basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P3

NOVEMBER 2012

MEMORANDUM

MARKS: 100

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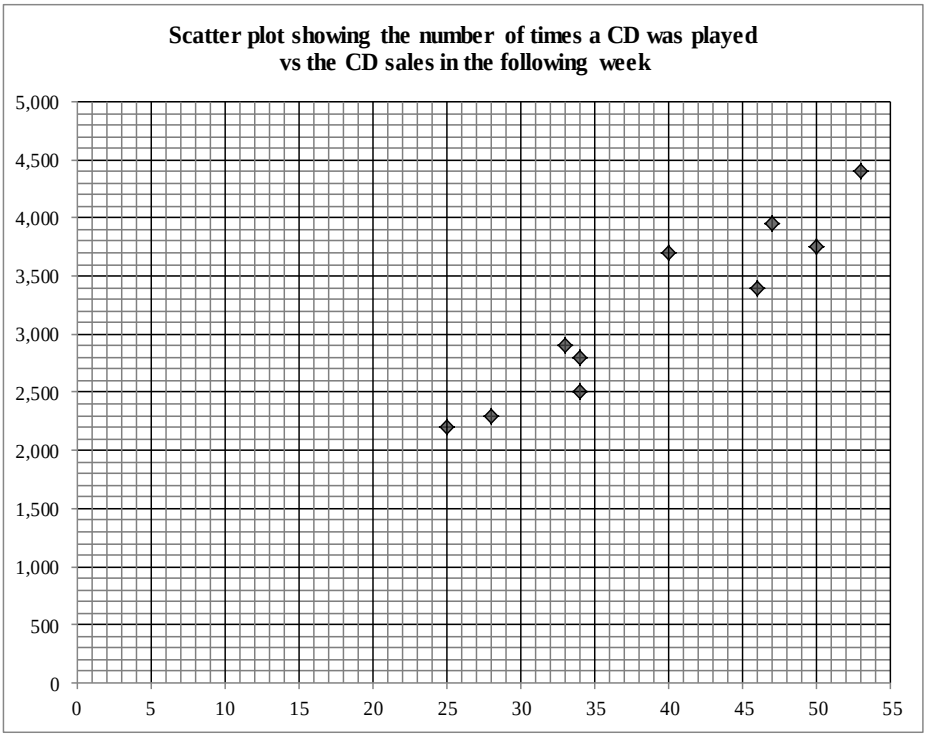
NOTE:

- If a candidate answered a question TWICE, mark the FIRST attempt ONLY.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed out question.
- Consistent accuracy applies in ALL aspects of the memorandum.

QUESTION 1

1.1	The number of times the CD was played. **Afrikaans:** Getalkerewat die CD gespeel is.	✓ answer (1)
1.2		✓✓✓ all 10 points plotted correctly **2 marks** if 5–9 points are plotted correctly **1 mark** if 1–4 points are plotted correctly. (3)
1.3	$a = 293{,}06$ (293,057554...) $b = 74{,}28$ (74,28057554...) $\hat{y} = 293{,}06 + 74{,}28x$	✓✓✓calculating a and b ✓ equation (4)
1.4	$r = 0{,}95$ (0,9458185...)	✓✓ answer (2)
1.5	$\hat{y} \approx 293{,}06 + 74{,}28(45)$ $\approx 3635{,}66$ ≈ 3635 ≈ 3650 (to the nearest 50) **Note:** Penalise 1 mark for incorrect rounding off.	✓ substitution ✓ answer (2)
1.6	There is a very **strong** positive relationship between the number of times that a CD was played and the sales of that CD in the following week.	✓strong (1) **[13]**

QUESTION 2

2.1

Yes. The events **Pass** and **Fail** are mutually exclusive.
It is not possible for pass and fail to take place at the same time. There is no intersection between the two sets.
P(Pass and Fail) = 0

Note:
If a candidate answers 'No' then award 0 marks

OR
P(Pass) = 0,59
P(Fail) = 0,41
P(Pass) + P(Fail)
= 0,59 + 0,41
=1
P(Pass and Fail) = 0 / No intersection of the sets
The events **Pass** and **Fail** are mutually exclusive.

Afrikaans
Ja. Die gebeurtenisse **Slaag** en **Druip** is onderling uitsluitend.
Dit is nie moontlik dat slaag en druip gelyktydig plaasvind nie. P(Slaag en Druip) = 0

✓Yes
✓ P(Pass and Fail) = 0 / no intersection between the sets. (2)

✓ Yes
✓P(Pass and Fail) = 0 / No intersection between the sets (2)

✓Ja
✓ P(Slaag en Druip) = 0 / geen snyding (2)

2.2

	PASS	FAIL	TOTAL
Males	46	32	78
Females	72	50	122
Total	118	82	200

$P(\text{Male}) = \frac{78}{200} = 0,39$

$P(\text{Pass}) = \frac{118}{200} = 0,59$

Note:
If a candidate answers 'No' then award 0 marks

$P(\text{Male and Pass}) = \frac{46}{200} = 0,23$

$P(\text{Male}) \times P(\text{Pass}) = 0,39 \times 0,59$
$= 0,23$ (0,2301)

$\therefore P(\text{Male}) \times P(\text{Pass}) = P(\text{Male and Pass})$
$\therefore$ Passing the competency test is independent of gender.

✓ $P(\text{Male}) = \frac{78}{200} = 0,39$ or $P(\text{Pass}) = \frac{118}{200} = 0,59$

✓ $P(\text{Male and Pass}) = 0,23$

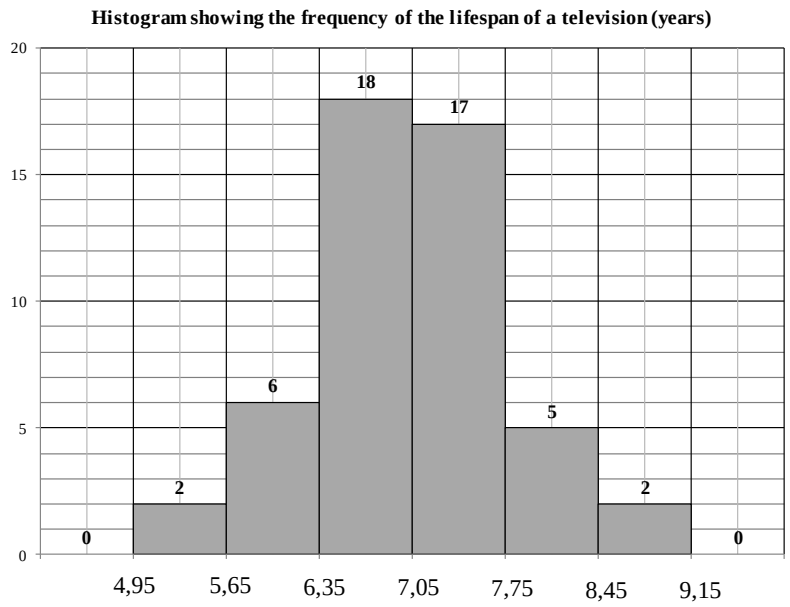
✓ $P(\text{Male}) \times P(\text{Pass}) = 0,23$

✓conclusion (4)

	OR $P(\text{Female}) = \frac{122}{200} = 0,61$ $P(\text{Pass}) = \frac{118}{200} = 0,59$ $P(\text{Female and Pass}) = \frac{72}{200} = 0,36$ $P(\text{Female}) \times P(\text{Pass}) = 0,61 \times 0,59$ $= 0,36$ (0,3599) $\therefore P(\text{Female}) \times P(\text{Pass}) = P(\text{Female and Pass})$ $\therefore$ Passing the competency test is independent of gender.	✓ $P(\text{Female}) = \frac{120}{200} = 0,61$ or $P(\text{Pass}) = \frac{118}{200} = 0,59$ ✓ $P(\text{Female and Pass}) = 0,36$ ✓ $P(\text{Female}) \times P(\text{Pass}) = 0,36$ ✓conclusion (4) **[6]**

QUESTION 3

3.1



✓ intervals
✓ 3 bars correct
✓ 6 bars correct
(3)

Note
If the candidate draws a bar graph, award max 2 marks

3.2

Lifespan (in years)	Frequency	Midpoint
$4,95 \leq x < 5,65$	2	5,3
$5,65 \leq x < 6,35$	6	6
$6,35 \leq x < 7,05$	18	6,7
$7,05 \leq x < 7,75$	17	7,4
$7,75 \leq x < 8,45$	5	8,1
$8,45 \leq x < 9,15$	2	8,8

Note:
If candidate works out average $(\bar{x})$ of midpoints, answer is 7,05 then 0 marks

$$\bar{x} = \frac{2 \times 5,3 + 6 \times 6 + 18 \times 6,7 + 17 \times 7,4 + 5 \times 8,1 + 2 \times 8,8}{50}$$
$$= \frac{351,1}{50}$$
$$= 7,02 \text{ years}$$
$$(\bar{x} = 7,022)$$

✓ frequencies × midpoints
✓ 50
✓ answer
(3)

3.3

The required area is 98% to the right of some value. This value is at 2 standard deviations on the left of the mean.

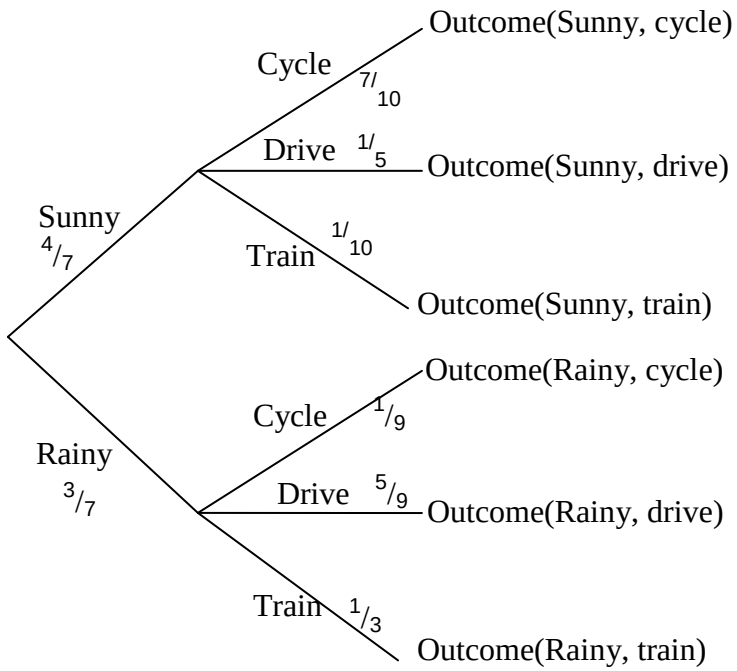
$\bar{x} - 2\sigma$
$= 7,02 - 2(0,76)$
$= 5,5$ years

✓ $\bar{x} - 2\sigma$
✓ $7,02 - 2(0,76)$
✓ answer
(3)

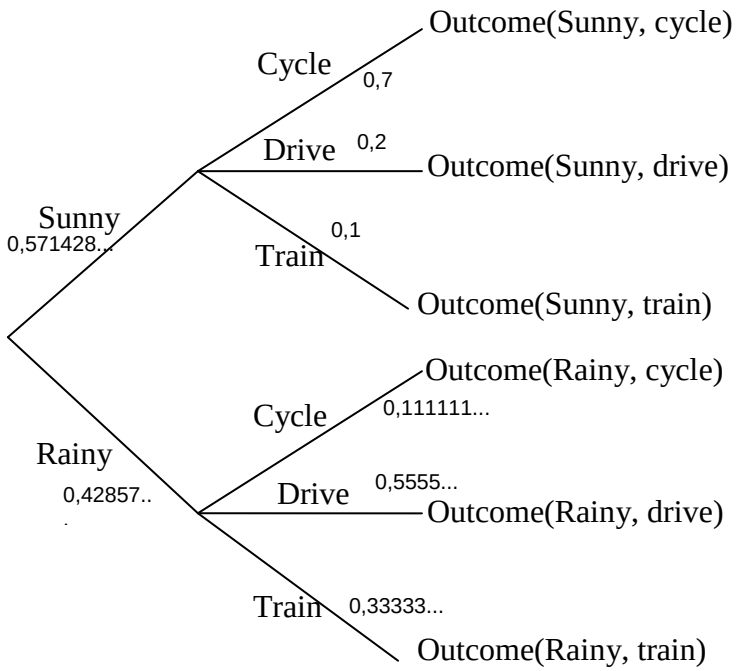
3.4	They can issue a 5-year guarantee. The average lifespan of a set is 7,02 years - which is in excess of 5 years. 98% of the sets lasted for more than 5,5 years. Very few sets have lasted less than 5 years. The number of sets of this brand that will be returnedshould be minimal if a 5-year guarantee is issued. **Afrikaans** Hullekan ‘n 5 jaar-waarborguitreik. Die gemiddelde lewens duur van 'n televisiestel is 7,02 jaar –wat 5 jaar oorskry. 98% van die stelle het langer as 5,5jaargehou. 'n Klein aantal stelle het vir minder as 5 jaar gehou. Die aantal stele wat terug geneem sal moet word sal minimal wees indien 'n 5 jaar-waarborg uitgereik word.	✓ Issue the 5-year guarantee ✓ reason (2) ✓kan ‘n 5 jaar-waarborg uitreik ✓rede (2) **[11]**

QUESTION 4

4.1



OR



✓Sunny branch
✓Rainybranch
✓ cycle, drive, train branches on both weather types
✓ probabilities listed
✓ outcomes listed

(5)

4.2.1	P(Rainy, Cycle) $=\frac{3}{7}\times\frac{1}{9}$ $=\frac{1}{21}$ **OR** P(Rainy, Cycle) $=0,428...\times 0,1111...$ $=0,04761904762$ $\approx 0,05$ or 4,76% **Note:** If $\frac{3}{7}+\frac{1}{9}$ then 0 marks	✓$\frac{3}{7}\times\frac{1}{9}$ ✓ answerin any form (must be from multiplication) (2)
4.2.2	P(Train) $=\frac{4}{7}\times 0,1+\frac{3}{7}\times\frac{1}{3}$ $=\frac{1}{5}$ $=0,2$ $=20\%$ **OR** P(Train) $=\frac{4}{7}\times 0,1+\frac{3}{7}\times\frac{1}{3}$ $=0,05714...+0,1428...$ $=\frac{1}{5}$ $=0,2$ $=20\%$	✓$\frac{4}{7}\times 0,1$ and $\frac{3}{7}\times\frac{1}{3}$ ✓ addition ✓ answer (in any form) (3)
4.3	P(Drive) $=\frac{4}{7}\times 0,2+\frac{3}{7}\times\frac{5}{9}$ $=\frac{37}{105}$ $=0,35238...$ Vusi drives for $\frac{37}{105}\times 245$ = 87 days (86,333...) Accept: 86 days **OR** P(Drive) $=\frac{4}{7}\times 0,2\times 245+\frac{3}{7}\times\frac{5}{9}\times 245$ $=28+58,333$ = 87 days (86,333...) Accept: 86 days	✓$\frac{4}{7}\times 0,2$ and $\frac{3}{7}\times\frac{5}{9}$ ✓ addition ✓$\frac{37}{105}$ ✓ answer (4) ✓$\frac{4}{7}\times 0,2$ and $\frac{3}{7}\times\frac{5}{9}$ ✓ addition ✓$28+58,333$ ✓ answer (4) **[14]**

QUESTION 5

5.1.1	Number of PIN codes $= 10 \times 10 \times 10 \times 10 \times 10$ $= 10^5$ $= 100\ 000$	✓10 ✓ answer (2)
5.1.2	Number of PIN codes $= 10 \times 9 \times 8 \times 7 \times 6$ $= 30\ 240$ **OR** Number of PIN codes $= \frac{10!}{5!}$ $= 30\ 240$	✓ multiplication ✓ answer (2) ✓$\frac{10!}{5!}$ ✓ answer (2)
5.2	Number of PINs that DO NOT contain 9s $= 9 \times 9 \times 9 \times 9 \times 9$ $= 59\ 049$ P(at least one 9) $= 1 - P(\text{no } 9s)$ $= 1 - \frac{59049}{100000}$ $= 0{,}41$ **OR** Number of PINs that DO NOT contain 9s $= 9 \times 9 \times 9 \times 9 \times 9$ $= 59\ 049$ Number of PINs that contain AT LEAST one 9 $= 100\ 000 - 59\ 049$ $= 40\ 951$ P(at least one 9) $= \frac{40951}{100000}$ $= 0{,}41$	✓ 9 ✓ 59 049 ✓$1 - \frac{59049}{100000}$ ✓ answer (4) ✓ 9 ✓ 59 049 ✓ 40951 ✓ answer (4) **[8]**

QUESTION 6

6.1

$T_{k+1} = 2T_k + 3$ where $T_1 = 1,\ k \geq 1$

✓✓
$T_{k+1} = 2T_k + 3$
✓$T_1 = 1$
✓$k \geq 1$
(4)

OR

$T_{k+1} = T_k + 2^{k+1}$ where $T_1 = 1,\ k \geq 1$

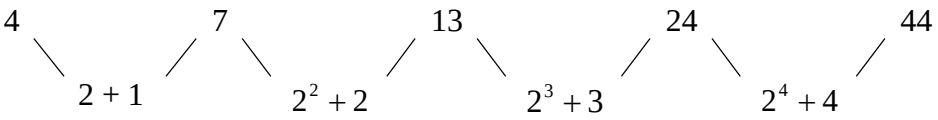
✓✓
$T_{k+1} = T_k + 2^{k+1}$
✓$T_1 = 1$
✓$k \geq 1$
(4)

OR

$T_{k+2} = 2(T_{k+1} - T_k) + T_{k+1}$ where $T_1 = 1,\ T_2 = 5, k \geq 1$

✓✓
$T_{k+2} = 2(T_{k+1} - T_k) + T_{k+1}$
✓$T_1 = 1\ T_2 = 5$
✓$k \geq 1$
(4)

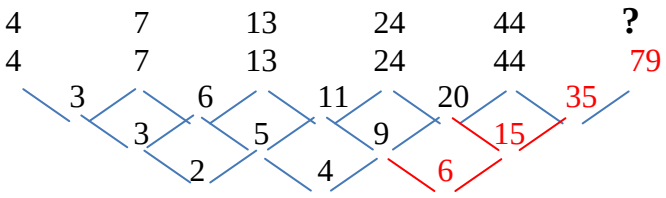
6.2



The next term of the sequence is
$44 + 2^5 + 5$
$= 81$

✓✓answer

OR



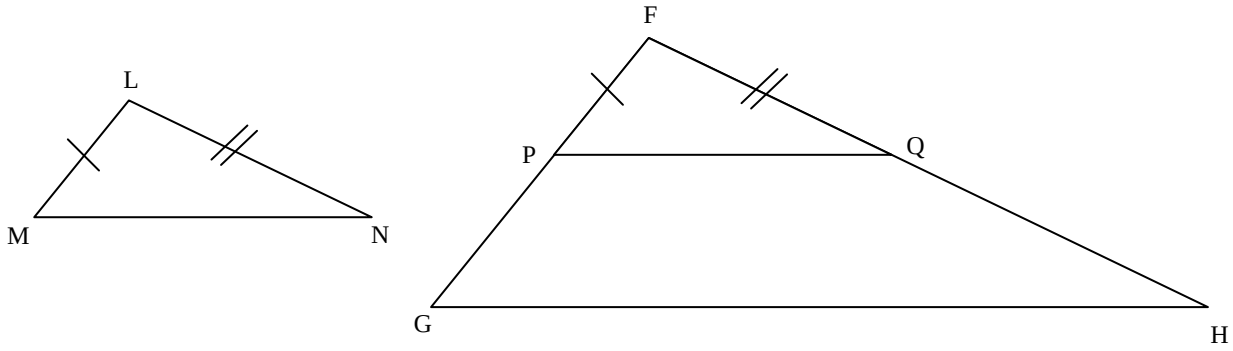
The next term of the sequence is 79.

✓✓answer

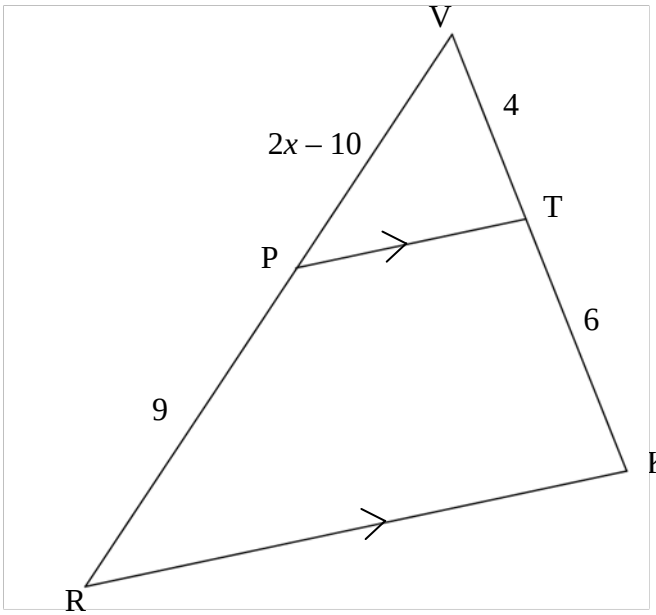
(2)
[6]

Note:
This sequence can be represented by the following recursive formula:
$T_{n+1} = T_n + \frac{1}{3}n^3 - n^2 + \frac{11}{3}n$ where $T_1 = 4$ and $n \geq 1$

QUESTION 7

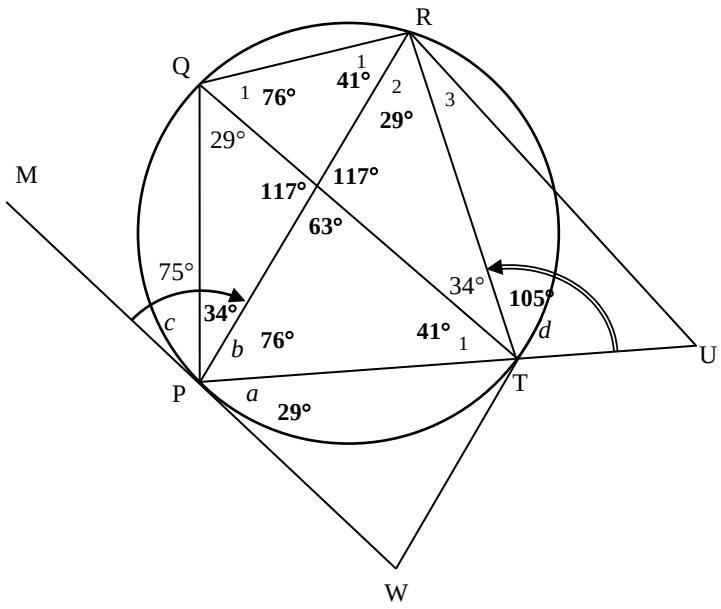


7.1	Draw a point P on FG such that FP = LM and a point Q on FH such that FQ = LN. **Note**: No construction constitutes a breakdown, hence no marks In ΔFPQ and ΔLMN 1. $\hat{F} = \hat{L}$ (given) 2. FP = LM (construction) 3. FQ = LN (construction) $\therefore \Delta FPQ \equiv \Delta LMN$ (SAS) $F\hat{P}Q = L\hat{M}N$ (≡Δs) But $F\hat{G}H = L\hat{M}N$ (given) $F\hat{P}Q = F\hat{G}H$ PQ ‖ GH (corresponding angles =) $\frac{FP}{FG} = \frac{FQ}{FH}$ (PQ ‖ GH ; Prop Th) $\frac{LM}{FG} = \frac{LN}{FH}$	✓ construction ✓All three statements must be given ✓ΔFPQ ≡ΔLMN (SAS) ✓ $F\hat{P}Q = L\hat{M}N$ ✓ $F\hat{P}Q = F\hat{G}H$ ✓ PQ ‖ GH ✓ $\frac{FP}{FG} = \frac{FQ}{FH}$ (7)

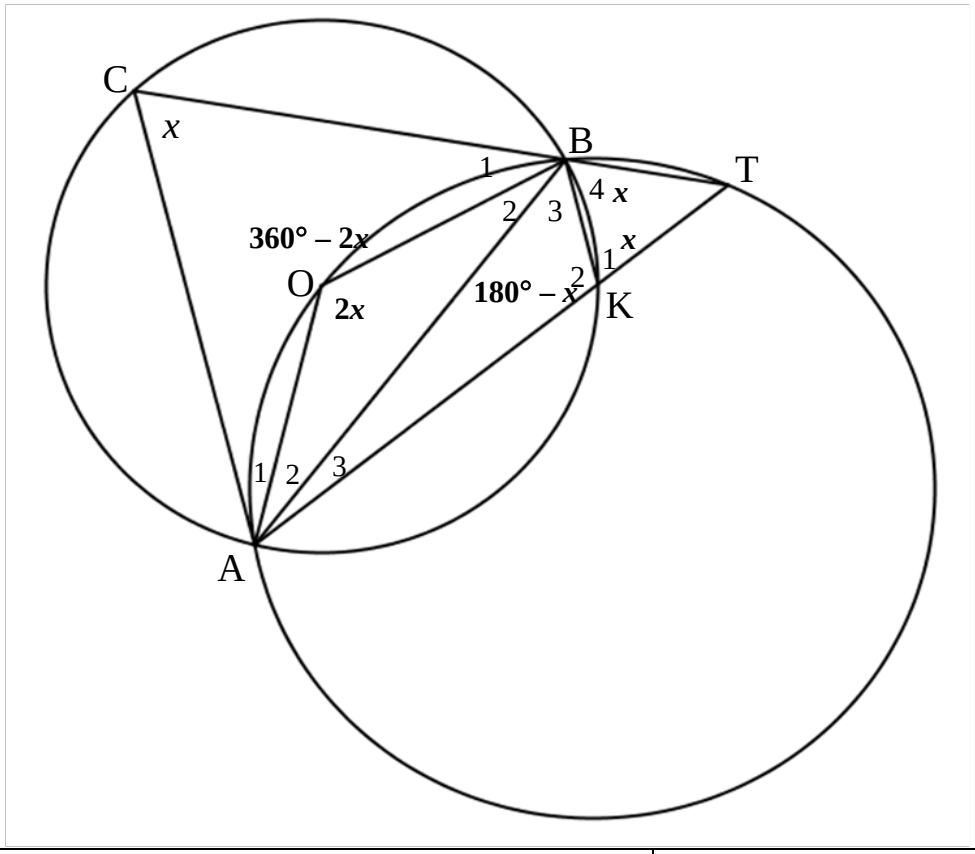


7.2	$\frac{VP}{PR}=\frac{VT}{TK}$ (PT || RK;Prop Th) $\frac{2x-10}{9}=\frac{4}{6}$ $2x-10=6$ $2x=16$ $x=8$ **OR** $\frac{VP}{VR}=\frac{VT}{VK}$ (PT || RK; Prop Th) $\frac{2x-10}{2x-1}=\frac{4}{10}$ $20x-100=8x-4$ $12x=96$ $x=8$	✓ $\frac{VP}{PR}=\frac{VT}{TK}$ ✓ (PT || RK; Prop Th) ✓ substitution ✓ answer (4) ✓ $\frac{VP}{VR}=\frac{VT}{VK}$ ✓ (PT || RK; Prop Th) ✓ substitution ✓ answer (4) **[11]**

QUESTION 8

8.1	... equal to the angle subtended by the chord in the alternate segment.	✓ answer (1)
8.2		✓ $a = 29°$ ✓ tan ch. thm ✓ $\hat{QPR} = 34°$ ✓ ∠s in same seg ✓ $c = 41°$ ✓ $b = 76°$ ✓ $\hat{Q}_1 = 76°$ ✓ $d = 105°$ ✓ ext∠ cyclic quad (9)
	$a = 29°$ (tan ch.thm) $\hat{QPR} = 34°$ (∠s in same seg) $c = 41°$ $b = 76°$ (adj∠s on str. line) $\hat{Q}_1 = 76°$ (∠s in same seg) $d = 105°$ (ext∠ cyclic quad)	
	OR $a = 29°$ (tan ch. thm) $\hat{T}_1 = c$ (tan ch. thm) $c + 34° = 75°$ (tan ch. thm) $c = 41°$ $b = 76°$ (adj∠s on str. line) $d = 105°$ (adj∠s on str. line)	✓ $a = 29°$ ✓ tan ch. thm ✓ $\hat{T}_1 = c$ ✓ tan ch. thm ✓ $c + 34° = 75°$ ✓ tan ch. thm ✓ $c = 41°$ ✓ $b = 76°$ ✓ $d = 105°$ (9)
	OR An alternative solution for calculating d: $\hat{Q}_1 = R\hat{P}T = 76°$ (∠s in same seg) $d + R\hat{T}Q = P\hat{Q}T + Q\hat{P}R + R\hat{P}T$ (ext∠Δ) $d + 34° = 29° + 34° + 76°$ $d = 105°$	**[10]**

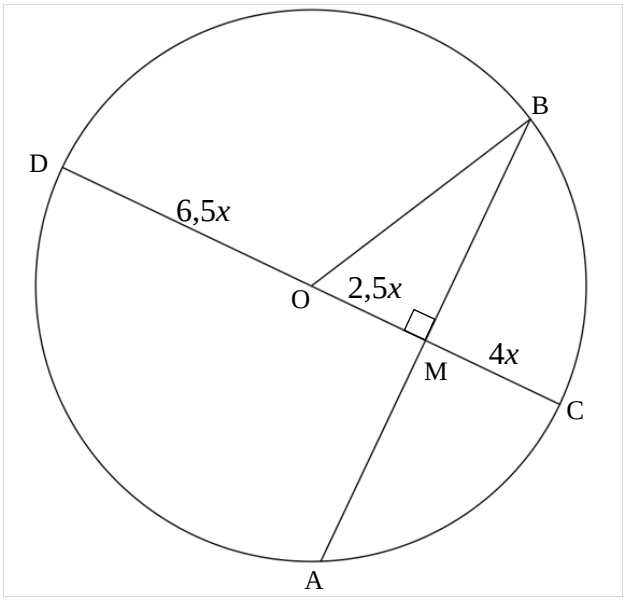
QUESTION 9



9.1	$A\hat{O}B = 2x$ (∠circ centre = 2 ∠ circumference) $\hat{T} = 180° - 2x$ (opp∠ cyclic quad suppl)	✓ $A\hat{O}B = 2x$ ✓∠circ centre = 2 ∠ circumference ✓opp∠ cyclic quad suppl (3)
9.2	$C\hat{A}T = x$ (∠ sum Δ) $\hat{K}_1 = x$ (ext∠ cyclic quad) $C\hat{A}T = \hat{K}_1$ BK || AC (corresponding ∠s =) **OR** $\hat{K}_1 = \hat{C} = x$ (ext∠ cyclic quad) $\hat{B}_4 = x$ (∠ sum Δ) $\hat{B}_4 = \hat{C} = x$ BK || CA (corresponding ∠s =) **OR** $C\hat{A}T = x$ (∠ sum Δ) $B\hat{K}A = 180° - x$ (opp∠ cyclic quad) $C\hat{A}T + B\hat{K}A = 180°$ BK || AC (coint∠s supp)	✓ $C\hat{A}T = x$ ✓∠ sum Δ ✓ $\hat{K}_1 = x$ ✓ext∠ cyclic quad ✓ corresponding ∠s = (5) ✓ $\hat{K}_1 = \hat{C} = x$ ✓ext∠ cyclic quad ✓ $\hat{B}_4 = x$ ✓∠ sum Δ ✓ corresponding ∠s = (5) ✓ $C\hat{A}T = x$ ✓∠ sum Δ ✓ $B\hat{K}A = 180° - x$ ✓opp∠ cyclic quad ✓co-int∠s supp (5)

9.3	In ΔBKT and ΔCAT 1. $C\hat{A}T = \hat{K}_1$ $(= x)$ 2. $\hat{T}$ is common 3. $A\hat{C}T = \hat{B}_4$ $(\angle \text{ sum } \Delta)$ $\Delta BKT \mid\mid\mid \Delta CAT$ $(\angle\angle\angle)$	✓ $C\hat{A}T = \hat{K}_1$ ✓ $\hat{T}$ is common ✓ $\angle\angle\angle$ (3)
9.4	$\frac{AC}{KB} = \frac{AT}{KT}$ $(\mid\mid\mid \Delta s)$ $\frac{AC}{KB} = \frac{7}{2}$	✓ $\frac{AC}{KB} = \frac{AT}{KT}$ ✓ $\mid\mid\mid \Delta s$ ✓ answer (3) **[14]**

QUESTION 10



10.1	DC = $13x$	✓ CD = $13\,x$ (1)
10.2	OD = $\frac{13}{2}x$ OM = $\frac{5}{2}x$	✓ OD = $\frac{13}{2}x$ ✓ answer (2)
10.3	BO = OD (radii) AM = MB = 12 units (line from circ cent ⊥ch) $12^2+\left(\frac{5}{2}x\right)^2=\left(\frac{13}{2}x\right)^2$ (Pythagoras) $144+\frac{25x^2}{4}=\frac{169x^2}{4}$ $144=\frac{144x^2}{4}$ $x^2=4$ $x=\pm 2$ $x=2$ $(x>0)$ The radius = $\frac{13}{2}(2)$ = 13 units.	✓MB = 12 ✓$12^2+\left(\frac{5}{2}x\right)^2=\left(\frac{13}{2}x\right)^2$ or $12^2+6{,}25x^2=42{,}25x^2$ or $12^2+\frac{25}{4}x^2=\frac{169}{4}x^2$ ✓ answer ✓ answer (4) **[7]**

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