basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P3

NOVEMBER 2011

POSSIBLE ANSWERS

MARKS: 100

This memorandum consists of 14 pages.

NOTE:

- If a candidate answers a question TWICE and does not delete any attempt, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in ALL aspects of the marking memorandum.
- A learner cannot use what s/he must prove to prove it (i.e. the circular argument.).

QUESTION 1

1.1	$T_{k+1} = T_k - 2;\ k \geq 1;\ T_1 = 12$ $T_1 = 12$ $T_2 = 12 - 2 = 10$ $T_3 = 10 - 2 = 8$ $T_4 = 8 - 2 = 6$	✓ 10 ✓ 8 ✓ 6 (3)
1.2	$12 + 10 + 8 + 6 + 4 + 2 + 0 + (-2) + (-4) + (-6) + (-8) + (-10) + (-12) = 0$ $\therefore$ 13 terms **Note:** If a learner writes out $12 + 10 + 8 + 6 + 4 + 2 + 0$ then 1/3 marks **Note:** Answer only: FULL marks **OR** There are 6 positive terms before the 7th term, which is 0. We need 6 negative terms of equal value to the positive terms so that the sum is zero 6 positive terms + 1 zero term + 6 negative terms = 13 terms **OR** $\frac{n}{2}[2(12) + (n-1)(-2)] = 0$ $\frac{n}{2}[24 + 2 - 2n] = 0$ $\frac{n}{2}[26 - 2n] = 0$ $13n - n^2 = 0$ $n(13 - n) = 0$ $n \neq 0$ *or* $n = 13$	✓✓ expansion ✓ 13 terms (3) ✓ $T_7 = 0$ ✓ 12 terms ✓ 13 terms (3) ✓ substitution into the arithmetic sum formula ✓ $\frac{n}{2}[26 - 2n] = 0$ ✓ 13 terms (3) **[6]**

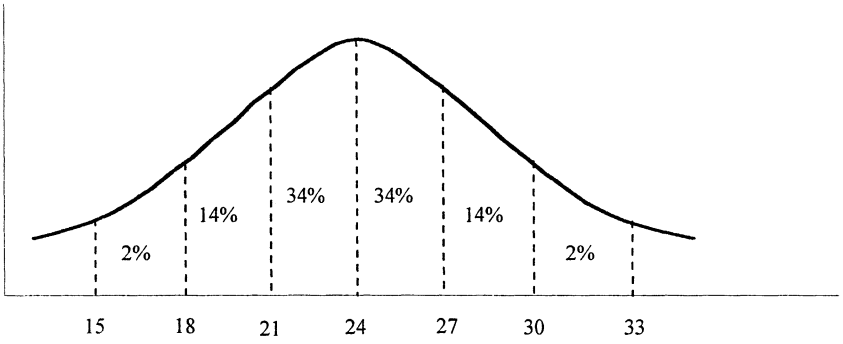
QUESTION 2

2.1	$42 - 28 = 14$	✓ answer (1)
2.2	Approximately 88 kg **NOTE:** Accept a range from 86 to 89 kg	✓ answer (1)
2.3	15 learners in the sample have a weight of less than 80 kg. One would expect $\frac{15}{50} \times 250 = 75$ learners in the grade to have a weight of less than 80 kg. **OR** 15 learners in the sample have a weight of less than 80 kg. One would expect $15 \times 5 = 75$ learners in the grade to have a weight of less than 80 kg. **NOTE:** • Accept $\frac{14}{50} \times 250 = 70$ • Answer as percentage: 1/2 marks • Answer only: 2/2 marks	✓ Cumulative Frequency value read off the graph when less than 80 ✓ answer (2) ✓ Cumulative Frequency value read off the graph when less than 80 ✓ answer (2)
2.4	This sampling method is biased towards those who arrive early on a Monday morning. In this way all the learners in the Grade do not have the same chance of being selected for the sample.	✓ sensible explanation of random sample (1) **[5]**

QUESTION 3

3.1	For mutually exclusive events P(A or B) = P(A) + P(B) $0,7 = 0,4 + k$ $k = 0,3$ **Note:** Answer only: FULL marks **NOTE:** If the candidate writes down $k = 1 - 0,7 = 0,3$: 0/2 marks	✓ $0,7 = 0,4 + k$ ✓ answer (2)
3.2	For independent events P(A and B) = P(A).P(B) $= 0,4k$ P(A or B) = P(A) + P(B) – P(A and B) $0,7 = 0,4 + k - 0,4k$ $0,3 = 0,6k$ $k = 0,5$ **Note:** • Answer only: 1/4 marks • Wrong formula: 0/4 marks **OR** $0,7 = 0,4 + k - 0,4k$ $0,3 = 0,6k$ $k = 0,5$	✓ P(A and B) = P(A).P(B) ✓ $0,4k$ ✓ $0,7 = 0,4 + k - 0,4k$ ✓ answer (4) ✓✓✓ $0,7 = 0,4 + k - 0,4k$ ✓ answer (4) **[6]**

QUESTION 4



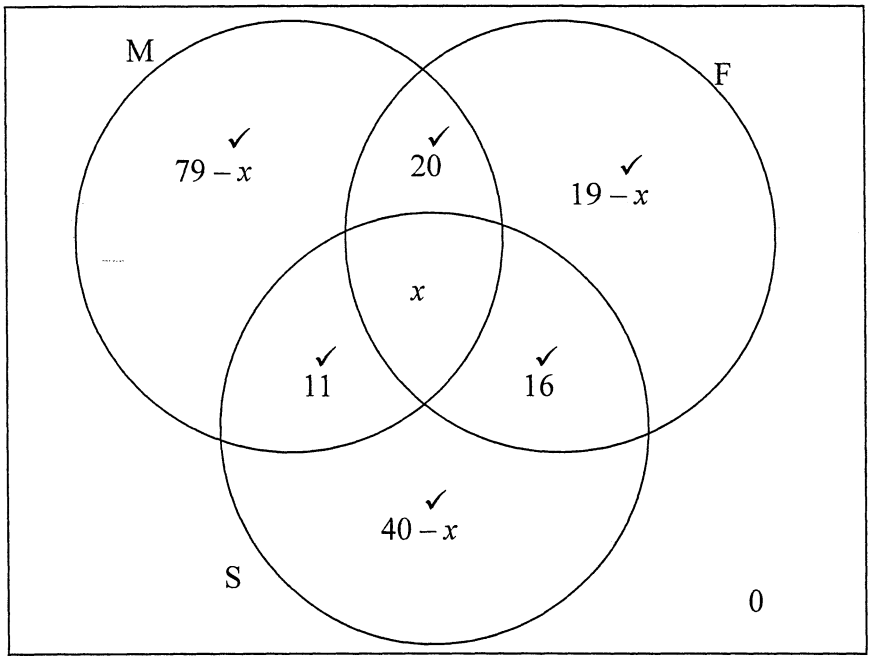
4.1	21 minutes is 1 standard deviation from the mean ∴ 34% of the pizzas are delivered between 21 and 24 minutes **Note:** Answer only: FULL marks	✓ 1 standard deviation ✓ 34% (2)
4.2	15 minutes is 3 standard deviations to the left of the mean ∴ 50% 27 minutes is 1 standard deviation to the right of the mean ∴ 34% 84% of the pizzas are delivered between 15 and 27 minutes **OR** 2% + 14% + 34% + 34% = 84% **Note:** Answer only: FULL marks	✓ 50% ✓ 34% ✓ 84% (3) ✓ 50% ✓ 34% ✓ 84% (3)
4.3	The required 2% is the area found to the right of 2 standard deviations on the right hand side of the mean. Maximum for delivery should be 24 + 2(3) = 30 minutes **Note:** Answer only: FULL marks	✓ 2 standard deviations ✓ 24 + 2(3) ✓ 30 (3) **[8]**

QUESTION 5

5.1	Number of unique codes = 7 × 7 × 7 = 7^3 = 343 **Note:** Answer only: FULL marks	✓ 7 × 7 × 7 ✓ answer (2)
5.2	Number of unique codes without repetition = 7 × 6 × 5 = 210 **Note:** Answer only: FULL marks **OR** $\frac{7!}{4!}$ = 210	✓ 7 × 6 × 5 ✓ answer (2) ✓ $\frac{7!}{4!}$ ✓ answer (2)
5.3	Number of codes with repetition that are greater than 300 and divisible by 5 = 4 × 7 × 2 – 1 = 55 **Note:** • No CA marking for the answer. • Answer only 3/3 marks **OR** For a 100 numbers there are 14 numbers divisible by 5 14 × 4 = 56 56 – 1 = 55	✓ 4 × 7 × 2 ✓ – 1 ✓ answer (3) ✓ 14× 4 ✓ – 1 ✓ answer (3) **[7]**

QUESTION 6

6.1



✓ $79-x$
✓ 20
✓ $19-x$
✓ 11
✓ 16
✓ $40-x$
(6)

6.2

$79-x+20+x+11+19-x+16+40-x=173$

$185-2x=173$

$x=6$

Note:
Check the reasonableness of the answer.

✓ addition
✓ 173
✓ answer
(3)

OR

232 complaints and 173 people in total
94 complaints from 47 people
138 complaints from remaining 126 people
For the two to be equal

$126-x=138-3x$

$2x=12$

$x=6$

✓ $126-x$ and $138-3x$
✓ $126-x=138-3x$
✓ answer
(3)

OR

$110+55+67=232$

$2x+20+11+16=232-173$

$2x+47=59$

$2x=12$

$x=6$

✓ 232
✓ $2x+20+11+16=232-173$
✓ answer
(3)

6.3

P(at least two complaints)

$=\frac{11+20+6+16}{173}$

$=\frac{53}{173}$

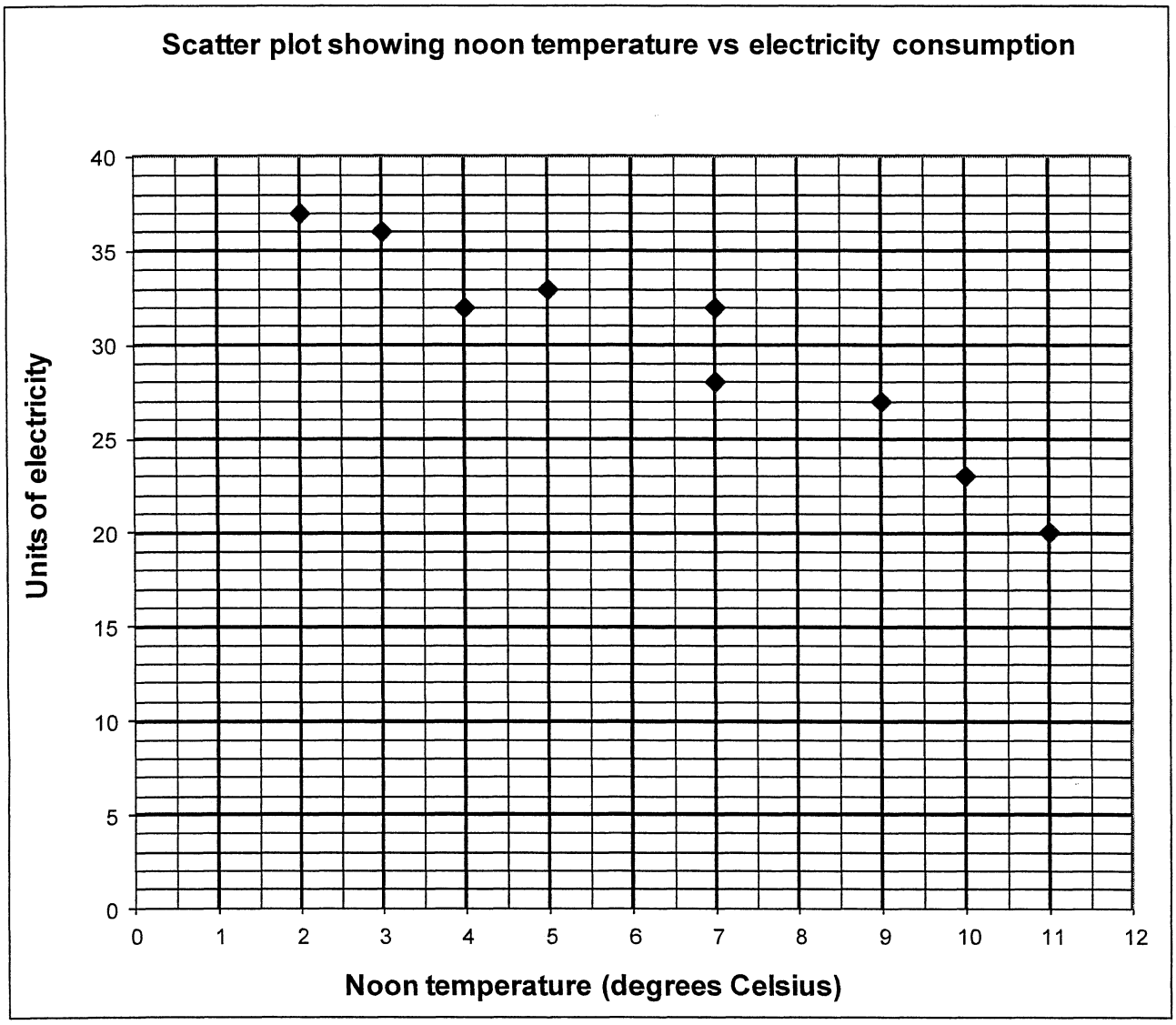
$=0,31$ (0,30635838...)

OR 30,64%

✓ $11+20+6+16$
✓ 173
✓ answer
(3)
[12]

QUESTION 7

Noon temperature (in °C)	2	3	4	5	7	7	9	10	11
Units of electricity used	37	36	32	33	32	28	27	23	20



7.1	See scatter plot above **Note:** Please ignore the point (0 ; 41).	✓✓✓ all 9 points plotted correctly 2 marks if 5 – 8 points are plotted correctly 1 mark if 1 – 4 points are plotted correctly. (3)

7.2	$a = 40,97$ (40,97108844...) $b = -1,74$ (–1,736394558...) $\hat{y} = 40,97 - 1,74x$ **Note:** • Penalise 1 mark for incorrect rounding to ONE decimal place in either 7.2 or 7.3 • Answer only: FULL marks **NOTE:** If the candidate works the coefficients out manually that $b = \frac{-204,2}{117,6}$ then 2 marks for b.	✓✓ a ✓ b ✓ equation (4)
7.3	$r = -0,97$ (–0,9699269087...) **NOTE:** If the candidate gives $b = \frac{6,139218}{3,42928}r$ and not simplified then 1 mark.	✓✓ answer (2)
7.4	There is a strong negative correlation between the noon temperature and the units of electricity used. **OR** As the noon temperature increases, the units of electricity used decreases. **OR** As the noon temperature decreases, the units of electricity used increases.	✓ strong ✓ negative (2) ✓✓ as noon temp increases & units decrease (2) ✓✓ as noon temp decreases & units increases (2)
7.5	$\hat{y} \approx 40,97 - 1,74(8)$ $\approx 27,05$ **OR** $\hat{y} \approx 27,0799 \approx 27,08$ **Note:** • Answer only: 2/2 marks • Accept a range of 26,5 – 27,5 if the least squares regression line is drawn and the answer is read off: 2/2 marks	✓ substitution ✓ answer (2) **[13]**

QUESTION 8

8.1

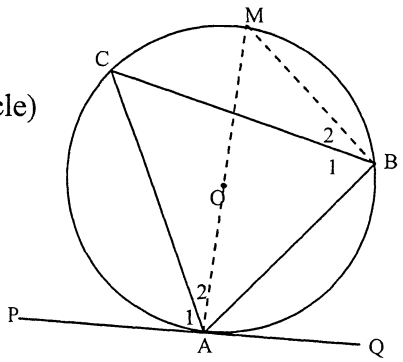
Draw diameter AM and join M to B.

$\hat{A}_1 + \hat{A}_2 = 90°$ (rad ⊥ tangent)

$\hat{B}_1 + \hat{B}_2 = 90°$ (∠s in a semi circle)

$\hat{B}_2 = \hat{A}_2$ (∠s in same seg)

$\hat{B}_1 = \hat{A}_1$



✓ construction
✓ S/R
✓ $\hat{B}_1 + \hat{B}_2 = 90°$
✓ ∠s in a semi circle
✓ S/R

(5)

OR

Draw radii OC and OA

Let $\hat{A}_2 = x$

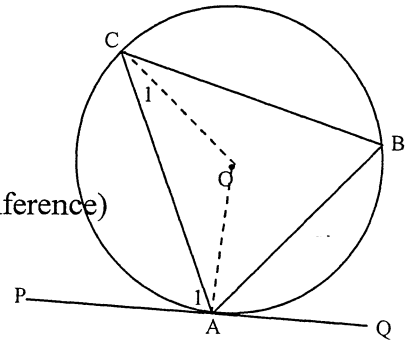
$\hat{C}_1 = x$ (∠ opp = radii)

$\hat{A}_1 = 90° - x$ (rad ⊥ tan)

$A\hat{O}C = 180° - 2x$ (∠ sum Δ)

$A\hat{B}C = 90° - x$ (∠ circ cent = 2 ∠ circumference)

$A\hat{B}C = \hat{A}_1$ (= $90° - x$)



✓ construction

✓ $\hat{A}_1 = 90° - x$
✓ rad ⊥ tan

✓ S/R
✓ S/R

(5)

NOTE:
If there is no construction: 0 / 5 marks

If candidate changes lettering and states "Similarly": full marks

OR

Draw QA extend to P. Draw tangent CP at C.

PC = PA (tan from comm pt)

$\hat{C}_2 = \hat{A}_1$ (∠s opp = sides)

$C\hat{O}A = 2A\hat{B}C$

(∠ circ cent = 2∠ circumf)

$\hat{A}_1 + \hat{A}_2 = 90°$ (tan ⊥ radius)

$C\hat{O}A = 180° - (90° - \hat{A}_1 + 90° - \hat{C}_2)$

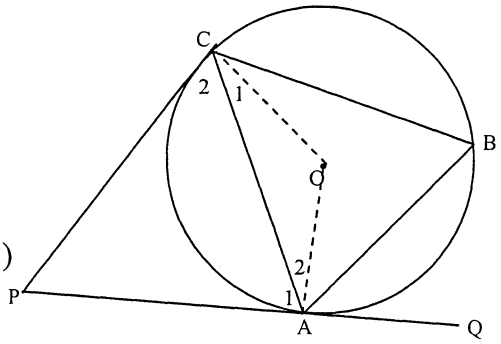
$= \hat{A}_1 + \hat{C}_2$

$= \hat{A}_1 + \hat{A}_1$

$= 2\hat{A}_1$

$\hat{A}_1 = \frac{1}{2} C\hat{O}A$

$= C\hat{B}A$



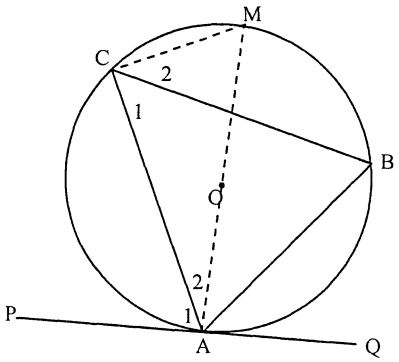
✓ construction
✓ S/R

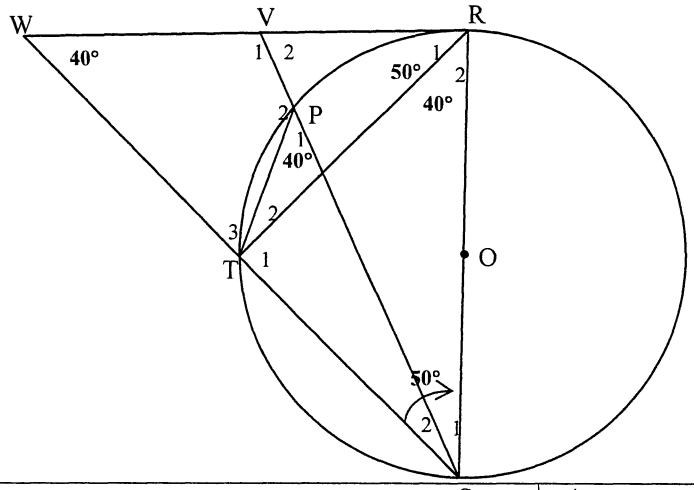
✓ S/R

✓ $\hat{A}_1 + \hat{A}_2 = 90°$
✓ tan ⊥ radius

(5)

OR

	Draw diameter AM and Join M and C $M\hat{C}A = 90°$ ($\angle$s in semi circle) $A\hat{M}C + \hat{A}_2 = 90°$ ($\angle$ sum Δ) $\hat{A}_1 + \hat{A}_2 = 90°$ (rad $\perp$ tangent) $A\hat{M}C = \hat{A}_1$ $A\hat{M}C = \hat{B}$ ($\angle$s in same seg) $\hat{A}_1 = \hat{B}$		✓ construction ✓ S/R ✓ S/R ✓ $\hat{A}_1 + \hat{A}_2 = 90°$ ✓ tan $\perp$ radius (5)

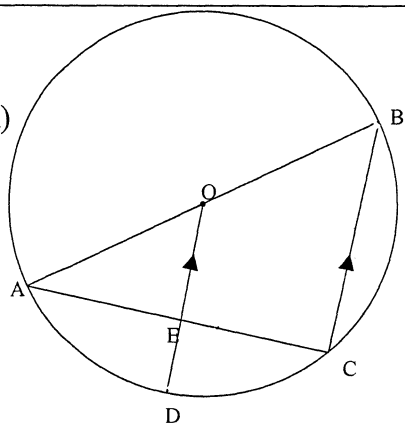


8.2.1	$W\hat{R}S = 90°$ (tan $\perp$ radius)		✓ statement (1)
8.2.2	$R\hat{S}T = 50°$ (tan ch th) $\hat{W} = 40°$ ($\angle$ sum Δ) **OR** $\hat{T}_1 = 90°$ ($\angle$s in semi circle) $\hat{W} + \hat{R}_1 = \hat{T}_1$ (ext $\angle$ Δ) $\hat{W} = 40°$		✓ S/R ✓ $\hat{W} = 40°$ (2) ✓ $\hat{W} + \hat{R}_1 = \hat{T}_1$ ✓ $\hat{W} = 40°$ (2)
8.2.3	$\hat{R}_2 = 40°$ (tan $\perp$ radius) $\hat{P}_1 = 40°$ ($\angle$s in same seg)		✓ $\hat{R}_2 = 40°$ ✓ $\hat{P}_1 = 40°$ ✓ $\angle$s in same seg (3)

8.2.4	$\hat{P}_1 = \hat{W}$ (= 40°) WVPT is a cyclic quadrilateral (ext ∠ = int opp) $\hat{V}_1 = P\hat{T}S$ (ext ∠ cyclic quad)	✓ $\hat{P}_1 = \hat{W}$ ✓ WVPT is a cyclic quadrilateral ✓ ext ∠ = in opp ✓ ext ∠ cyclic quad (4)
	OR $\hat{T}_1 = 90°$ (∠s in semi circle) $P\hat{T}S = 90° + \hat{T}_2$ $\hat{T}_2 = \hat{S}_1$ (∠s in same seg) $P\hat{T}S = 90° + \hat{S}_1$ $\hat{V}_1 = 90° + \hat{S}_1$ (ext ∠ Δ) $\hat{V}_1 = P\hat{T}S$	✓ ∠s in semi circle ✓ $P\hat{T}S = 90° + \hat{T}_2$ ✓ $\hat{T}_2 = \hat{S}_1$ ✓ ∠s in same seg (4)
	OR $\hat{P}_2 = 140°$ (∠s on str line) $\hat{W} + \hat{P}_2 = 180°$ WVPT is cyclic quad (opp ∠s suppl) $\hat{V}_1 = P\hat{T}S$ (ext ∠ cyclic quad)	✓ $\hat{W} + \hat{P}_2 = 180°$ ✓ WVPT is a cyclic quadrilateral ✓ opp ∠ suppl ✓ ext ∠ cyclic quad (4)
	OR $\hat{V}_1 = \hat{R}_1 + \hat{R}_2 + \hat{S}_1$ (ext ∠ Δ) $\hat{V}_1 = 90° + \hat{S}_1$ $P\hat{T}S = 90° + \hat{T}_2$ But $\hat{T}_2 = \hat{S}_1$ (∠s in same seg) $\hat{V}_1 = P\hat{T}S$	✓ $\hat{V}_1 = 90° + \hat{S}_1$ ✓ $P\hat{T}S = 90° + \hat{T}_2$ ✓ $\hat{T}_2 = \hat{S}_1$ ✓ ∠s in same seg (4)
	OR In ΔPTS and ΔWVS $\hat{P}_1 = \hat{W}$ (= 40°) $\hat{S}_2$ is common $\hat{V}_1 = P\hat{T}S$ (∠ sum Δ)	✓ identification of triangles ✓ $\hat{P}_1 = \hat{W}$ ✓ $\hat{S}_2$ is common ✓ ∠ sum Δ (4) **[15]**

QUESTION 9

9.



$\hat{C} = 90°$ (∠s in semi circle)
$O\hat{E}A = 90°$ (corres ∠s; OD || BC)
AE = 8 cm (line from circ cent ⊥ ch bis ch)
OE = 6 cm (Pythagoras)
ED = 10 – 6
= 4 cm

✓ $\hat{C} = 90°$
✓ $O\hat{E}A = 90°$
✓ line from circ cent ⊥ ch bis ch
✓ OE = 6 cm
✓ ED = 4 cm

OR

$\hat{C} = 90°$ (∠s in semi circle)
$O\hat{E}A = 90°$ (corres ∠s; OD || BC)
OE || BC (given)
OA = OB (radii)
AE = EC = 8cm (midpoint theorem)
OE = 6 cm (Pythagoras)
ED = 10 – 6
= 4 cm

✓ $\hat{C} = 90°$
✓ $O\hat{E}A = 90°$
✓ midpoint theorem
✓ OE = 6 cm
✓ ED = 4 cm

OR

$\hat{C} = 90°$ (∠s in semi circle)
$BC^2 = (20)^2 - (16)^2$
$BC^2 = 144$
$BC = 12$
$OE = \frac{1}{2}BC$ (midpoint theorem)
OE = 6 cm
OD = 10cm
ED = 10 – 6
= 4 cm

✓ $\hat{C} = 90°$
✓ BC = 12
✓ reason
✓ OE = 6 cm
✓ ED = 4 cm

[5]

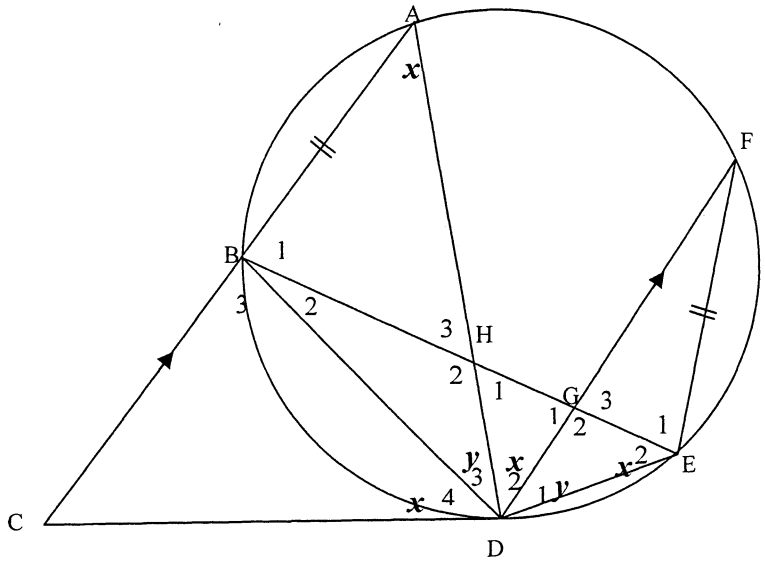
OR

$\hat{C} = 90°$ (∠s in semi circle)
$BC^2 = (20)^2 - (16)^2$
$BC^2 = 144$
$BC = 12$
$OE = \frac{1}{2}BC$ (midpoint theorem)
OE = 6 cm
ED = 4cm

✓ $\hat{C} = 90°$
✓ BC = 12
✓ reason
✓ OE = 6 cm
✓ ED = 4 cm

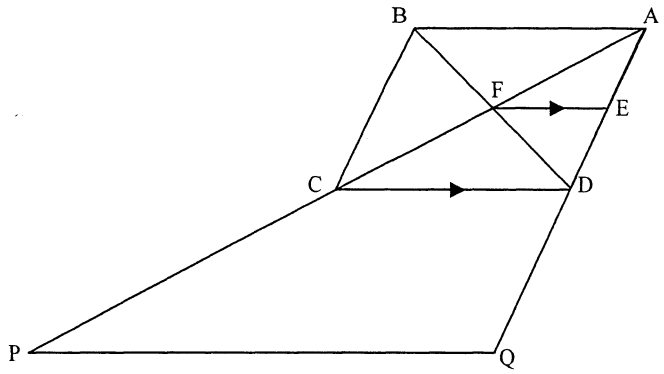
[5]

QUESTION 10



10.1	$\hat{A} = \hat{D}_4 = x$ (tan ch th) $\hat{E}_2 = x$ (tan ch th) **OR** ($\angle$s in same seg) $\hat{D}_2 = \hat{A} = x$ (alt $\angle$s; CA $\parallel$ DF)	✓ $\hat{A} = x$ ✓ tan ch th ✓ $\hat{E}_2 = x$ ✓ reason ✓ $\hat{D}_2 = x$ ✓ alt $\angle$s; CA $\parallel$ DF (6)
10.2	In ΔBHD and Δ FED 1. $\hat{B}_2 = \hat{F}$ ($\angle$s in same seg) 2. $\hat{D}_3 = \hat{D}_1$ (= chs subt = $\angle$s) ΔBHD $\|\|\|$ Δ FED ($\angle\angle\angle$)	✓ $\hat{B}_2 = \hat{F}$ ✓ $\angle$s in same seg ✓ $\hat{D}_3 = \hat{D}_1$ ✓ = chs subt = $\angle$s ✓ $\angle\angle\angle$ (5)
10.3	$\frac{FE}{BH} = \frac{FD}{BD}$ ($\|\|\|$ Δs) But FE = AB (given) $\frac{AB}{BH} = \frac{FD}{BD}$ AB.BD = FD.BH	✓ $\frac{FE}{BH} = \frac{FD}{BD}$ ✓ FE = AB (2) **[13]**

QUESTION 11



11.1	AF = FC FE ∥ CD AE = ED	(diags of parallelogram bisect) (Prop Th; FE ∥ CD) **OR** (Midpoint Theorem)	✓ AF = FC ✓ reason (2)
11.2	$\frac{AC}{CP}=\frac{1}{2}$ $\frac{AD}{DQ}=\frac{1}{2}$ $\frac{AC}{CP}=\frac{AD}{DQ}$ CD ∥ PQ CD ∥ FE ∴ PQ ∥ FE	(given) (given) (converse proportionality theorem) (given)	 ✓ ratios equal ✓ CD ∥ PQ ✓ reason: **converse** prop th and conclusion (3)
	OR $\frac{AC}{AP}=\frac{1}{3}$ $\frac{AD}{AQ}=\frac{1}{3}$ $\frac{AC}{AP}=\frac{AD}{AQ}$ CD ∥ PQ CD ∥ FE ∴ PQ ∥ FE	 (converse proportionality theorem) (given)	 ✓ ratios equal ✓ CD ∥ PQ ✓ reason: **converse** prop th and conclusion (3)
	OR $\frac{AF}{AP}=\frac{1}{6}$ $\frac{AE}{AQ}=\frac{1}{6}$ $\frac{AF}{AP}=\frac{AE}{AQ}$ ∴ PQ ∥ FE	 (converse proportionality theorem)	✓ $\frac{AF}{AP}=\frac{1}{6}$ ✓ $\frac{AF}{AP}=\frac{AE}{AQ}$ ✓ conv prop theorem

11.3	In ΔAEF and ΔAPQ 1. $\hat{A}$ is common 2. $A\hat{E}F = A\hat{Q}P$ (corres $\angle$s; FE $\parallel$ PQ) 3. $A\hat{F}E = A\hat{P}Q$ (corres $\angle$s; FE $\parallel$ PQ) $\therefore \Delta AEF \,|||\, \Delta AQP$ ($\angle\angle\angle$) $\frac{FE}{PQ} = \frac{AF}{AP}$ ($|||$ Δs) $\frac{FE}{60} = \frac{1}{6}$ FE = 10 cm **NOTE**: If the similarity has not been proven, then max 3/5 marks **OR** In ΔADC and ΔAPQ 1. $\hat{A}$ is common 2. $A\hat{D}C = A\hat{Q}P$ (corres $\angle$s; CD $\parallel$ PQ) 3. $A\hat{C}D = A\hat{P}Q$ (corres $\angle$s; CD $\parallel$ PQ) $\therefore \Delta ADC \,|||\, \Delta AQP$ ($\angle\angle\angle$) $\frac{AC}{AP} = \frac{AD}{AQ} = \frac{1}{3}$ ($|||$ Δs) $CD = \frac{1}{3}PQ$ CD = 20 cm But AF = FC AE = ED (Midpoint Theorem) $FE = \frac{1}{2}CD$ FE = 10 cm	✓ first pair of angles equal with reason ✓ second pair of angles equal with reason ✓ $\frac{AF}{AP} = \frac{1}{6}$ ✓ $\frac{FE}{PQ} = \frac{AF}{AP}$ ✓ answer (5) ✓ first pair of angles equal with reason ✓ second pair of angles equal with reason ✓ $CD = \frac{1}{3}PQ$ ✓ $FE = \frac{1}{2}CD$ ✓ answer (5) **[10]**

TOTAL: 100