MATHEMATICS P3
NOVEMBER 2011
POSSIBLE ANSWERS

MARKS: 100

This memorandum consists of 14 pages.
NOTE:
- If a candidate answers a question TWICE and does not delete any attempt, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in ALL aspects of the marking memorandum.
- A learner cannot use what s/he must prove to prove it (i.e. the circular argument.).

QUESTION 1

1.1 \[ T_{k+1} = T_k - 2; \quad k \geq 1; \quad T_1 = 12 \]

- \[ T_1 = 12 \]
- \[ T_2 = 12 - 2 = 10 \]
- \[ T_3 = 10 - 2 = 8 \]
- \[ T_4 = 8 - 2 = 6 \]

\[ \checkmark \quad 10 \]
\[ \checkmark \quad 8 \]
\[ \checkmark \quad 6 \]

1.2 \[ 12 + 10 + 8 + 6 + 4 + 2 + 0 + (-2) + (-4) + (-6) + (-8) + (-10) + (-12) = 0 \]
\[ \therefore 13 \text{ terms} \]

**Note:**
- If a learner writes out \[ 12 + 10 + 8 + 6 + 4 + 2 + 0 \] then 1/3 marks

**OR**

There are 6 positive terms before the 7th term, which is 0. We need 6 negative terms of equal value to the positive terms so that the sum is zero.

6 positive terms + 1 zero term + 6 negative terms = 13 terms

**OR**

\[ \frac{n}{2} [2(12) + (n-1)(-2)] = 0 \]

\[ \frac{n}{2} [24 + 2 - 2n] = 0 \]

\[ \frac{n}{2} [26 - 2n] = 0 \]

\[ 13n - n^2 = 0 \]

\[ n(13 - n) = 0 \]

\[ n \neq 0 \quad \text{or} \quad n = 13 \]

\[ \checkmark \quad T_7 = 0 \]

\[ \checkmark \quad 12 \text{ terms} \]

\[ \checkmark \quad 13 \text{ terms} \]

\[ \checkmark \quad \text{substitution into the arithmetic sum formula} \]

\[ \frac{n}{2} [26 - 2n] = 0 \]

\[ \checkmark \quad 13 \text{ terms} \]
### QUESTION 2

2.1 42 – 28 = 14

2.2 Approximately 88 kg

NOTE: Accept a range from 86 to 89 kg

2.3 15 learners in the sample have a weight of less than 80 kg. One would expect \( \frac{15}{50} \times 250 = 75 \) learners in the grade to have a weight of less than 80 kg.

OR

15 learners in the sample have a weight of less than 80 kg. One would expect \( 15 \times 5 = 75 \) learners in the grade to have a weight of less than 80 kg.

NOTE:
- Accept \( \frac{14}{50} \times 250 = 70 \)
- Answer as percentage: 1/2 marks
- Answer only: 2/2 marks

2.4 This sampling method is biased towards those who arrive early on a Monday morning. In this way all the learners in the Grade do not have the same chance of being selected for the sample.

### QUESTION 3

3.1 For mutually exclusive events

\[
P(A \text{ or } B) = P(A) + P(B) = 0,7 + 0,4 + k
\]

\[
k = 0,3
\]

NOTE:
If the candidate writes down \( k = 1 - 0,7 = 0,3 \): 0/2 marks

3.2 For independent events

\[
P(A \text{ and } B) = P(A) \times P(B) = 0,4k
\]

\[
P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0,7 + 0,4 + k - 0,4k
\]

\[
0,3 = 0,6k
\]

\[
k = 0,5
\]

OR

\[
0,7 = 0,4 + k - 0,4k
\]

\[
0,3 = 0,6k
\]

\[
k = 0,5
\]

Note:
- Answer only: 1/4 marks
- Wrong formula: 0/4 marks

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QUESTION 4

4.1 21 minutes is 1 standard deviation from the mean
∴ 34% of the pizzas are delivered between 21 and 24 minutes

Note: Answer only: FULL marks

4.2 15 minutes is 3 standard deviations to the left of the mean ∴ 50%
27 minutes is 1 standard deviation to the right of the mean ∴ 34%
84% of the pizzas are delivered between 15 and 27 minutes

Note: Answer only: FULL marks

4.3 The required 2% is the area found to the right of 2 standard deviations on the right hand side of the mean.
Maximum for delivery should be
24 + 2(3)
= 30 minutes

Note: Answer only: FULL marks

QUESTION 5

5.1 Number of unique codes
= 7 × 7 × 7
= 7³
= 343

Note: Answer only: FULL marks

5.2 Number of unique codes without repetition
= 7 × 6 × 5
= 210

Note: Answer only: FULL marks

5.3 Number of codes with repetition that are greater than 300 and divisible by 5
= 4 × 7 × 2 – 1
= 55

Note: No CA marking for the answer.
Answer only 3/3 marks

For a 100 numbers there are 14 numbers divisible by 5
14 × 4 = 56
56 – 1 = 55

Note: Answer only: FULL marks

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QUESTION 6

6.1

\[ \begin{align*} 
79 - x & \\
20 & \\
19 - x & \\
x & \\
11 & \\
16 & \\
40 - x & \\
0 & 
\end{align*} \]

\[ \sqrt{79 - x} \]
\[ \sqrt{20} \]
\[ \sqrt{19 - x} \]
\[ \sqrt{x} \]
\[ \sqrt{11} \]
\[ \sqrt{16} \]
\[ \sqrt{40 - x} \]

6.2

\[ 79 - x + 20 + x + 11 + 19 - x + 16 + 40 - x = 173 \]
\[ 185 - 2x = 173 \]
\[ x = 6 \]

OR

232 complaints and 173 people in total
94 complaints from 47 people
138 complaints from remaining 126 people
For the two to be equal
\[ 126 - x = 138 - 3x \]
\[ 2x = 12 \]
\[ x = 6 \]

OR

\[ 110 + 55 + 67 = 232 \]
\[ 2x + 20 + 11 + 16 = 232 - 173 \]
\[ 2x + 47 = 59 \]
\[ 2x = 12 \]
\[ x = 6 \]

6.3

\[ \begin{align*} 
P(\text{at least two complaints}) & = \frac{11 + 20 + 6 + 16}{173} \\
& = \frac{53}{173} \\
& = 0.31 \ (0.30635838...) \\
\end{align*} \]

OR \ 30.64\%
**QUESTION 7**

<table>
<thead>
<tr>
<th>Noon temperature (in °C)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>7</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units of electricity used</td>
<td>37</td>
<td>36</td>
<td>32</td>
<td>33</td>
<td>32</td>
<td>28</td>
<td>27</td>
<td>23</td>
<td>20</td>
</tr>
</tbody>
</table>

**Scatter plot showing noon temperature vs electricity consumption**

7.1 See scatter plot above

**Note:**
Please ignore the point (0 ; 41).

☑️☑️☑️ all 9 points plotted correctly
2 marks if 5 – 8 points are plotted correctly
1 mark if 1 – 4 points are plotted correctly.

(3)
### Question 7.2

\[
\begin{align*}
\text{a} &= 40.97 \quad (40.97108844...) \\
b &= -1.74 \quad (-1.736394558...) \\
\hat{y} &= 40.97 - 1.74x
\end{align*}
\]

**Note:**
- Penalise 1 mark for incorrect rounding to ONE decimal place in either 7.2 or 7.3
- Answer only: FULL marks

**NOTE:**
If the candidate works the coefficients out manually that
\[
b = \frac{-204.2}{117.6}
\]
then 2 marks for \(b\).

![Image](image.png)

### Question 7.3

\[
r = -0.97 \quad (-0.9699269087...)
\]

**NOTE:** If the candidate gives \(b = \frac{6.139218}{3.42928} r\) and not simplified then 1 mark.

![Image](image.png)

### Question 7.4

There is a strong negative correlation between the noon temperature and the units of electricity used.

**OR**

As the noon temperature increases, the units of electricity used decreases.

**OR**

As the noon temperature decreases, the units of electricity used increases.

![Image](image.png)

### Question 7.5

\[
\hat{y} \approx 40.97 - 1.74(8) \\
\approx 27.05
\]

**Note:**
- Answer only: 2/2 marks
- Accept a range of 26.5 – 27.5 if the least squares regression line is drawn and the answer is read off: 2/2 marks

![Image](image.png)
8.1 Draw diameter AM and join M to B.
\( \hat{A}_1 + \hat{A}_2 = 90^\circ \) (rad \( \perp \) tangent)
\( \hat{B}_1 + \hat{B}_2 = 90^\circ \) (\( \angle \)s in a semi circle)
\( \hat{B}_2 = \hat{A}_2 \) (\( \angle \)s in same seg)
\( \hat{B}_1 = \hat{A}_1 \)

\( \hat{C}_1 = x \) (\( \angle \) opp = radii)
\( \hat{A}_1 = 90^\circ - x \) (rad \( \perp \) tan)
\( AOC = 180^\circ - 2x \) (\( \angle \) sum \( \Delta \))
\( \hat{A}_1 + \hat{A}_2 = 90^\circ \) (\( \angle \) circ cent = 2 \( \angle \) circumference)
\( ABC = A_1 \) (= 90\(^\circ\) - \( x \))

**NOTE:**
If there is no construction: 0 / 5 marks
If candidate changes lettering and states “Similarly”: full marks

**OR**
Draw radii OC and OA
Let \( \hat{A}_2 = x \)
\( \hat{C}_1 = x \) (\( \angle \) opp = radii)
\( \hat{A}_1 = 90^\circ - x \) (rad \( \perp \) tan)
\( AOC = 180^\circ - 2x \) (\( \angle \) sum \( \Delta \))
\( \hat{A}_1 = 90^\circ - x \) (rad \( \perp \) tan)
\( S/R \)

\( \hat{A}_1 + \hat{A}_2 = 90^\circ \)
\( S/R \)

\( \hat{A}_1 + \hat{A}_2 = 90^\circ \)
\( \angle \) circ cent = 2 \( \angle \) circumference
Draw diameter AM and Join M and C

$\hat{M}CA = 90^\circ$ (\(\angle s\) in semi circle)

$\hat{A}_M + \hat{A}_2 = 90^\circ$ (\(\angle s\) sum \(\Delta\))

$\hat{A}_1 + \hat{A}_2 = 90^\circ$ (rad \perp tangent)

$\hat{A}_M = \hat{A}_1$

$\hat{A}_M = \hat{B}$ (\(\angle s\) in same seg)

$\hat{A}_1 = \hat{B}$

8.2.1 \[
\widehat{WRS} = 90^\circ \quad (\tan \perp radius)
\]

8.2.2 \[
\widehat{RST} = 50^\circ \quad (\tan ch th)
\hat{W} = 40^\circ \quad (\angle s\ sum \Delta)
\]

\textbf{OR}

$\hat{T}_1 = 90^\circ$ (\(\angle s\) in semi circle)

$\hat{W} + \hat{R}_1 = \hat{T}_1$ (ext \(\angle \Delta\))

$\hat{W} = 40^\circ$

8.2.3 \[
\hat{R}_2 = 40^\circ \quad (\tan \perp radius)
\hat{P}_1 = 40^\circ \quad (\angle s\ in\ same\ seg)
\]

\(\widehat{R}_2 = 40^\circ\)

\(\hat{P}_1 = 40^\circ\)

\(\angle s\ in\ same\ seg\)
8.2.4  \( \hat{P}_1 = \hat{W} \quad (= 40^\circ) \)
WVPT is a cyclic quadrilateral \( \text{ (ext } \angle = \text{ int opp) } \)
\( \hat{V}_1 = P\hat{T}\hat{S} \quad \text{(ext } \angle \text{ cyclic quad) } \)

**OR**
\( \hat{T}_1 = 90^\circ \quad \text{(} \angle \text{s in semi circle) } \)
\( P\hat{T}\hat{S} = 90^\circ + \hat{T}_2 \)
\( \hat{T}_2 = \hat{S}_1 \quad \text{(} \angle \text{s in same seg) } \)
\( P\hat{T}\hat{S} = 90^\circ + \hat{T}_1 \)
\( \hat{V}_1 = 90^\circ + \hat{S}_1 \quad \text{(ext } \angle \Delta ) \)
\( \hat{V}_1 = P\hat{T}\hat{S} \)

**OR**
\( \hat{P}_2 = 140^\circ \quad \text{(} \angle \text{s on str line) } \)
\( \hat{W} + \hat{P}_2 = 180^\circ \)
WVPT is cyclic quad \( \text{ (opp } \angle \text{s suppl) } \)
\( \hat{V}_1 = P\hat{T}\hat{S} \quad \text{(ext } \angle \text{ cyclic quad) } \)

**OR**
\( \hat{V}_1 = \hat{R}_1 + \hat{R}_2 + \hat{S}_1 \quad \text{(ext } \angle \Delta ) \)
\( \hat{V}_1 = 90^\circ + \hat{S}_1 \)
\( P\hat{T}\hat{S} = 90^\circ + \hat{T}_2 \)
But \( \hat{T}_2 = \hat{S}_1 \quad \text{(} \angle \text{s in same seg) } \)
\( \hat{V}_1 = P\hat{T}\hat{S} \)

**OR**
In \( \triangle P\hat{T}\hat{S} \) and \( \triangle W\hat{V}\hat{S} \)
\( \hat{P}_1 = \hat{W} \quad (= 40^\circ) \)
\( \hat{S}_2 \text{ is common} \)
\( \hat{V}_1 = P\hat{T}\hat{S} \quad \text{(} \angle \text{ sum } \Delta ) \)

[15]
QUESTION 9

9. \( \hat{C} = 90^\circ \) (\( \angle s \) in semi circle)
   O\( \hat{E}A = 90^\circ \) (corres \( \angle s \); OD \( \parallel \) BC)
   AE = 8 cm (line from circ cent \( \perp \) ch bis ch)
   OE = 6 cm (Pythagoras)
   ED = 10 \(- 6\)
   = 4 cm

   OR
   \( \hat{C} = 90^\circ \) (\( \angle s \) in semi circle)
   O\( \hat{E}A = 90^\circ \) (corres \( \angle s \); OD \( \parallel \) BC)
   OE \parallel BC (given)
   OA = OB (radii)
   AE = EC = 8cm (midpoint theorem)
   OE = 6 cm (Pythagoras)
   ED = 10 \(- 6\)
   = 4 cm

   OR
   \( \hat{C} = 90^\circ \) (\( \angle s \) in semi circle)
   BC\(^2\) = (20\(^2\) \(- (16\(^2\)
   BC = 144
   BC = 12
   OE = \( \frac{1}{2} \)BC (midpoint theorem)
   OE = 6 cm
   OD = 10cm
   ED = 10 \(- 6\)
   = 4 cm

   OR
   \( \hat{C} = 90^\circ \) (\( \angle s \) in semi circle)
   BC\(^2\) = (20\(^2\) \(- (16\(^2\)
   BC = 144
   BC = 12
   OE = \( \frac{1}{2} \)BC (midpoint theorem)
   OE = 6 cm
   ED = 4 cm

[5]
### QUESTION 10

#### 10.1
\[
\hat{A} = \hat{D}_4 = x \quad \text{(tan ch th)}
\]
\[
\hat{E}_2 = x \quad \text{(tan ch th) OR (} \angle s \text{ in same seg)}
\]
\[
\hat{D}_2 = \hat{A} = x \quad \text{(alt } \angle s; \text{ CA } \parallel \text{ DF)}
\]

#### 10.2
In \(\triangle BHD\) and \(\triangle FED\)
1. \(\hat{B}_2 = \hat{F}\) \(\quad (\angle s \text{ in same seg})\)
2. \(\hat{D}_3 = \hat{D}_1\) \(\quad (= \text{ chs subt } = \angle s)\)

\(\triangle BHD \parallel \triangle FED\, (\angle \angle \angle)\)

#### 10.3
\[
\frac{FE}{BH} = \frac{FD}{BD} \quad (\parallel \Delta s)
\]
But \(FE = AB\) \(\text{ (given)}\)
\[
\frac{AB}{BH} = \frac{FD}{BD}
\]
\[
AB \cdot BD = FD \cdot BH
\]

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QUESTION 11

11.1 \[ \frac{AF}{FC} = 1 \] (diags of parallelogram bisect)  
\[ FE \parallel CD \]  
\[ AE = ED \] (Prop Th; FE \parallel CD) OR (Midpoint Theorem)  
\[ \checkmark AF = FC \]  
\[ \checkmark \text{reason} \]  

11.2 \[ \frac{AC}{CP} = 1 \] (given)  
\[ \frac{AD}{DQ} = 1 \] (given)  
\[ \frac{AC}{CP} = \frac{AD}{DQ} \]  
\[ CD \parallel PQ \] (converse proportionality theorem)  
\[ CD \parallel FE \] (given)  
\[ \therefore PQ \parallel FE \]  
\[ \checkmark \text{ratios equal} \]  
\[ \checkmark CD \parallel PQ \]  
\[ \checkmark \text{reason: converse} \]  
prop th and conclusion  

\[ \checkmark \text{prop th and conclusion} \]  

\[ \checkmark \frac{AF}{AP} = 1 \]  
\[ \checkmark \frac{AE}{AQ} = \frac{AF}{AP} \]  
\[ \checkmark \text{conv prop theorem} \]  

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11.3 In $\triangle AEF$ and $\triangle APQ$
1. $\angle A$ is common
2. $\angle AEF = \angle AQP$ (corres $\angle$s; $FE \parallel PQ$)
3. $\angle AFE = \angle APQ$ (corres $\angle$s; $FE \parallel PQ$)

$\therefore \triangle AEF \sim \triangle AQP$ ($\angle \angle $)

\[
\frac{FE}{PQ} = \frac{AF}{AP} \quad (\parallel \Delta s)
\]

\[
\frac{FE}{PQ} = \frac{1}{6}
\]

$FE = 10 \text{ cm}$

NOTE: If the similarity has not been proven, then max 3/5 marks

OR

In $\triangle ADC$ and $\triangle APQ$
1. $\angle A$ is common
2. $\angle ADC = \angle AQP$ (corres $\angle$s; $CD \parallel PQ$)
3. $\angle ACD = \angle APQ$ (corres $\angle$s; $CD \parallel PQ$)

$\therefore \triangle ADC \sim \triangle AQP$ ($\angle \angle $)

\[
\frac{AC}{AD} = \frac{1}{3} \quad (\parallel \Delta s)
\]

\[
\frac{CD}{PQ} = \frac{1}{3}
\]

$CD = 20 \text{ cm}$

But $AF = FC$

$AE = ED$ (Midpoint Theorem)

$FE = \frac{1}{2} CD$

$FE = 10 \text{ cm}$

✓ first pair of angles equal with reason
✓ second pair of angles equal with reason

\[
\frac{AF}{AP} = \frac{1}{6}
\]

\[
\frac{FE}{PQ} = \frac{AF}{AP}
\]

✓ answer

\[5\]

\[
\frac{CD}{PQ} = \frac{1}{3}
\]

✓ $FE = \frac{1}{2} CD$

✓ answer

\[5\]

TOTAL: 100