basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2012

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x in each of the following:

1.1.1 $(2x-1)(x+4)=0$ (2)

1.1.2 $3x^2 - x = 5$ (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $x^2 + 7x - 8 < 0$ (4)

1.2 Given: $4y - x = 4$ and $xy = 8$

1.2.1 Solve for x and y simultaneously. (6)

1.2.2 The graph of $4y - x = 4$ is reflected across the line having equation $y = x$. What is the equation of the reflected line? (2)

1.3 The solutions of a quadratic equation are given by $x = \dfrac{-2 \pm \sqrt{2p+5}}{7}$

For which value(s) of p will this equation have:

1.3.1 Two equal solutions (2)

1.3.2 No real solutions (1)
[21]

QUESTION 2

2.1 $3x + 1$; $2x$; $3x - 7$ are the first three terms of an arithmetic sequence. Calculate the value of x. (2)

2.2 The first and second terms of an arithmetic sequence are 10 and 6 respectively.

2.2.1 Calculate the 11^{th} term of the sequence. (2)

2.2.2 The sum of the first n terms of this sequence is –560. Calculate n. (6)
[10]

QUESTION 3

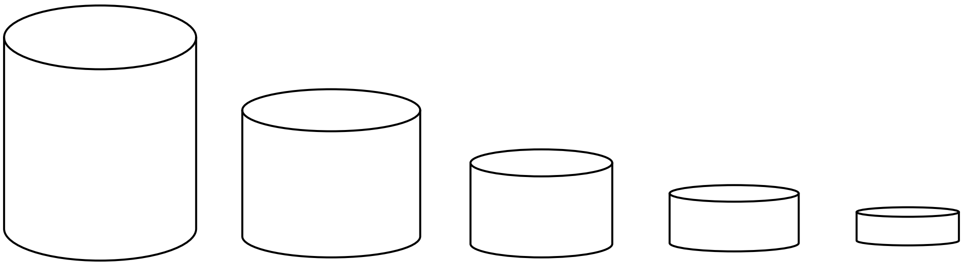
3.1 Given the geometric sequence: 27 ; 9 ; 3 …

3.1.1 Determine a formula for T_n, the n^{th} term of the sequence. (2)

3.1.2 Why does the sum to infinity for this sequence exist? (1)

3.1.3 Determine S_∞. (2)

3.2 Twenty water tanks are decreasing in size in such a way that the volume of each tank is $\frac{1}{2}$ the volume of the previous tank. The first tank is empty, but the other 19 tanks are full of water.



Would it be possible for the first water tank to hold all the water from the other 19 tanks? Motivate your answer. (4)

3.3 The n^{th} term of a sequence is given by $T_n = -2(n-5)^2 + 18$.

3.3.1 Write down the first THREE terms of the sequence. (3)

3.3.2 Which term of the sequence will have the greatest value? (1)

3.3.3 What is the second difference of this quadratic sequence? (2)

3.3.4 Determine ALL values of n for which the terms of the sequence will be less than –110. (6)

[21]

QUESTION 4

4.1 Consider the function $f(x) = 3.2^x - 6$.

4.1.1 Calculate the coordinates of the *y*-intercept of the graph of *f*. (1)

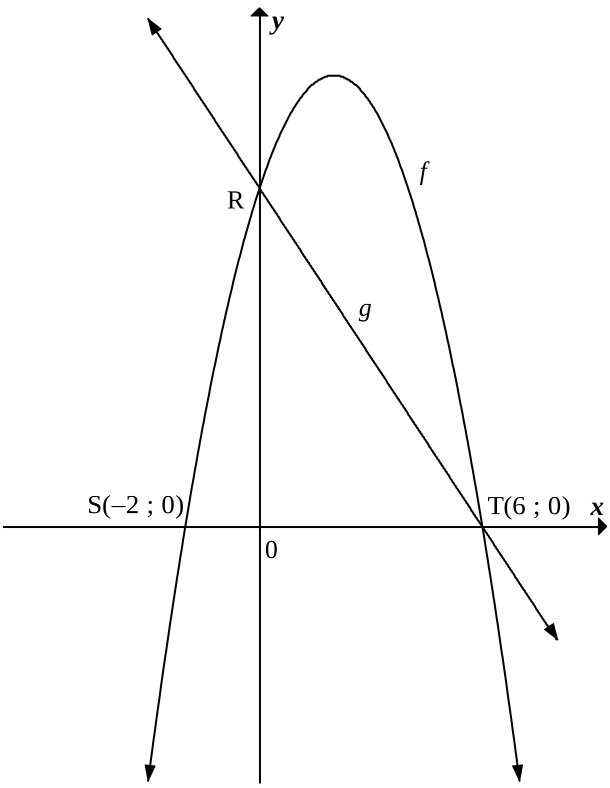
4.1.2 Calculate the coordinates of the *x*-intercept of the graph of *f*. (2)

4.1.3 Sketch the graph of f in your ANSWER BOOK.

Clearly show ALL asymptotes and intercepts with the axes. (3)

4.1.4 Write down the range of *f*. (1)

4.2 S(–2 ; 0) and T(6 ; 0) are the *x*-intercepts of the graph of $f(x) = ax^2 + bx + c$ and R is the *y*-intercept. The straight line through R and T represents the graph of $g(x) = -2x + d$.



4.2.1 Determine the value of *d*. (2)

4.2.2 Determine the equation of f in the form $f(x) = ax^2 + bx + c$. (4)

4.2.3 If $f(x) = -x^2 + 4x + 12$, calculate the coordinates of the turning point of *f*. (2)

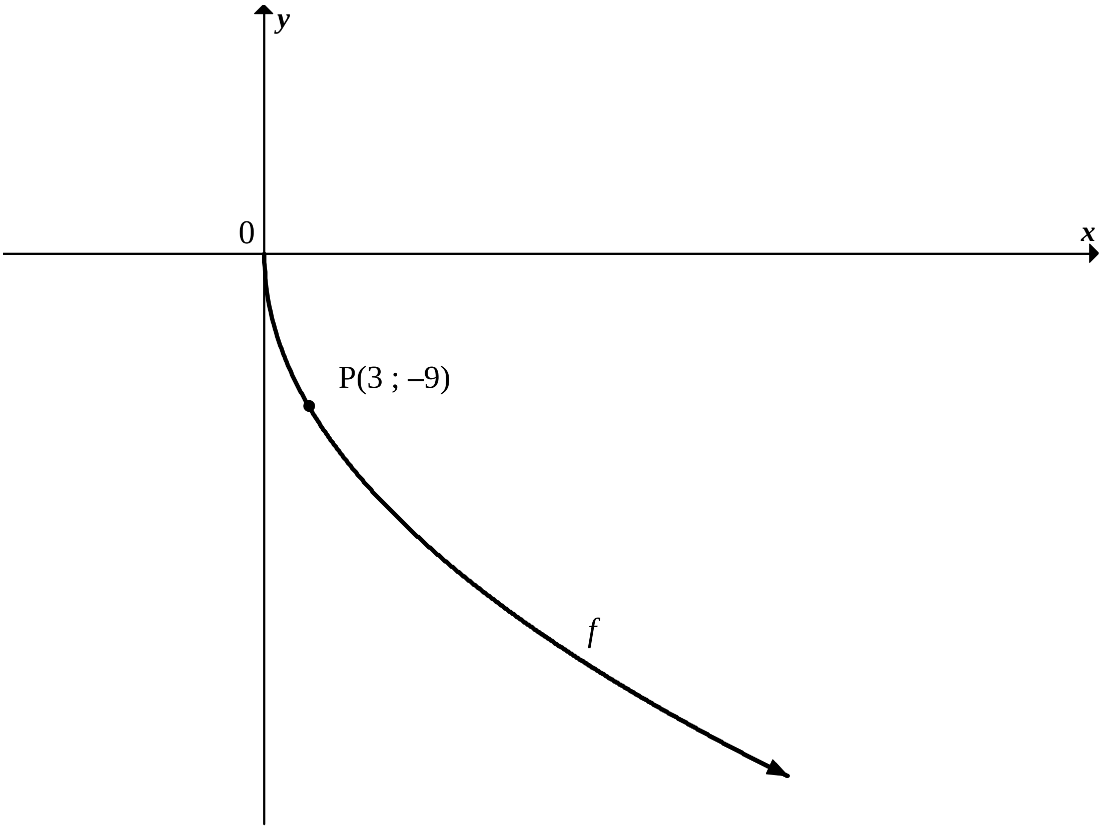
4.2.4 For which values of *k* will $f(x) = k$ have two distinct roots? (2)

4.2.5 Determine the maximum value of $h(x) = 3^{f(x)-12}$. (3)

[20]

QUESTION 5

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched below.
The point P(3 ; –9) lies on the graph of f.



5.1 Use your graph to determine the values of x for which $f(x) \geq -9$. (2)

5.2 Write down the equation of f^{-1} in the form $y = \ldots$ Include ALL restrictions. (3)

5.3 Sketch f^{-1}, the inverse of f, in your ANSWER BOOK.
Indicate the intercept(s) with the axes and the coordinates of ONE other point. (3)

5.4 Describe the transformation from f to g if $g(x) = \sqrt{27x}$, where $x \geq 0$. (1)
[9]

QUESTION 6

The graph of a hyperbola with equation $y = f(x)$ has the following properties:

- Domain: $x \in \mathbf{R}$, $x \neq 5$
- Range: $y \in \mathbf{R}$, $y \neq 1$
- Passes through the point (2 ; 0)

Determine $f(x)$. **[4]**

QUESTION 7

7.1 A business buys a machine that costs R120 000. The value of the machine depreciates at 9% per annum according to the diminishing-balance method.

7.1.1 Determine the scrap value of the machine at the end of 5 years. (3)

7.1.2 After five years the machine needs to be replaced. During this time, inflation remained constant at 7% per annum. Determine the cost of the new machine at the end of 5 years. (3)

7.1.3 The business estimates that it will need R90 000 by the end of five years. A sinking fund for R90 000, into which equal monthly instalments must be paid, is set up. Interest on this fund is 8,5% per annum, compounded monthly. The first payment will be made immediately and the last payment will be made at the end of the 5-year period.

Calculate the value of the monthly payment into the sinking fund. (5)

7.2 Lorraine receives an amount of R900 000 upon her retirement. She invests this amount immediately at an interest rate of 10,5% per annum, compounded monthly.

She needs an amount of R18 000 per month to maintain her current lifestyle. She plans to withdraw the first amount at the end of the first month.

For how many months will she be able to live from her investment? (6)
[17]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 2x^2 - 5$. (5)

8.2 Evaluate $\frac{dy}{dx}$ if $y = x^{-4} + 2x^3 - \frac{x}{5}$. (3)

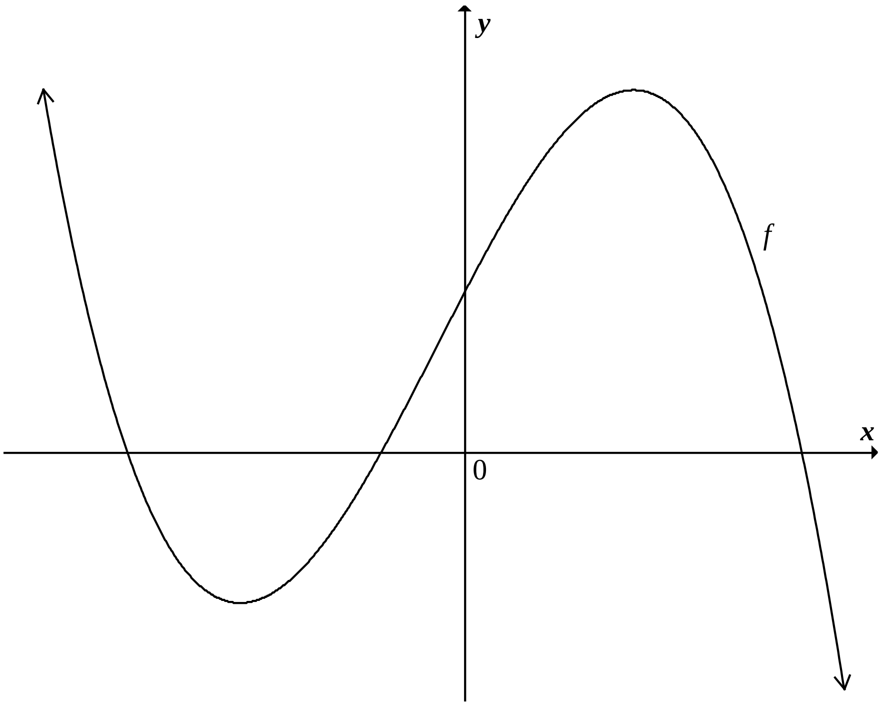
8.3 Given: $g(x) = \frac{x^2 + x - 2}{x - 1}$

8.3.1 Calculate $g'(x)$ for $x \neq 1$. (2)

8.3.2 Explain why it is not possible to determine $g'(1)$. (1)
[11]

QUESTION 9

9.1 The graph of the function $f(x) = -x^3 - x^2 + 16x + 16$ is sketched below.



9.1.1 Calculate the x-coordinates of the turning points of f. (4)

9.1.2 Calculate the x-coordinate of the point at which $f'(x)$ is a maximum. (3)

9.2 Consider the graph of $g(x) = -2x^2 - 9x + 5$.

9.2.1 Determine the equation of the tangent to the graph of g at $x = -1$. (4)

9.2.2 For which values of q will the line $y = -5x + q$ not intersect the parabola? (3)

9.3 Given: $h(x) = 4x^3 + 5x$

Explain if it is possible to draw a tangent to the graph of h that has a negative gradient. Show ALL your calculations. (3)

[17]

QUESTION 10

A particle moves along a straight line. The distance, s, (in metres) of the particle from a fixed point on the line at time t seconds ($t \geq 0$) is given by $s(t) = 2t^2 - 18t + 45$.

10.1 Calculate the particle's initial velocity. (Velocity is the rate of change of distance.) (3)

10.2 Determine the rate at which the velocity of the particle is changing at t seconds. (1)

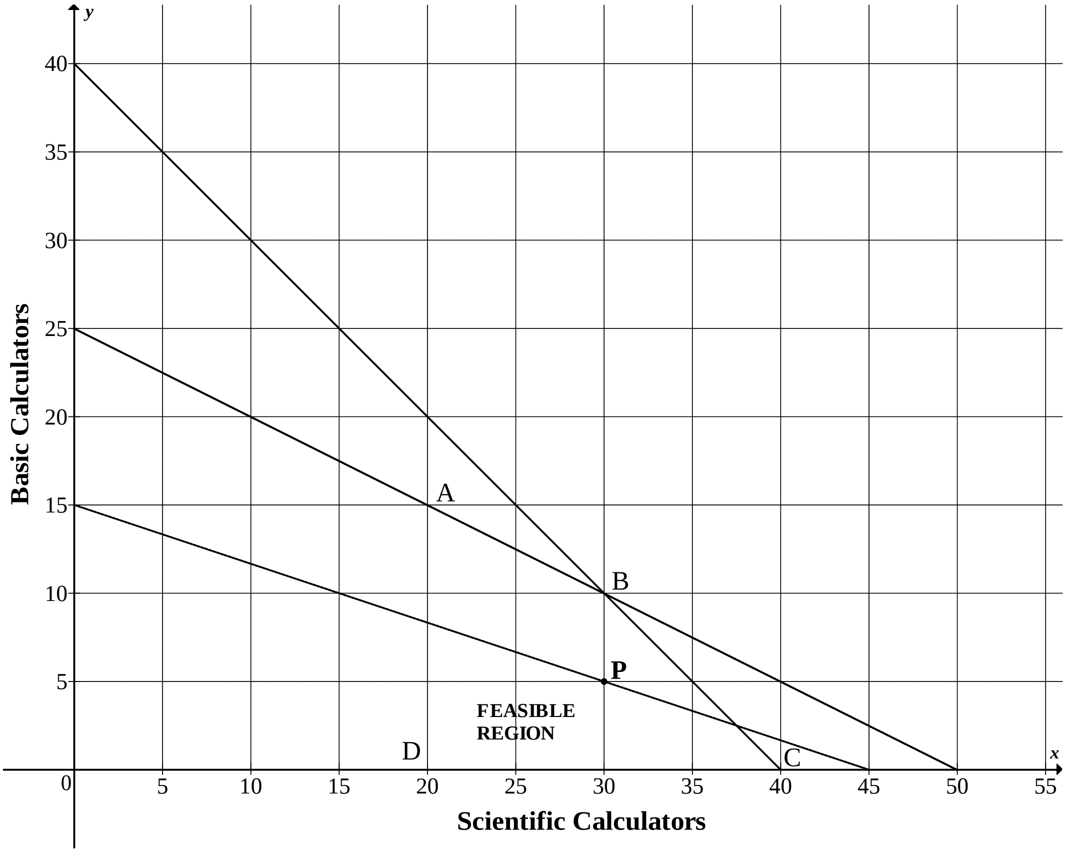
10.3 After how many seconds will the particle be closest to the fixed point? (2)

[6]

QUESTION 11

A calculator company manufactures two kinds of calculators: scientific and basic. The company is able to sell all the calculators that it produces. A system of constraints has been developed for the production of the calculators. The feasible region is shaded below.

Let x and y respectively be the number of scientific and basic calculators produced each day.



11.1 Is it possible for the company to manufacture 15 scientific calculators and 5 basic calculators in one day according to their system of constraints? Motivate your answer. (1)

11.2 Write down all the algebraic inequalities which describe the constraints related to the manufacturing of the calculators. (6)

11.3 The profit Q (in hundreds of rands) is given by Q = $x+3y$. The dotted line on the graph is a search line associated with the profit function.

11.3.1 Identify the point in the region where the profit is a maximum. Use only A, B, C or D. (1)

11.3.2 Write down the coordinates of a point on the dotted line (if the point exists) at which the profit is greater than the profit at P. (2)

11.3.3 Given that the profit, when given by Q $= ax + by$ ($a > 0$; $b > 0$), is a maximum at B, determine the maximum value of $\frac{a}{b}$. (4)

[14]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2} ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$$(x ; y) \to (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$