$\hat{N}_2 = 90^\circ$ [\angle in semi-circle/halfsirkel] \therefore TPLN is a cyclic quad/ 'n koordevh [opp \angle s of quad is suppl/ teenoor \angle e v vh is suppl] OR $\hat{N}_2 = 90^\circ$ [\angle in semi-circle/halfsirkel] \therefore TPLN is a cyclic quad [ext $\angle =$ int opp \angle /buite $\angle =$ to binne \angle]	\checkmark S \checkmark R \checkmark R (3) \checkmark S \checkmark R \checkmark R (3)
11.2	$\hat{T}_2 = \hat{PLN} = x$ [ext \angle of cyclic quad/buite \angle van koordevh] $\hat{K} = 90^\circ - x$ [sum of/som v \angle s/e in Δ] $\hat{N}_1 = \hat{K} = 90^\circ - x$ [tan chord theorem/raakl-koordst] OR/OF $\hat{K} = 90^\circ - x$ [sum of/som v \angle s/e in Δ] $\hat{N}_1 = \hat{K} = 90^\circ - x$ [tan chord theorem/raakl-koordst] OR/OF $\hat{N}_3 = x$ [tan chord theorem/raakl-koordst] $\hat{N}_2 = 90^\circ$ [\angle in semi circle/ halfsirkel] $\hat{N}_1 = 90^\circ - x$ [straight line/reguitlyn]	\checkmark R \checkmark S \checkmark R (3) \checkmark R \checkmark S \checkmark R (3) \checkmark R \checkmark S \checkmark S (3)

<p>11.3.1</p>	<p>In ΔKTP and ΔKLN: $\hat{P}KT = \hat{L}KN$ [common/<i>gemeen</i>] $\hat{K}PT = \hat{K}NL = 90^\circ$ [given/<i>gegee</i>] $\therefore \Delta KTP \parallel \Delta KLN$ [$\angle\angle\angle$] OR/OF In ΔKTP and ΔKLN: $\hat{P}KT = \hat{L}KN$ [common/<i>gemeen</i>] $\hat{K}PT = \hat{K}NL = 90^\circ$ [given/<i>gegee</i>] $\hat{T}_2 = \hat{P}LN = x$ [proved in 11.2 OR sum of \angles in Δ] $\therefore \Delta KTP \parallel \Delta KLN$</p>	<p>✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)</p>
<p>11.3.2</p>	<p>$\frac{KT}{KL} = \frac{KP}{KN}$ [$\parallel \Delta$s] $\therefore KT \cdot KN = KP \cdot KL$ But $KL = 2KP$ [radii: $PK = LP$] $\therefore KT \cdot KN = KP \cdot 2KP$ $\qquad = 2KP^2$ $\qquad = 2(KT^2 - TP^2)$ [Theorem of Pythagoras] $\qquad = 2KT^2 - 2TP^2$</p>	<p>✓ S/R ✓ S ✓ S ✓ S ✓ S (5) [14]</p>

TOTAL/TOTAAL: 150