basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2012

MEMORANDUM

MARKS: 150

This memorandum consists of 29 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in **ALL** aspects of the marking memorandum unless indicated otherwise

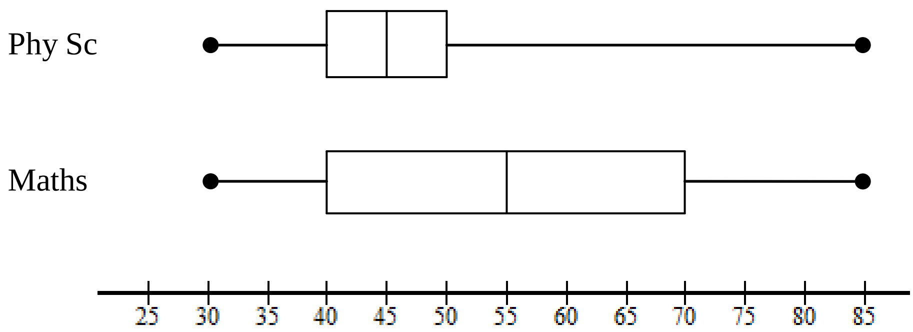
QUESTION 1

1.1	Approximately 121cm (Accept 120 – 122)	✓answer (1)
1.2	As the age increases, the height increases **OR** Every year the height increases by approximately 6,2 cm **OR** Straight line (linear) with a positive gradient **OR** Strong positive correlation **OR** Increase in height: increase in age is a constant	✓ description (1)
1.3	Approximate increase in average height $= \frac{169-88}{15-2}$ $= 6{,}23$ Range for numerator (87 – 89 ; 167 – 170) (Accept any answer between 6 and 6,4 cm)	✓ reading off from graph ✓ numerator ✓answer (3)
1.4	Children stop growing when they reach adulthood. **OR** If the trend continues the boys would reach impossible heights **OR** The trend will start approaching a constant value. **OR** People cannot grow indefinitely	✓comment (1) **[6]**

QUESTION 2

2.1	Average number of runs $\bar{x} = \frac{\sum x}{n} = \frac{128}{8} = 16$	✓ 128 ✓16 (2)
2.2	Standard deviation = 7,55 **NOTE**: Penalty of 1 mark for incorrect rounding off	✓✓ 7,55 (2)
2.3	Standard deviation = 9,71 Standard deviation increases. **OR** 2 and 35 are far from the mean, namely 16. Since the standard deviation depends on how far data points are from the mean, the standard deviation would be expected to increase.	✓ 9,71 ✓ increases (2) ✓2 and 35 far from mean ✓increase (2)
2.4	Total number of runs required is 20 x 16 = 320 Total number of runs to be scored in last five games = 320 – 59 – 128 = 133 Average number of runs for last five games is $\frac{133}{5} = 26{,}6$ **OR** $\frac{128 + 59 + x}{16} = 20$ $187 + x = 320$ $\therefore x = 133$ $\therefore \frac{133}{5} = 26{,}6$ **OR** $\frac{128 + 59 + 5x}{16} = 20$ $5x = 133$ $\therefore x = 26{,}6$	✓ 320 ✓ 133 ✓ 26,6 (3) ✓ 320 ✓ 133 ✓ 26,6 (3) ✓ 320 ✓ 133 ✓ 26,6 (3) **[9]**

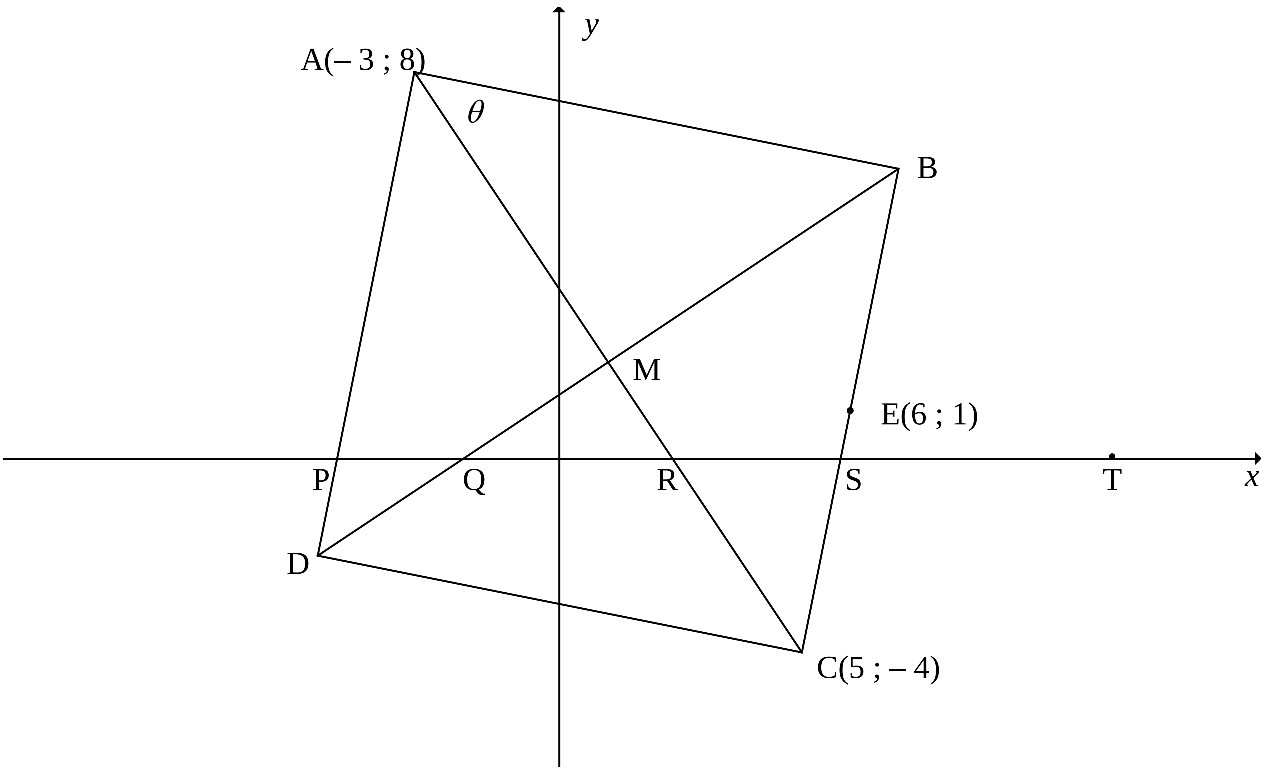
QUESTION 3

3.1	Range = 85 – 30 = 55	✓ 55 (1)
3.2		✓ max 85 ✓ $Q_3 = 70$ ✓ $Q_1 = 40$ ✓ Median = 55 (4)
3.3	From the information given for Mathematics, the value of the third quartile is 70%. Therefore 75% of learners got below 70%. Number of learners below 70% is expected to be $\frac{75}{100} \times 60 = \frac{3}{4} \times 60 = 45$ learners	✓ 75% of learners ✓ 45 learners (2)
3.4	No, Joe's claim is invalid. 50% of the learners scored between 30% and 45% in Physical Sciences. 50% of the learners scored between 30% and 55% in Mathematics. Therefore the numbers will be equal. **OR** No, Joe's claim is invalid. Same number of learners (between min and median)	✓ invalid/no ✓ median represents 50% of learners (2) **[9]**

QUESTION 4

4.1	Modal class is $50 \leq x < 60$ **OR** $50 < x \leq 60$ **OR** 50 to 60	✓Correct class (1)
4.2	Median position is 15 learners (grouped data). Approximate weight is about 53 kg. (Accept from 52 kg to 54 kg)	✓ 53 kg (1)
4.3	30 – 23 = 7 learners collected more than 60 kg.	✓ ✓ 7 learners (2) **[4]**

QUESTION 5



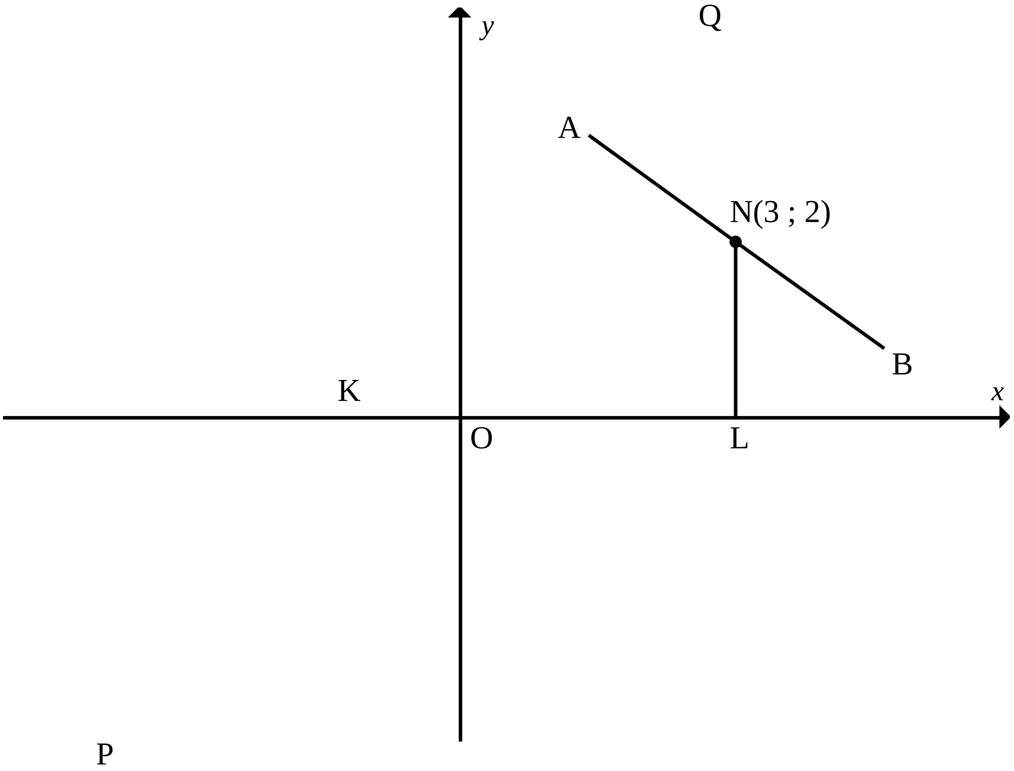
5.1	Diagonals bisect each other at M: $x_M = \frac{-3+5}{2} = 1$; $y_M = \frac{8+(-4)}{2} = 2$ M(1 ; 2)	✓ $x_M = 1$ ✓ $y_M = 2$ (2)
5.2	$m_{BC} = \frac{1+4}{6-5}$ $m_{BC} = 5$ **OR** $m_{BC} = \frac{-4-1}{5-6}$ $m_{BC} = 5$	✓ substitution into gradient formula ✓ 5 (2) ✓ $m_{BC} = \frac{-4-1}{5-6}$ ✓ 5 (2)
5.3	$y - y_1 = m(x - x_1)$ $y - 8 = m(x+3)$ $m_{AD} = m_{BC} = 5$ Lines parallel $y - 8 = 5(x+3)$ $y = 5x + 23$ **OR**	✓ substitute (–3 ; 8) ✓ gradients equal ✓ equation (3)

	$m_{AD} = m_{BC}$ Lines parallel $m_{AD} = 5$ $y = 5x + c$ $8 = 5(-3) + c$ $c = 23$ $y = 5x + 23$	✓ gradients equal ✓ substitute (–3 ; 8) ✓ equation (3)
5.4	ABCD is a rhombus, therefore AB = BC $\theta = B\hat{C}A = A\hat{R}S - R\hat{S}C$ $= A\hat{R}S - B\hat{S}T$ $\tan A\hat{R}S = m_{AC} = \frac{8+4}{-3-5}$ $\tan A\hat{R}S = -\frac{3}{2}$ $A\hat{R}S = 180° - 56,3099...$ $A\hat{R}S = 123,69°$ $\tan B\hat{S}T = m_{BC} = 5$ $B\hat{S}T = 78,69°$ $\theta = B\hat{C}A = 123,69° - 78,69°$ $\theta = 45°$ **OR** $\tan A\hat{R}S = m_{AC} = \frac{8+4}{-3-5} = -\frac{3}{2}$ $A\hat{R}S = 123,69°$ $\tan A\hat{P}R = m_{AD} = 5$ $A\hat{P}R = 78,69°$ $P\hat{A}R = A\hat{R}S - A\hat{P}R$ Exterior angle of a triangle $= 123,69° - 78,69°$ $= 45°$ $\theta = P\hat{A}R$ Diagonals of the rhombus bisect opposite angles $= 45°$	✓ $\theta = B\hat{C}A$ ✓ $\tan A\hat{R}S = -\frac{3}{2}$ ✓ 123,69° ✓ $\tan B\hat{S}T = m_{BC} = 5$ ✓ 78,69° ✓ $\theta = 45°$ (6) ✓ $\tan A\hat{R}S = -\frac{3}{2}$ ✓ 123,69° ✓ $\tan A\hat{P}R = m_{AD} = 5$ ✓ 78,69° ✓ $P\hat{A}R = 45°$ ✓ $\theta = 45°$ (6)

	OR $\tan A\hat{R}S = m_{AC} = \frac{8+4}{-3-5} = -\frac{3}{2}$ $A\hat{R}S = 123{,}69°$ $\tan A\hat{P}R = 5$ $A\hat{P}R = 78{,}69°$ $\theta = P\hat{A}R$ Diagonals of the rhombus bisect opposite angles $\theta = A\hat{R}S - A\hat{P}R$ Exterior angle of a triangle $\theta = 123{,}69° - 78{,}69°$ $\theta = 45°$	✓ $\tan A\hat{R}S = -\frac{3}{2}$ ✓ $123{,}69°$ ✓ $\tan A\hat{P}R = m_{AD} = 5$ ✓ $78{,}69°$ ✓ $\theta = P\hat{A}R$ ✓ $\theta = 45°$ (6)
	OR $\tan A\hat{R}S = m_{AC} = \frac{8+4}{-3-5} = -\frac{3}{2}$ $A\hat{R}S = 123{,}69°$ $\tan B\hat{S}T = 5$ $B\hat{S}T = 78{,}69°$ $\theta = R\hat{C}S$ BA=BC $R\hat{C}S + B\hat{S}T = R\hat{C}S + R\hat{S}C$ $= A\hat{R}S$ $\theta = A\hat{R}S - B\hat{S}T$ $= 123{,}69° - 78{,}69°$ $= 45°$	✓ $\tan A\hat{R}S = -\frac{3}{2}$ ✓ $123{,}69°$ ✓ $\tan B\hat{S}T = 5$ ✓ $78{,}69°$ ✓ $\theta = R\hat{C}S$ ✓ $\theta = 45°$ (6)
	OR ABCD is a rhombus, therefore AB = BC $\therefore A\hat{C}B = B\hat{A}C$ $\tan\theta = \tan A\hat{C}B$ $= \tan(A\hat{R}S - B\hat{S}T)$ $= \frac{\tan A\hat{R}S - \tan B\hat{S}T}{1 + \tan A\hat{R}S . \tan B\hat{S}T}$ $= \frac{\left(\frac{12}{-8}\right) - \left(\frac{-5}{-1}\right)}{1 + \left(\frac{12}{8}\right)\left(\frac{5}{1}\right)}$ $= 1$ $\theta = 45°$	✓ $A\hat{C}B = B\hat{A}C$ ✓ $\tan\theta = \tan A\hat{C}B$ ✓ formula ✓ substitution ✓ $\tan\theta = 1$ ✓ $\theta = 45°$ (6)

	OR From 5.1, M has coordinates (1 ; 2) Join ME $m_{ME} = \frac{2-1}{1-6} = -\frac{1}{5}$ From 5.2, $m_{BC} = 5$ $\therefore m_{ME} \times .m_{BC} = -1$ $\therefore M\hat{E}C = 90°$ $ME = \sqrt{(1-6)^2 + (2-1)^2} = \sqrt{26}$ $EC = \sqrt{(5-6)^2 + (-4-1)^2} = \sqrt{26}$ $\therefore$ MEC is a right-angled triangle. $E\hat{C}M = 45°$ ABCD is a rhombus, therefore AB = BC $\therefore \theta = B\hat{C}M = 45°$	✓gradient of ME ✓gradient of BC ✓ $M\hat{E}C = 90°$ ✓ $ME = \sqrt{26}$ ✓ $EC = \sqrt{26}$ ✓ $E\hat{C}M = 45°$ (6)
	OR $AM = \sqrt{(-3-1)^2 + (8-2)^2} = 2\sqrt{13}$ Now to calculate the coordinates of B: $m_{AC} = \frac{8+4}{-3-5} = -\frac{3}{2}$ $m_{BD} \times m_{AC} = -1$ diagonals bisect at right angles $m_{BD} = \frac{2}{3}$ Equation of BD is $y = \frac{2}{3}x + \frac{4}{3}$ Equation of BC is $y = 5x - 29$ BD and BC intersect at B. Solve equations simultaneously to get B(7 ; 6). $BM = \sqrt{(7-1)^2 + (6-2)^2} = \sqrt{52} = 2\sqrt{13}$ $\therefore BM = AM$ Since $A\hat{M}B = 90°$ $\tan\theta = \frac{BM}{AM}$ $\therefore \tan\theta = 1$ $\theta = 45°$	✓ $AM = 2\sqrt{13}$ ✓ $y = \frac{2}{3}x + \frac{4}{3}$ ✓ $y = 5x - 29$ ✓ B(7 ; 6) ✓ $BM = 2\sqrt{13}$ ✓ 45° (6) **[13]**

QUESTION 6



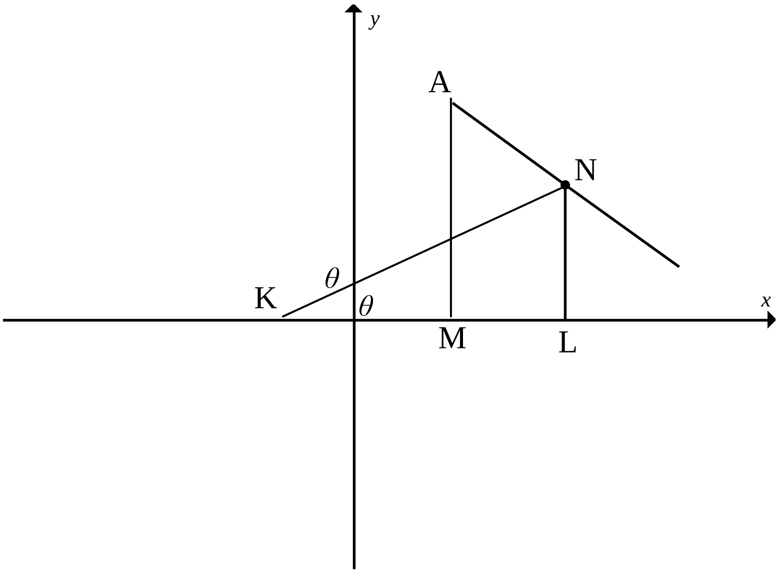
6.1	The radius (NL) of a circle is perpendicular to the tangent (OL) at the point of contact.	✓ radius ⊥ tangent (1)
6.2	L(3 ; 0)	✓ (3 ; 0) (1)
6.3	Centre N (3 ; 2) and r = NL = 2 Equation of the circle N: $(x-a)^2+(y-b)^2=r^2$ $(x-3)^2+(y-2)^2=4$	✓r = 2 ✓ $(x-3)^2+(y-2)^2$ ✓ 4 (3)
6.4	Coordinates of K. K is the x-intercept of the tangent. $y=\frac{4}{3}x+\frac{4}{3}$ $0=\frac{4}{3}x+\frac{4}{3}$ $0=4x+4$ $4x=-4$ $x=-1$ $K(-1;0)$ KL = 3 – (–1) **OR** KL = 3 + 1 KL = 4	✓ substitute y = 0 into equation of tangent ✓ x = – 1 ✓ KL = 4 (3)

	OR $y = \frac{4}{3}x + \frac{4}{3}$ $0 = \frac{4}{3}x + \frac{4}{3}$ $0 = 4x + 4$ $4x = -4$ $x = -1$ $K(-1;0)$ $KL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $KL = \sqrt{(3+1)^2 + (0-0)^2}$ $KL = \sqrt{16}$ $KL = 4$	✓ substitute $y = 0$ into equation of tangent ✓ $x = -1$ ✓ $KL = 4$ (3)
	OR For AK, $m = \frac{4}{3}$, $c = \frac{4}{3}$ $\frac{\frac{4}{3}}{OK} = \tan A\hat{K}O = \frac{4}{3}$ $OK = 1$ $\therefore KL = 4$	✓ $\frac{\frac{4}{3}}{OK} = \frac{4}{3}$ ✓ OK = 1 ✓ $KL = 4$ (3)
	OR $y = \frac{4}{3}x + \frac{4}{3}$ $0 = \frac{4}{3}x + \frac{4}{3}$ $0 = 4x + 4$ $4x = -4$ $x = -1$ $K(-1;0)$ $KN^2 = NL^2 + KL^2$ Theorem of Pythagoras $(-1-3)^2 + (0-2)^2 = 4 + KL^2$ $20 = 4 + KL^2$ $16 = KL^2$ $KL = 4$	✓ $x = -1$ ✓ $KN^2 = NL^2 + KL^2$ ✓ $KL = 4$ (3)

6.5	$m_{AB} \times m_{AK} = -1$ tangent ⊥ radius $m_{AK} = \frac{4}{3}$ $\therefore m_{AB} = -\frac{3}{4}$ $y - y_1 = m(x - x_1)$ $y - 2 = -\frac{3}{4}(x - 3)$ $y = -\frac{3}{4}x + \frac{9}{4} + \frac{8}{4}$ $y = -\frac{3}{4}x + \frac{17}{4}$		✓ $m_{AK} = \frac{4}{3}$ ✓ $m_{AB} = -\frac{3}{4}$ ✓ substitution of point (3;2) into equation ✓ equation (4)
	OR		
	$m_{AB} \times m_{AK} = -1$ tangent ⊥ radius $m_{AK} = \frac{4}{3}$ $\therefore m_{AB} = -\frac{3}{4}$ $y = -\frac{3}{4}x + c$ $2 = \left(-\frac{3}{4}\right)(3) + c$ $c = \frac{8}{4} + \frac{9}{4}$ $c = \frac{17}{4}$ $y = -\frac{3}{4}x + \frac{17}{4}$		✓ $m_{AK} = \frac{4}{3}$ ✓ $m_{AB} = -\frac{3}{4}$ ✓ substitution of point (3;2) into equation ✓ equation (4)

6.6	Point A lies on PQ and AB. Therefore $\frac{4}{3}x+\frac{4}{3}=-\frac{3}{4}x+\frac{17}{4}$ $16x+16=-9x+51$ $25x = 35$ $x = \frac{7}{5}$ $y = -\frac{3}{4}\left(\frac{7}{5}\right)+\frac{17}{4}$ $y = \frac{16}{5}$ $A\left(\frac{7}{5}\,;\,\frac{16}{5}\right)$ **OR** Point A lies on PQ and the circle. Therefore $(x-3)^2+(\frac{4}{3}x+\frac{4}{3}-2)^2=4$ $(x-3)^2+(\frac{4}{3}x-\frac{2}{3})^2=4$ $25x^2-70x+49=0$ $(5x-7)^2=0$ $x=\frac{7}{5}$ $y = -\frac{3}{4}\left(\frac{7}{5}\right)+\frac{17}{4}$ $y = \frac{16}{5}$ **OR**	✓ equation ✓ $25x = 35$ ✓ substitution of x (3) ✓ equation ✓ $(5x-7)^2 = 0$ ✓ substitution of x (3)

	Point A lies on the circle and line AB $(x-3)^2+(y-2)^2=4$ ---------- (1) $y=-\frac{3}{4}x+\frac{17}{4}$ ---------- (2) Subs (2) in (1) : $x^2-6x+9+(-\frac{3}{4}x+\frac{17}{4}-2)^2=4$ $x^2-6x+9+(-\frac{3}{4}x+\frac{9}{4})^2=4$ $25x^2-150x+161=0$ $(5x-23)(5x-7)=0$ $x=\frac{7}{5}$ $y=-\frac{3}{4}\left(\frac{7}{5}\right)+\frac{17}{4}$ $y=\frac{16}{5}$	✓ equation ✓ $(5x-23)(5x-7)=0$ ✓substitution of x (3)
	OR Using rotation: Let $\theta=A\hat{K}N=L\hat{K}N$ Move diagram 1 unit to the right. Then $A^{/}$ is $L^{/}$ rotated through 2θ . $\tan\theta=\frac{AN}{KA}=\frac{2}{4}=\frac{1}{2}$ $\therefore \sin 2\theta=2\sin\theta\cos\theta=2(\frac{1}{\sqrt{5}})(\frac{2}{\sqrt{5}})=\frac{4}{5}$ $\cos 2\theta=\cos^2\theta-\sin^2\theta=(\frac{2}{\sqrt{5}})^2-(\frac{1}{\sqrt{5}})^2=\frac{3}{5}$ $\therefore x_{A''}=x_{L'}\cos 2\theta-y_{L'}\sin 2\theta=4(\frac{3}{5})-(0)(\frac{4}{5})=\frac{12}{5}$ $y_{A''}=x_{L'}\sin 2\theta+y_{L'}\cos 2\theta=4(\frac{4}{5})-(0)(\frac{3}{5})=\frac{16}{5}$ $A'(\frac{12}{5}\ ;\ \frac{16}{5})$ Now to get back to A, move back 1 unit to the left. $\therefore A(\frac{7}{5}\ ;\ \frac{16}{5})$	✓values of $\sin 2\theta$ and $\cos 2\theta$ ✓substitution into rotation formulae ✓ $A'(\frac{12}{5}\ ;\ \frac{16}{5})$ (3)
	OR	

		
	Let $N\hat{K}L = \theta$. So, $\tan\theta = \frac{NL}{KN} = \frac{2}{4} = \frac{1}{2}$.	✓$\tan\ \theta = \frac{1}{2}$
	Hence $\sin\theta = \frac{1}{\sqrt{5}}$ and $\cos\theta = \frac{2}{\sqrt{5}}$	
	Let $AM \perp x$ – axis with M on x - axis $\Delta NAK \equiv \Delta NLK$ $A\hat{K}N = N\hat{K}L = \theta$ $\therefore\ A\hat{K}L = 2\theta$	
	$y_A = AM = AK\sin 2\theta = KL\sin 2\theta = 4\sin 2\theta$ $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right) = \frac{4}{5}$ $y_A = 4\left(\frac{4}{5}\right) = \frac{16}{5}$	✓ $\sin 2\theta = \frac{4}{5}$
	$x_A = OL - NA\sin M\hat{A}N$ $= 3 - 2\sin\left(90° - M\hat{A}K\right)$ $= 3 - 2\sin 2\theta$ $= 3 - \frac{8}{5}$ $= \frac{7}{5}$	✓ solve for x and y (3)

6.7	$KA=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=\sqrt{\left(\frac{7}{5}+1\right)^2+\left(\frac{16}{5}-0\right)^2}$ $=4$ **OR** $KN=\sqrt{4^2+2^2}=\sqrt{20}$ $KA^2=KN^2-AN^2$ $=20-4$ $=16$ $KA=4$ **OR** KA = KL Tangents from a common point are equal KA = 4	✓ distance formula ✓ substitution ✓ 4 (3) ✓ $KN=\sqrt{20}$ ✓ $KA^2=KN^2-AN^2$ ✓ 4 (3) ✓ KA=KL ✓ reason ✓ 4 (3)
6.8	AN = NL Radii are equal KA = KL $\therefore$KLNA is a kite two pairs of adjacent sides are equal.	✓ AN = NL ✓ KA = KL (2)
6.9	AB = AN + NB = 2 + 2 = 4 AK = 4= AB $K\hat{A}B=90^\circ$ tangent $\perp$ radius $\therefore \Delta$AKB is a right – angled isosceles triangle $A\hat{K}B+A\hat{B}K=90^\circ$ $2A\hat{B}K=90^\circ$ $\therefore A\hat{B}K=45^\circ$ **OR**	✓ AB = 4 ✓ AK = AB ✓ $K\hat{A}B=90^\circ$ (3)

<table>
<tr>
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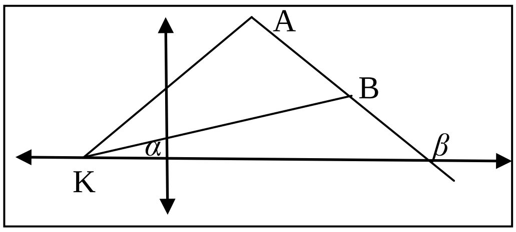
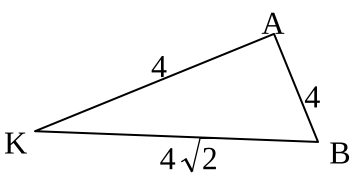
<td>
N is midpoint of AB
Let B be $\left(x_B\,;y_B\right)$
$\dfrac{x_B+\frac{7}{5}}{2}=3$ $\dfrac{y_B+\frac{16}{5}}{2}=2$
$\therefore x_B=\dfrac{23}{5}$ $\therefore y_B=\dfrac{4}{5}$
$\therefore B\left(\dfrac{23}{5};\dfrac{4}{5}\right)$


$\tan\beta=m_{AB}=-\dfrac{3}{4}$
$\beta=180°-36{,}87°$
$\beta=143{,}13°$
$\tan\alpha=m_{KB}=\dfrac{\frac{4}{5}-0}{\frac{23}{5}+1}=\dfrac{1}{7}$
$\alpha=8{,}13°$
$A\hat{B}K=\alpha+(180°-\beta)$
$=8{,}13°+36{,}87°$
$=45°$

OR

N is midpoint of AB
Let B be $\left(x_B\,;y_B\right)$
$\dfrac{x_B+\frac{7}{5}}{2}=3$ $\dfrac{y_B+\frac{16}{5}}{2}=2$
$\therefore x_B=\dfrac{23}{5}$ $\therefore y_B=\dfrac{4}{5}$
$\therefore B\left(\dfrac{23}{5};\dfrac{4}{5}\right)$


$KB=\sqrt{(\frac{23}{5}+1)^2+(\frac{4}{5})^2}=4\sqrt{2}$
$4^2=4^2+(\sqrt{32})^2-2(4)(\sqrt{32})\cos\theta$
$\cos\theta=\dfrac{\sqrt{2}}{2}$
$\therefore\theta=45°$
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✓ $143{,}13°$

✓ $8{,}13°$
✓ $A\hat{B}K=\alpha+(180°-\beta)$
(3)

✓ $4\sqrt{2}$

✓ substitution into cosine formula
✓ $\cos\theta=\dfrac{\sqrt{2}}{2}$
(3)
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<td>6.10</td>
<td>$N^{/}(3;-2)$</td>
<td>✓ $N^{/}(3;-2)$ (1)
[24]</td>
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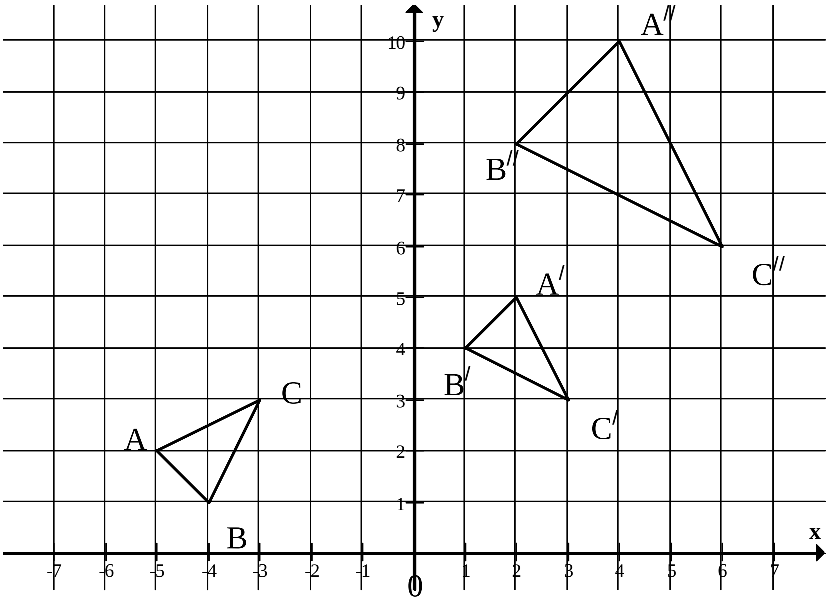
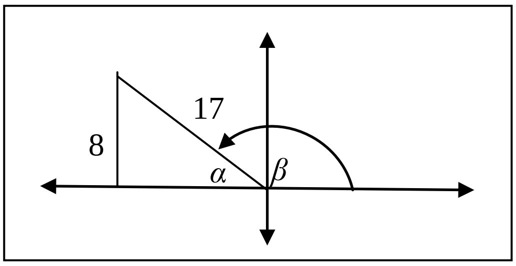
QUESTION 7
NOTE: CA not applicable in this question

7.1	Rotation about the origin through 90° in a clockwise direction. **OR** Rotation about the origin through 270° in an anti-clockwise direction. **OR** Rotation about the origin through -90°.	✓ rotation of 90° ✓ clockwise direction (2) ✓ rotation of 270° ✓ anti-clockwise direction (2) ✓✓statement (2)
7.2	$(x;y)\rightarrow(y;-x)$	✓ ✓(both) $(x;y)\rightarrow(y;-x)$ (2)
7.3		✓ one point correct ✓ all points correct and triangle drawn (2)
7.4	$(x;y)\rightarrow(2x;2y)$	✓ $(2x;2y)$ (1)
7.5.1	$A(-5;2)\rightarrow(-5;-2)\rightarrow D(5;-2)$	✓ 5 ✓ -2 (2)
7.5.2	$(x;y)\rightarrow(x;-y)\rightarrow(-x;-y)$	✓ $(x;-y)$ ✓ $(-x;-y)$ (2)
7.5.3	Rotation of 180° through the origin in either direction. **OR** Reflection about the origin.	✓ rotation ✓ 180° (2) ✓ reflection ✓ origin (2) **[13]**

QUESTION 8
No calculator allowed in this question

8.1.1	OT = k , PT = 8 and OP = 17 $k^2+8^2=17^2$ $k^2=289-64$ $k^2=225$ $k=\pm 15$ $k>0$ $k=15$ **OR** $k^2=17^2-8^2$ $k^2=(17-8)(17+8)$ $=25\times 9$ $=225$ $k=\pm 15$ $k>0$ $k=15$	✓ substitution into Pythagoras ✓ k = 15 (2) ✓ substitution into Pythagoras ✓ k = 15 (2)
8.1.2	$\cos\alpha=\frac{15}{17}$	✓ $\frac{15}{17}$ (1)
8.1.3	$\alpha+\beta=180°$ $\beta=180°-\alpha$ $\therefore \cos\beta=\cos(180°-\alpha)$ $=-\cos\alpha$ $=-\frac{15}{17}$ **OR**  $\therefore \cos\beta=\cos(180°-\alpha)$ $=-\cos\alpha$ $=-\frac{15}{17}$	✓ $\cos(180°-\alpha)$ or $-\cos\alpha$ ✓ $-\frac{15}{17}$ (2) ✓ $\cos(180°-\alpha)$ or $-\cos\alpha$ ✓ $-\frac{15}{17}$ (2)

8.1.4	$\sin(\beta-\alpha)$ $=\sin\beta\cos\alpha-\cos\beta\sin\alpha$ $=\left(\frac{8}{17}\right)\left(\frac{15}{17}\right)-\left(-\frac{15}{17}\right)\left(\frac{8}{17}\right)$ $=\frac{120}{289}+\frac{120}{289}$ $=\frac{240}{289}$ **OR** $\beta-\alpha=(180°-\alpha)-\alpha$ $=180°-2\alpha$ $\sin(\beta-\alpha)=\sin(180°-2\alpha)$ $=\sin 2\alpha$ $=2\sin\alpha.\cos\alpha$ $=2\left(\frac{8}{17}\right)\left(\frac{15}{17}\right)$ $=\frac{240}{289}$	✓ expansion ✓ $\sin\beta=\frac{8}{17}$ ✓ $\sin\alpha=\frac{8}{17}$ ✓ $\frac{240}{289}$ (4) ✓ substitute β ✓ *2*sinαcosα ✓ $\sin\alpha=\frac{8}{17}$ ✓ $\frac{240}{289}$ (4)
8.2.1	$LHS=\frac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ $=\frac{1-(1-2\sin^2 x)-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{2\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{\sin x(2\sin x-1)}{\cos x(2\sin x-1)}$ $=\frac{\sin x}{\cos x}$ $=\tan x$ $=RHS$ **OR**	✓ $1-2\sin^2 x$ ✓ $2\sin x\cos x$ ✓ either $\sin x(2\sin x-1)$ or $\cos x(2\sin x-1)$ ✓ $\frac{\sin x}{\cos x}$ (4)

	$LHS = \dfrac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ $= \dfrac{1-(2\cos^2 x-1)-\sin x}{2\sin x\cos x-\cos x}$ $= \dfrac{2-\cos^2 x-\sin x}{2\sin x\cos x-\cos x}$ $= \dfrac{2(1-\cos^2 x)-\sin x}{2\sin x\cos x-\cos x}$ $= \dfrac{2\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $= \dfrac{\sin x(2\sin x-1)}{\cos x(2\sin x-1)}$ $= \dfrac{\sin x}{\cos x}$ $= \tan x$ $= RHS$	✓ $2\cos^2 x-1$ ✓ $2\sin x\cos x$ ✓ either $\sin x(2\sin x-1)$ or $\cos x(2\sin x-1)$ ✓ $\dfrac{\sin x}{\cos x}$ (4)
	OR	
	$LHS = \dfrac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ $= \dfrac{1-(\cos^2 x-\sin^2 x)-\sin x}{2\sin x\cos x-\cos x}$ $= \dfrac{1-\cos^2 x+\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $= \dfrac{\sin^2 x+\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $= \dfrac{2\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $= \dfrac{\sin x(2\sin x-1)}{\cos x(2\sin x-1)}$ $= \dfrac{\sin x}{\cos x}$ $= \tan x$ $= RHS$	✓ $\cos^2 x-\sin^2 x$ ✓ $2\sin x\cos x$ ✓ either $\sin x(2\sin x-1)$ or $\cos x(2\sin x-1)$ ✓ $\dfrac{\sin x}{\cos x}$ (4)

8.2.2	$\sin 2x - \cos x = 0$ $2\sin x\cos x - \cos x = 0$ $\cos x(2\sin x - 1) = 0$ $\cos x = 0$ $x = 90° + 360°k$ or $x = 270° + 360°k$ $k \in Z$ or $\sin x = \frac{1}{2}$ $x = 30° + 360°k$ or $x = 150° + 360°k$ $x = 90°$ or $x = 270°$ or $x = 30°$ or $x = 150°$	✓ $2\sin x\cos x$ ✓ $\cos x = 0$ and $\sin x = \frac{1}{2}$ ✓ for two correct answers ✓ for four correct answers (4)
	OR $\sin 2x = \cos x$ $\sin 2x = \sin(90° - x)$ $2x = 90° - x + 360°k\,;k \in Z$ or $2x = 180° - (90° - x) + 360°k$ $3x = 90° + 360°k$ $2x = 90° + x + 360°k$ $x = 30° + 120°k$ $x = 90° + 360°k$ $x = 30°$ or $x = 150°$ or $x = 270°$ or $x = 90°$	✓ $\sin(90° - x)$ ✓ $x = 30° + 120°.k$ and $x = 90° + 360°.k$ ✓ for two correct answers ✓ for four correct answers (4) **[17]**

QUESTION 9

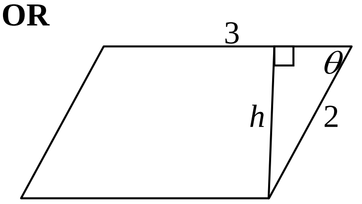
9.1	$\dfrac{\sin^2\theta}{\sin(180°-\theta).\cos(90°+\theta)+\tan 45°}$ $=\dfrac{\sin^2\theta}{(\sin\theta)(-\sin\theta)+1}$ $=\dfrac{\sin^2\theta}{-\sin^2\theta+1}$ $=\dfrac{\sin^2\theta}{\cos^2\theta}$ $=\tan^2\theta$	✓ $\sin\theta$ ✓ $-\sin\theta$ ✓ 1 ✓ $\cos^2\theta$ ✓ $\tan^2\theta$ (5)
9.2	$\dfrac{\sin 104°(2\cos^2 15°-1)}{\tan 38°\sin^2 412°}$ $=\dfrac{\sin 76°.\cos 30°}{\tan 38°.(\sin 52°)^2}$ $=\dfrac{2\sin 38°\cos 38°\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sin 38°}{\cos 38°}\right)(\cos 38°)^2}$ $=\dfrac{\sqrt{3}\sin 38°\cos 38°}{\sin 38°\cos 38°}$ $=\sqrt{3}$ **NOTE:** • If cos 30° is missing: deduct 1 mark • Answer only: 0/8 **OR** $\dfrac{\sin 104°(2\cos^2 15°-1)}{\tan 38°\sin^2 412°}$ $=\dfrac{\sin 2(52°).(2\cos^2 15°-1)}{\frac{\sin 38°}{\cos 38°}.(\sin 52°)^2}$ $=\dfrac{2\sin 52°\cos 52°.\cos 30°}{\left(\frac{\cos 52°}{\sin 52°}\right)(\sin 52°)^2}$ $=2\cos 30°$ $=2.\dfrac{\sqrt{3}}{2}$ $=\sqrt{3}$	✓ sin 76° ✓ cos30° ✓ $\dfrac{\sin 38°}{\cos 38°}$ ✓ sin52° ✓ 2sin38°cos38° ✓ $\dfrac{\sqrt{3}}{2}$ ✓ sin52°=cos38° ✓ $\sqrt{3}$ (8) ✓ sin2(52°) ✓ $\dfrac{\sin 38°}{\cos 38°}$ ✓ sin52° ✓ 2sin52°cos52° ✓ cos30° ✓ cos52°=sin38° and sin52°=cos38° ✓ $\dfrac{\sqrt{3}}{2}$ ✓ $\sqrt{3}$ (8)

	OR $\frac{\sin 104°(2\cos^2 15° - 1)}{\tan 38° \sin^2 412°}$ $= \frac{\sin 104°.\cos 30°}{\left(\frac{\sin 38°}{\cos 38°}\right)(\sin 52°)^2}$ $= \frac{(\sin 104°)\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sin 38°}{\cos 38°}\right)(\cos^2 38)}$ $= \frac{\sqrt{3}\sin 104°}{2\sin 38° \cos 38°}$ $= \frac{\sqrt{3}\sin 104°}{\sin 76°}$ $= \frac{\sqrt{3}\sin 76°}{\sin 76°}$ or $\frac{\sqrt{3}\cos 14°}{\cos 14°}$ $= \sqrt{3}$	✓cos30° ✓ $\frac{\sin 38°}{\cos 38°}$ ✓ sin52° ✓$\cos^2 38°$ ✓ $\frac{\sqrt{3}}{2}$ ✓✓ sin76° ✓ $\sqrt{3}$ (8)
	OR $\frac{\sin 104°(2\cos^2 15° - 1)}{\tan 38° \sin^2 412°}$ $= \frac{\sin 104°.\cos 30°}{\frac{\sin 38°}{\cos 38°}.(\sin 52°)^2}$ $= \frac{\sin 104°.\frac{\sqrt{3}}{2}}{\left(\frac{\cos 52°}{\sin 52°}\right)(\sin 52°)^2}$ $= \frac{\sin 104°.\frac{\sqrt{3}}{2}}{\cos 52°(\sin 52°)}$ $= \frac{\sin 104°.\frac{\sqrt{3}}{2}}{\frac{1}{2}\sin 104°}$ $= \sqrt{3}$	✓ cos30° ✓ $\frac{\sin 38°}{\cos 38°}$ ✓ sin52° ✓ $\frac{\sqrt{3}}{2}$ ✓ cos52°=sin38° and sin52°=cos38° ✓ $\cos 52^0.\sin 52^0$ ✓ $\frac{1}{2}\sin 104°$ ✓ $\sqrt{3}$ (8) **[13]**

QUESTION 10

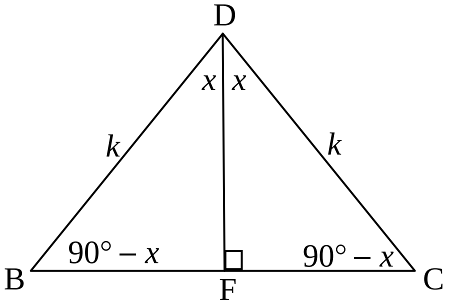
10.1	$f(0)-g(0)=0{,}5-(-2)=2{,}5$	✓ 2,5 (1)
10.2	$\sin(x+30°)=-2\cos x$ $\sin x.\cos 30°+\cos x.\sin 30°=-2\cos x$ $\left(\frac{\sqrt{3}}{2}\right)\sin x+\left(\frac{1}{2}\right)\cos x=-2\cos x$ $\sqrt{3}\sin x+\cos x=-4\cos x$ $\sqrt{3}\sin x=-5\cos x$ $\tan x=-\frac{5}{\sqrt{3}}$ $x=109{,}11°+180°k\ ;\ k\in Z$ $x_P=-70{,}89°$ *and* $x_Q=109{,}11°$ **OR** $\sin(x+30°)=-2\cos x$ $\cos(90°-x-30°)=-2\cos x$ $\cos(60°-x)=-2\cos x$ $\cos 60°\cos x+\sin 60°\sin x=-2\cos x$ $\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x=-2\cos x$ $\cos x+\sqrt{3}\sin x=-4\cos x$ $\sqrt{3}\sin x=-5\cos x$ $\tan x=-\frac{5}{\sqrt{3}}$ $x=109{,}11°+180°.k\ ;\ k\in Z$ $x_P=-70{,}89°$ *and* $x_Q=109{,}11°$	✓ equation ✓ expansion of $\sin(x+30°)$ ✓ substitution of special angles ✓ simplification ✓ $\tan x=-\frac{5}{\sqrt{3}}$ ✓ $x_P=-70{,}89°$ ✓ $x_Q=109{,}11°$ (7) ✓ equation ✓ expansion of $\cos(60°-x)$ ✓ substitution of special angles ✓ simplification ✓ $\tan x=-\frac{5}{\sqrt{3}}$ ✓ $x_P=-70{,}89°$ ✓ $x_Q=109{,}11°$ (7)
10.3	$-70{,}89°\le x\le 109{,}11°$ **OR** [–70,89° ; 109,11°] **OR** $x_P\le x\le x_Q$	✓ angles ✓ correct interval (2)
10.4	$h(x)=2\sin(x+60°+30°)=2\sin(x+90°)=2\cos x=-g(x)$ *h* is the reflection of *g* about the *x*-axis. **OR** *f* is shifted to the left through 60° and then doubled. ∴ *h* is the reflection of *g* about the *x*-axis.	✓✓ reflection about the *x*-axis or line *y* = 0 (2) ✓✓ reflection about the *x*-axis or line *y* = 0 (2) **[12]**

QUESTION 11

11.1	Area parallelogram ABCD = $2 \times$ Area ΔABC
	$= 2\left[\left(\frac{1}{2}\right)(3)(2)\sin\theta\right]$
	$= 6\sin\theta$
	
	NOTE: If no working is shown, then 0/3
	
	OR
	
	
	$\frac{h}{2} = \sin\theta$
	$h = 2\sin\theta$
	$\therefore$ Area $ABCD =$ base $\times$ height $= 3h = 3.2\sin\theta = 6\sin\theta$
	
	OR
	
	Area of parallelogram ABCD = area of ΔABC + area of ΔADC
	$= \left(\frac{1}{2}\right)(3)(2)\sin\theta + \left(\frac{1}{2}\right)(3)(2)\sin\theta$
	= 6 sin θ
	OR
	Area = $\frac{1}{2}$(sum of // sides)× h
	$= \frac{1}{2}(3 + 3) \times 2\sin\theta$
	= 6 sin θ
		✓✓ 2area ΔABC
		✓ substitution into area rule
		(3)
		
		✓ $\frac{h}{2} = \sin\theta$
		✓ $h = 2\sin\theta$
		✓ $b.h$
		(3)
		
		✓ sum of areas
		✓✓equal sides and equal angles
		(3)
		
		✓ formula
		✓ $h = 2\sin\theta$
		✓substitution
		(3)

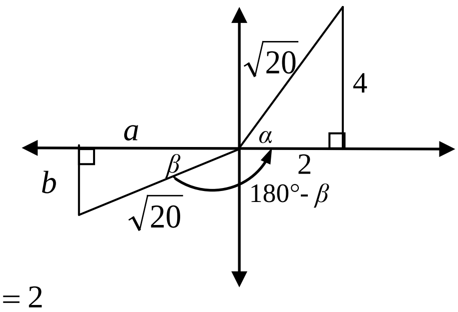
11.2	Area of parallelogram ABCD = $3\sqrt{3}$ $6\sin\theta = 3\sqrt{3}$ $\sin\theta = \frac{\sqrt{3}}{2}$ $\theta = 60°$ **NOTE:** Deduct 1 mark if both 60° and 120° are given as answers **OR** 6 sin 60° = $3\sqrt{3}$ $\therefore$ θ = 60°	✓ $6\sin\theta = 3\sqrt{3}$ ✓ $\sin\theta = \frac{\sqrt{3}}{2}$ ✓ 60° (3) ✓✓ $6\sin\theta = 3\sqrt{3}$ ✓60° (3)
11.3	Maximum area of parallelogram occurs when sin $\theta = 1$, that is when $\theta = 90°$	✓ sin $\theta = 1$ ✓ $\theta = 90°$ (2) **[8]**

QUESTION 12

12.1	$\frac{CB}{\sin B\hat{D}C} = \frac{CD}{\sin C\hat{B}D}$ $\frac{CB}{\sin 2x} = \frac{k}{\sin(90° - x)}$ $CB = \frac{k.\sin 2x}{\sin(90° - x)}$ $CB = \frac{k.2\sin x\cos x}{\cos x}$ $= 2k\sin x$	✓ Using the sine rule in triangle CBD ✓ $\frac{CB}{\sin 2x} = \frac{k}{\sin(90° - x)}$ ✓ $\frac{k.\sin 2x}{\sin(90° - x)}$ ✓ 2sin*x*.cos*x* ✓ cos *x* (5)
	OR $D\hat{C}B = 180° - (90° - x + 2x) = 90° - x$ $\therefore DC = DB = k$  Draw DF ⊥ BC $\frac{CF}{CD} = \sin x$ $CF = k\sin x$ $CB = 2CF$ $CB = 2k\sin x$	✓ $D\hat{C}B = D\hat{B}C = 90° - x$ ✓ $DC = DB = k$ ✓ $C\hat{D}F = x$ ✓ $CF = k\sin x$ ✓ *CB=2 CF* (5)
	OR $D\hat{C}B = 180° - (90° - x + 2x) = 90° - x$ $\therefore DC = DB = k$ $CB^2 = CD^2 + BD^2 - 2.CD.BD.\cos 2x$ $CB^2 = k^2 + k^2 - 2k^2\cos 2x$ $= 2k^2(1 - \cos 2x)$ $= 2k^2(1 - (1 - 2\sin^2 x))$ $= 2k^2(2\sin^2 x)$ $= 4k^2\sin^2 x$ $= (2k\sin x)^2$ $CB = 2k\sin x$	✓ $D\hat{C}B = D\hat{B}C = 90° - x$ ✓ $DC = DB = k$ ✓ using cosine rule in triangle CDB ✓ factors ✓ simplification (5)

12.2	$\cos x = \frac{\text{BC}}{\text{HC}}$ $\text{HC} = \frac{\text{BC}}{\cos x}$ $= \frac{2k \sin x}{\cos x}$ $= 2k \tan x$ **OR** $\frac{HC}{\sin 90°} = \frac{BC}{\sin(90° - x)}$ $HC = \frac{BC}{\sin(90° - x)}$ $= \frac{2k \sin x}{\cos x}$ $= 2k \tan x$	✓ $\cos x = \frac{BC}{HC}$ ✓ $HC = \frac{BC}{\cos x}$ ✓ substitution of BC (3) ✓ $HC = \frac{BC}{\sin(90° - x)}$ ✓ substitution of BC ✓ $\sin(90° - x) = \cos x$ (3)
12.3	HC = $2k \tan x = 2(40).\tan(23°) = 33{,}9579...$ In ΔHCD: $CD^2 = HC^2 + HD^2 - 2HC.HD.\cos\theta$ $\cos\theta = \frac{HC^2 + HD^2 - CD^2}{2HC.HD}$ $= \frac{(33{,}9579...)^2 + 31{,}8^2 - 40^2}{2(33{,}9579...)(31{,}8)}$ $\cos\theta = 0{,}2613...$ $\therefore \theta = 74{,}85°$	✓value of HC ✓substitution into cos formula ✓cos $\theta = 0{,}2613...$ ✓ 74,85° (4) **[12]**

QUESTION 13

13.1	Angle that minute hand moves is: $\frac{37}{60} \times 360°$ $= 222°$ **OR** 60 min : 360° 1 min : 6° 37 min : 37 × 6 = 222° P is rotated by 360° - 222° = 138° in an **anti-clockwise** direction: $a = 2\cos 138° - 4\sin 138°$ and $b = 4\cos 138° + 2\sin 138°$ $= -4{,}16$ $= -1{,}63$	✓✓ $\frac{37}{60} \times 360°$ ✓ 222° ✓ substitution of 138^0 into formula for *x* and *y* ✓ $-4{,}16$ ✓ $-1{,}63$ (6)
	OR Angle that minute hand moves is: $\frac{37}{60} \times 360°$ $= 222°$ P is rotated by 222° in a **clockwise** direction: $a = 2\cos 222° + 4\sin 222°$ and $b = 4\cos 222° - 2\sin 222°$ $= -4{,}16$ $= -1{,}63$	✓✓ $\frac{37}{60} \times 360°$ ✓ 222° ✓ substitution of 222^0 into formula for *x* and *y* ✓ $-4{,}16$ ✓ $-1{,}63$ (6)
	OR  $\tan\alpha = 2$ $\alpha = 63{,}43°$ $\alpha + 180° - \beta = 222°$ $\beta = 63{,}43° + 180° - 222°$ $= 21{,}43°$ $\therefore a = -\sqrt{20}\cos 21{,}43° = -4{,}16$ $b = -\sqrt{20}\sin 21{,}43° = -1{,}63$	✓ $\tan\alpha = 2$ ✓ $\alpha = 63{,}43°$ ✓ $\alpha + 180° - \beta = 222°$ ✓ $\beta = 21{,}43°$ ✓ $-4{,}16$ ✓ $-1{,}63$ (6)

13.2	The minute hand moves through 360° in 60 minutes. The hour hand moves through 30° in 60 minutes, that is, $\frac{1}{12}$ that of the minute hand. So when the minute hand moves through 222°, the hour hand moves through $\frac{222°}{12} = 18{,}5°$ **OR** The hour hand moves through $\frac{360°}{12} = 30°$ in 60 minutes $\therefore$ it moves through $\frac{37}{60} \times 30° = 18{,}5°$ in 37 minutes	✓ 360° ✓ 30° ✓ $\frac{1}{12}$ ✓ 18,5° (4) ✓ 360° ✓ 30° ✓ $\frac{37}{60} \times 30°$ ✓ 18,5° (4) **[10]**

TOTAL : 150