## basic education

Department:
Basic Education REPUBLIC OF SOUTH AFRICA

## NATIONAL SENIOR CERTIFICATE

## GRADE 12



MARKS: 150

TIME: 3 hours

This question paper consists of $\mathbf{1 2}$ pages, $\mathbf{3}$ diagram sheets and 1 information sheet.

## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagram sheets for QUESTION 1.4, QUESTION 2.1, QUESTION 2.2 and QUESTION 6.2.2(b) are attached at the end of this question paper. Write your centre number and examination number on these diagram sheets in the spaces provided and insert the diagram sheets inside the back cover of your ANSWER BOOK.
8. An information sheet with formulae is included at the end of this question paper.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

## QUESTION 1

The tuck shop at Great Future High School sells cans of soft drinks. The Environmental Club at the school decided to have a can-collection project for three weeks to make learners aware of the effects of litter on the environment.

The data below shows the number of cans collected on each school day of the three-week project.

| 58 | 83 | 85 | 89 | 94 |
| :---: | :---: | :---: | :---: | :---: |
| 97 | 98 | 100 | 105 | 109 |
| 112 | 113 | 114 | 120 | 145 |

1.1 Calculate the mean number of cans collected over the three-week period.
1.2 Calculate the standard deviation.
1.3 Determine the lower and upper quartiles of the data.
1.4 Use the scaled line on DIAGRAM SHEET 1 to draw a box and whisker diagram to represent the data.
1.5 On how many days did the number of cans collected lie outside ONE standard deviation of the mean?

## QUESTION 2

The histogram below shows the time, in minutes, spent by customers while shopping at Excellent Supermarket.

2.1 Complete the frequency column and cumulative frequency column in the table on DIAGRAM SHEET 1.
2.2 Use the grid on DIAGRAM SHEET 2 to draw the ogive of the above data.
2.3 Use the ogive to estimate the median time that customers spent at this supermarket.
2.4 Comment on the skewness of the data.

## QUESTION 3

The scatter plot below shows the age and the time taken for each of the first ten swimmers of a swimming club to complete an open water swimming event. The time taken is rounded to the nearest half-minute.

3.1 Write down the coordinates of an outlier in the scatter plot.
3.2 Which of the following functions will best fit the data: linear, quadratic or exponential?
3.3 Give an explanation for the trend observed in this set of data.
3.4 If the two worst (longest) times are disregarded from the set of data, how will this affect the following:
3.4.1 The standard deviation of the original set of data
3.4.2 The mean of the original set of data

## QUESTION 4

In the diagram below, $\mathrm{A}(-1 ;-3), \mathrm{B}$ and C are the vertices of a triangle.
$\mathrm{P}(2,5 ; 1)$ is the midpoint of AB . CA extended cuts the $y$-axis at D .
The equation of CD is $y=-3 x+k . \quad \mathrm{CAB}=\theta$.
$\alpha$ and $\beta$ are the angles that AB and AC respectively make with the $x$-axis.

4.1 Determine the value of $k$.
4.2 Determine the coordinates of B.
4.3 Determine the gradient of AB .
4.4 Calculate the size of $\theta$.
4.5 Calculate the length of AD. Leave your answer in surd form.
4.6 If $\mathrm{AC}=2 \mathrm{AD}$ and $\mathrm{AB}=\sqrt{113}$, calculate the length of CB .

## QUESTION 5

In the diagram below, the equation of the circle with centre M is $(x-8)^{2}+(y+4)^{2}=45$. PT is a tangent to this circle at T and PT is parallel to OM. Another circle, having centre O, touches the circle having centre M at N .

5.1 Write down the coordinates of M.
5.2 Calculate the length of OM. Leave your answer in simplest surd form.
5.3 Calculate the length of ON. Leave your answer in simplest surd form.
5.4 Calculate the size of OMT .
5.5 Determine the equation of MT in the form $y=m x+c$.
5.6 Calculate the coordinates of T.

## QUESTION 6

6.1 The diagram below shows polygons $\mathrm{A}, \mathrm{B}$ and C .
$P(3 ; 5)$ is a vertex of polygon A.

6.1.1 Fully describe the transformation from polygon A to polygon B.
6.1.2 Write down the rule that transforms polygon B to polygon C .
6.1.3 $\quad \mathrm{P}^{\prime}$ is the image of P when polygon A is rotated about the origin through $180^{\circ}$. Write down the coordinates of $\mathrm{P}^{\prime}$.
6.2 $\mathrm{P}(3 ; 2), \mathrm{Q}(-1 ; 2), \mathrm{R}(-2 ; 1)$ and $\mathrm{S}(-1 ; 0)$ are the vertices of quadrilateral PQRS .
6.2.1 PQRS is enlarged through the origin by a scale factor of 2 to obtain $\mathrm{P}^{\prime} \mathrm{Q}^{\prime} \mathrm{R}^{\prime} \mathrm{S}^{\prime}$. Write down the coordinates of $\mathrm{Q}^{\prime}$.
6.2.2 $\quad \mathrm{P}^{\prime /} \mathrm{Q}^{\prime /} \mathrm{R}^{\prime /} \mathrm{S}^{\prime \prime}$ is the image when $\mathrm{P}^{\prime} \mathrm{Q}^{\prime} \mathrm{R}^{\prime} \mathrm{S}^{\prime}$ is reflected about the $y$-axis and then translated 3 units to the right and 1 unit upwards.
(a) Write down the single rule that transforms PQRS to $\mathrm{P}^{/ /} \mathrm{Q}^{/ /} \mathrm{R}^{/ /} \mathrm{S}^{\prime \prime}$.
(b) Use the grid on DIAGRAM SHEET 3 to draw $\mathrm{P}^{/ /} \mathrm{Q}^{/ /} \mathrm{R}^{/ /} \mathrm{S}^{\prime \prime}$. Label the vertices.
6.2.3 Solve for $t$ in the equation:

Perimeter of $\mathrm{PQRS}=t \times$ perimeter of $\mathrm{P}^{/ /} \mathrm{Q}^{\prime} \mathrm{R}^{/} \mathrm{S}^{/ /}$

## QUESTION 7




The STOP sign is a regular octagon. Consider $\mathrm{A}(8,42 ; 20,33)$ as a vertex on a STOP sign having its centre at the origin.

If the STOP sign is rotated about the origin in an anticlockwise direction such that A coincides with point $B$, determine the coordinates of $B$.

## QUESTION 8

8.1 If $\sin \mathrm{A}=\frac{3}{5}$ and $\cos \mathrm{A}<0$, determine, WITHOUT using a calculator, the value of:

$$
\begin{equation*}
\text { 8.1.1 } \sin (-A) \tag{2}
\end{equation*}
$$

$$
\text { 8.1.2 } \tan \mathrm{A}
$$

8.2 If $\cos 34^{\circ}=p$, WITHOUT using a calculator, write down the following in terms of $p$ :
8.2.1 $\quad \cos 214^{\circ}$
8.2.2 $\cos 68^{\circ}$
8.2.3 $\tan 56^{\circ}$
8.3 WITHOUT using a calculator, determine the value of the following expression:
$\cos 350^{\circ} \sin 40^{\circ}-\cos 440^{\circ} \cos 40^{\circ}$

## QUESTION 9

The graphs of $f(x)=\cos \left(x-45^{\circ}\right)$ and $g(x)=-2 \sin x$ are drawn below for $x \in\left[-180^{\circ} ; 180^{\circ}\right]$. The point T is an $x$-intercept of $f$ as indicated on the diagram.

9.1 Show that $\cos \left(x-45^{\circ}\right)=-2 \sin x$ can be written as $\tan x=-0,2612$.
9.2 Solve the equation: $\cos \left(x-45^{\circ}\right)=-2 \sin x$ for $x \in\left[-180^{\circ} ; 180^{\circ}\right]$.
9.3 Write down the coordinates of point T.
9.4 Write down the interval for which $f(x) \geq g(x)$.
9.5 Write down the interval for which both $f$ and $g$ are strictly increasing.
9.6 The graph $h$ is obtained when the graph $f$ is shifted $45^{\circ}$ to the right. Write down the equation of $h$ in its simplest form.

## QUESTION 10

In the diagram below, $R S$ is the height of a vertical tower. $T$ and $Q$ are two points in the same horizontal plane as the foot $S$ of the tower. From point $T$ the angle of elevation to the top of the tower is $60^{\circ} . \mathrm{RTQ}=\theta, \mathrm{R} \hat{\mathrm{Q}}=60^{\circ}$ and $\mathrm{TQ}=k$ metres.

10.1 Express TR in terms of $\theta$ and $k$.
10.2 Show that $\mathrm{RS}=\frac{3 k}{2(\sqrt{3} \cos \theta+\sin \theta)}$.

## QUESTION 11

11.1 Consider the function: $f(x)=3-2 \sin ^{2} x$
11.1.1 Determine the range of $f$.
11.1.2 For which value(s) $x, x \in\left[-180^{\circ} ; 180^{\circ}\right]$, will $f$ have a minimum value?
11.2 11.2.1 Show that $1-\cos 2 Q=2 \sin ^{2} Q$.
11.2.2 Given: $\quad \hat{\mathrm{P}}+\hat{\mathrm{Q}}+\hat{\mathrm{R}}=180^{\circ}$
(a) Show that $\sin 2 R=-\sin (2 P+2 Q)$.
(b) Hence, show that
$\sin 2 \mathrm{P}+\sin 2 \mathrm{Q}+\sin 2 \mathrm{R}=4 \sin \mathrm{P} \sin \mathrm{Q} \sin \mathrm{R}$.

## CENTRE NUMBER:

$\square$
EXAMINATION NUMBER:


## DIAGRAM SHEET 1

## QUESTION 1.4



## QUESTION 2.1

| Time (in minutes) | Frequency | Cumulative <br> frequency |
| :---: | :--- | :---: |
| $0<x \leq 10$ |  |  |
| $10<x \leq 20$ |  |  |
| $20<x \leq 30$ |  |  |
| $30<x \leq 40$ |  |  |
| $40<x \leq 50$ |  |  |
| $50<x \leq 60$ |  |  |

CENTRE NUMBER:


EXAMINATION NUMBER: $\square$

## DIAGRAM SHEET 2

## QUESTION 2.2



## CENTRE NUMBER:



EXAMINATION NUMBER:


## DIAGRAM SHEET 3

QUESTION 6.2.2(b)


## INFORMATION SHEET

$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

$$
(x-a)^{2}+(y-b)^{2}=r^{2}
$$

$$
\text { In } \triangle A B C: \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \quad a^{2}=b^{2}+c^{2}-2 b c \cdot \cos A \quad \text { area } \triangle A B C=\frac{1}{2} a b \cdot \sin C
$$

$\sin (\alpha+\beta)=\sin \alpha \cdot \cos \beta+\cos \alpha \cdot \sin \beta$

$$
\sin (\alpha-\beta)=\sin \alpha \cdot \cos \beta-\cos \alpha \cdot \sin \beta
$$

$\cos (\alpha+\beta)=\cos \alpha \cdot \cos \beta-\sin \alpha \cdot \sin \beta$ $\cos (\alpha-\beta)=\cos \alpha \cdot \cos \beta+\sin \alpha \cdot \sin \beta$
$\cos 2 \alpha=\left\{\begin{array}{l}\cos ^{2} \alpha-\sin ^{2} \alpha \\ 1-2 \sin ^{2} \alpha \\ 2 \cos ^{2} \alpha-1\end{array} \quad \sin 2 \alpha=2 \sin \alpha \cdot \cos \alpha\right.$
$(x ; y) \rightarrow(x \cos \theta-y \sin \theta ; y \cos \theta+x \sin \theta)$
$\bar{x}=\frac{\sum f x}{n}$
$\sigma^{2}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n}$
$P(A)=\frac{n(A)}{n(S)}$
$P(A$ or $B)=P(A)+P(B)-P(A$ and $B)$
$\hat{y}=a+b x$

$$
b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^{2}}
$$

$$
\begin{aligned}
& A=P(1+n i) \quad A=P(1-n i) \quad A=P(1-i)^{n} \quad A=P(1+i)^{n} \\
& \sum_{i=1}^{n} 1=n \quad \sum_{i=1}^{n} i=\frac{n(n+1)}{2} \quad T_{n}=a+(n-1) d \quad \mathrm{~S}_{n}=\frac{n}{2}(2 a+(n-1) d) \\
& T_{n}=a r^{n-1} \quad S_{n}=\frac{a\left(r^{n}-1\right)}{r-1} ; r \neq 1 \quad S_{\infty}=\frac{a}{1-r} ;-1<r<1 \\
& \begin{array}{l}
F=\frac{x\left[(1+i)^{n}-1\right]}{i} \\
f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}
\end{array} \\
& d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \quad \mathrm{M}\left(\frac{x_{1}+x_{2}}{2} ; \frac{y_{1}+y_{2}}{2}\right) \\
& y=m x+c \quad y-y_{1}=m\left(x-x_{1}\right) \quad m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \quad m=\tan \theta
\end{aligned}
$$

