basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2013

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 2 diagram sheets and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 13 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.

4. Answers only will not necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. TWO diagram sheets for QUESTION 2.1, QUESTION 2.2 and QUESTION 12 are attached at the end of this question paper. Write your centre number and examination number on these diagram sheets in the spaces provided and insert the diagram sheets inside the back cover of your ANSWER BOOK.

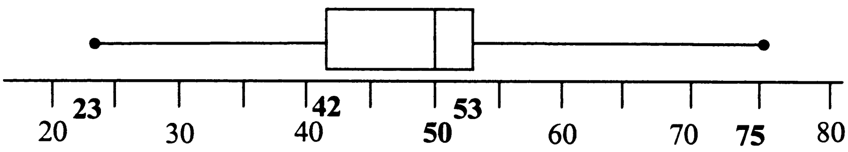
8. An information sheet with formulae is included at the end of this question paper.

9. Number the answers correctly according to the numbering system used in this question paper.

10. Write neatly and legibly.

QUESTION 1

The five-number summary of the heights of trees three months after they were planted is (23 ; 42 ; 50 ; 53 ; 75). This information is shown in the box and whisker diagram below.



1.1 Determine the interquartile range. (2)

1.2 What percentage of plants has a height in excess of 53 cm? (2)

1.3 Between which quartiles do the heights of the trees have the least variation? Explain. (2)
[6]

QUESTION 2

The relationship between blood alcohol levels and the risk of having a car accident has been studied for years. Research has shown the following results:

BLOOD ALCOHOL LEVEL (%)	RELATIVE RISK OF HAVING A CAR ACCIDENT (%)
0,00	1,0
0,05	2,9
0,10	8,5
0,15	24,8
0,20	72,2
0,21	89,5

2.1 Draw a scatter plot on DIAGRAM SHEET 1 to represent the data. (3)

2.2 Draw a line (or curve) of best fit on DIAGRAM SHEET 1. (1)

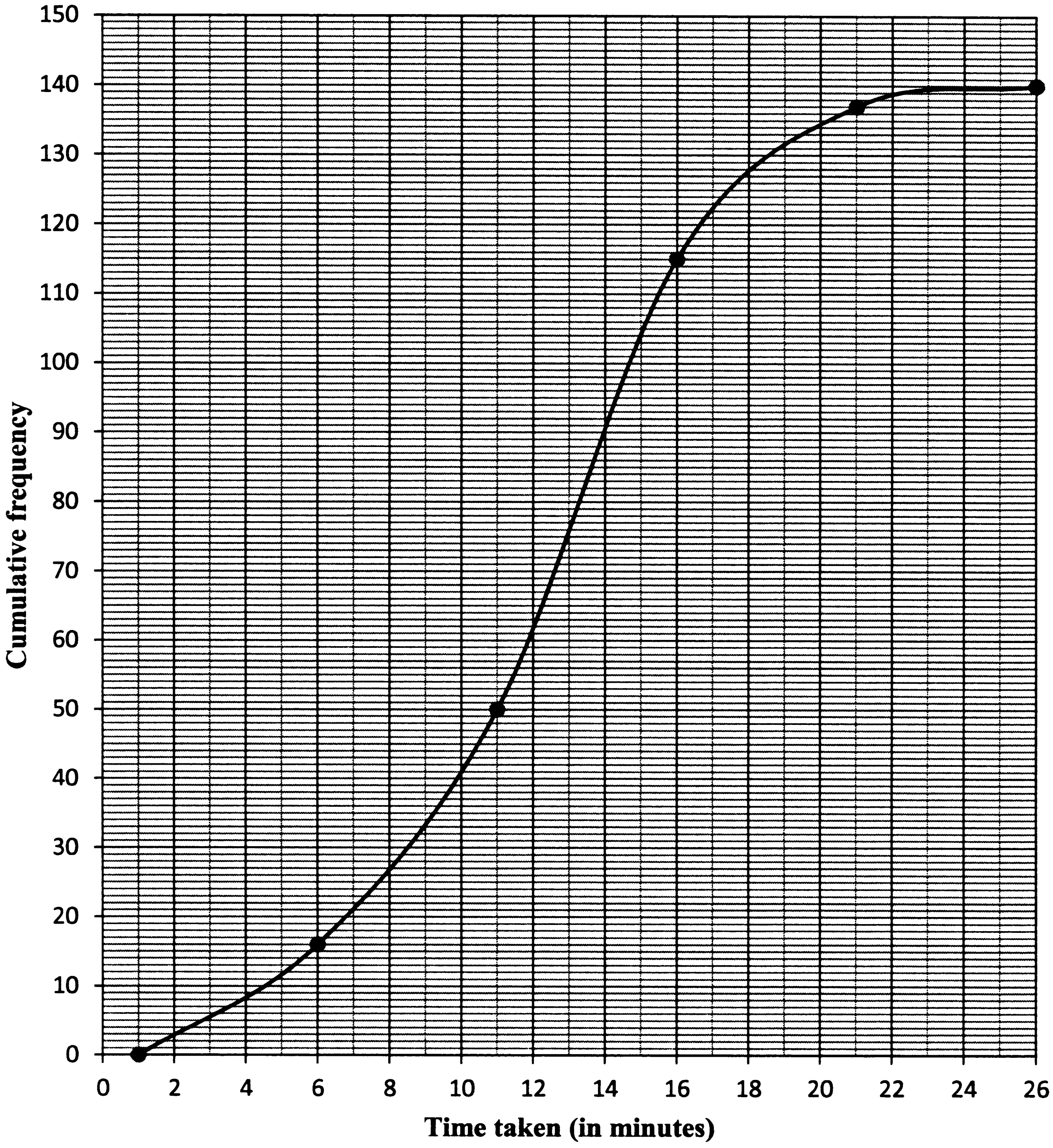
2.3 Describe the trend of the data. (1)

2.4 Estimate the probability of having a car accident when one's blood alcohol level is 0,18%. (The legal limit of the blood alcohol level is 0,05%.) (2)
[7]

QUESTION 3

The cumulative frequency curve (ogive) drawn below shows the time taken (in minutes) for 140 patrons to leave an auditorium after watching a show.

Cumulative frequency curve showing the time taken to leave an auditorium



3.1 Estimate the number of people who took more than 15 minutes to leave the auditorium. (2)

3.2 Estimate the number of people who took between 8 and 12 minutes to leave the auditorium. (2)

3.3 Write down the modal class for the data. (1)

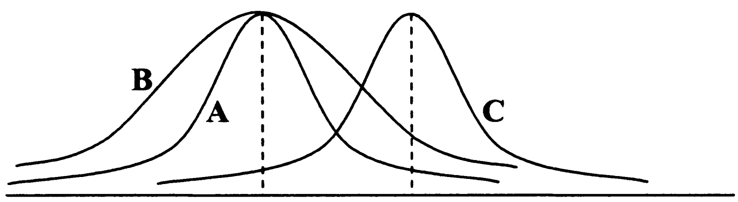
[5]

QUESTION 4

The Grade 10 classes of three schools wrote a term test. All three schools have the same number of learners in Grade 10. The results of the tests have been summarised in the table below.

	SCHOOL A	SCHOOL B	SCHOOL C
Mean	9,8	9,8	14,8
Standard deviation	2,3	3,1	2,3

The distribution of the results is shown in the diagram below.



4.1 In which school (A, B or C) is the majority of the results more widely spread around the mean? Give a reason for your answer. (2)

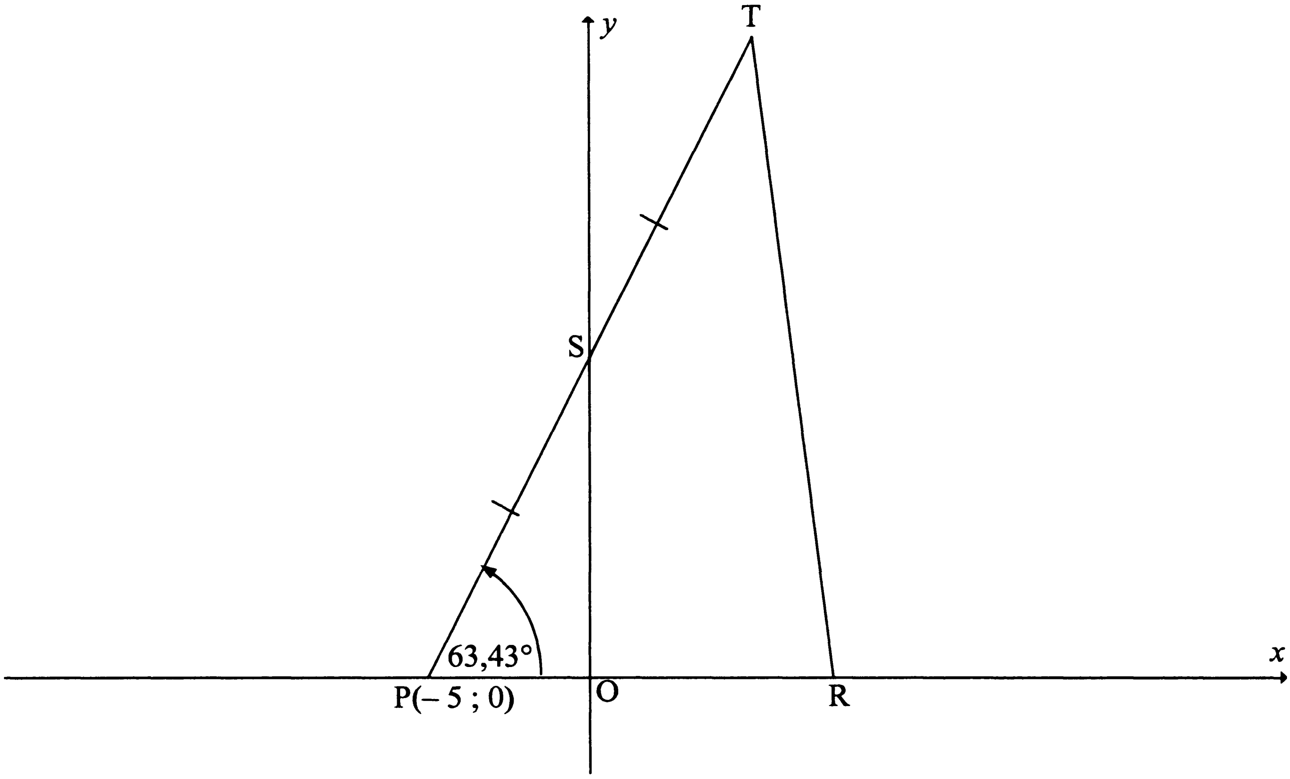
4.2 What is the difference in the spread around the respective means of the marks in School A and School C? (1)

4.3 Explain how the marks of School A must be adjusted to match the marks of School C. (2)

4.4 If each mark in School C is lowered by 10%, explain the effect it will have on the mean and standard deviation of this school. (2)
[7]

QUESTION 5

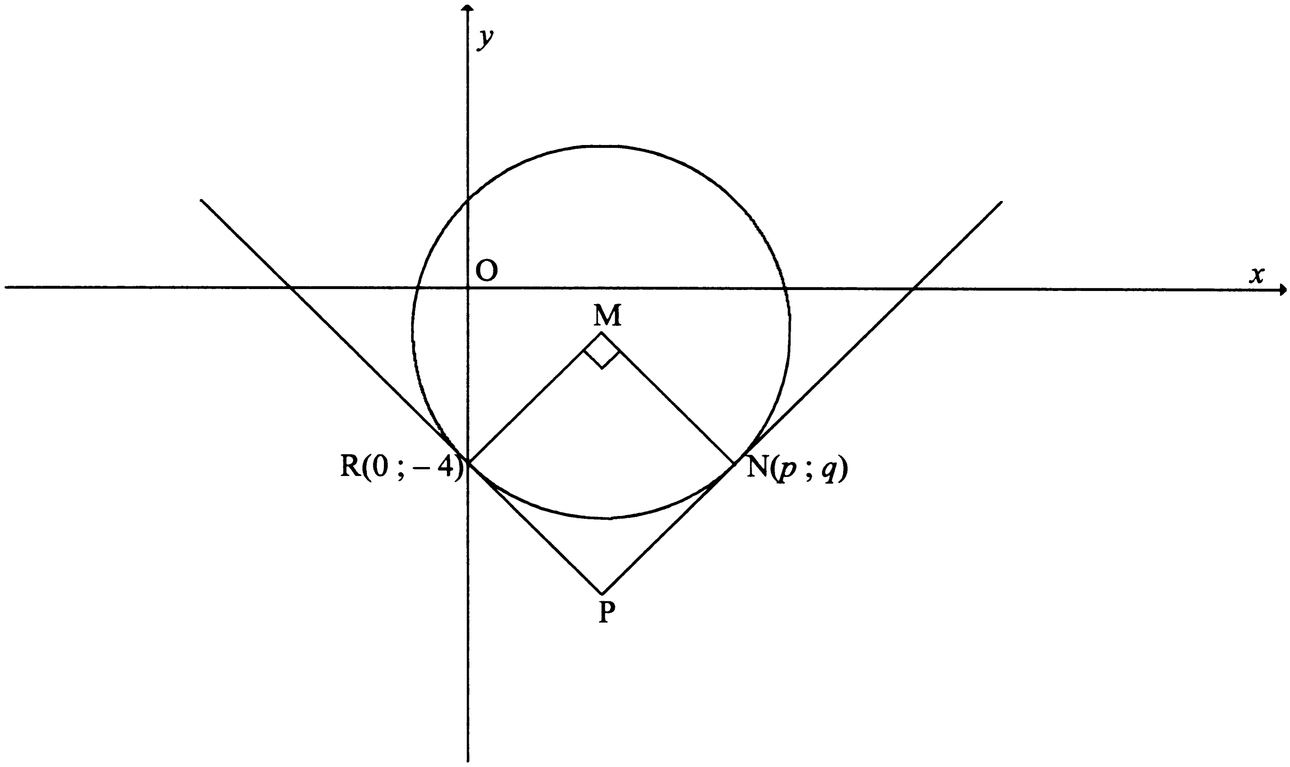
In the diagram below, P is a point (– 5 ; 0). The inclination of line PT is 63,43°. S is the midpoint and the y-intercept of PT. R is a point on the x-axis such that PO : OR = 2 : 3.



5.1	Determine:		
	5.1.1	The gradient of PT, correct to the nearest integer value	(2)
	5.1.2	The equation of PT in the form $y = mx + c$	(2)
	5.1.3	The distance PS in surd form	(3)
	5.1.4	The coordinates of T	(2)
5.2	Determine the coordinates of R.		(2)
5.3	Calculate the area of ΔPTR.		(4) **[15]**

QUESTION 6

In the diagram below, M is the centre of the circle having the equation $x^2 + y^2 - 6x + 2y - 8 = 0$. The circle passes through R(0 ; – 4) and N(p ; q). $R\hat{M}N = 90°$. The tangents drawn to the circle at R and N meet at P.



6.1 Show that M is the point (3 ; – 1). (4)

6.2 Determine the equation of MR in the form $y = mx + c$. (3)

6.3 Show that $q = 2 - p$. (4)

6.4 Determine the values of p and q. (5)

6.5 Determine the equation of the circle having centre O and passing through N. (2)

6.6 Calculate the area of the circle centred at M. (2)

6.7 Calculate the ratio in its simplest form: $\frac{NP}{MP}$ (4)

[24]

QUESTION 7

7.1 Determine the image of $P(x ; y)$ if P is rotated through 90° about the origin in a clockwise direction and then reflected about the y-axis. (2)

7.2 Determine the image of $P(x ; y)$ if P is reflected about the y-axis and then rotated through 90° about the origin in a clockwise direction. (2)

7.3 Mo and Ziya argue about the image of $P(x ; y)$ under the following transformations:

- Rotation through 90° about the origin in a clockwise direction
- Reflection about the y-axis

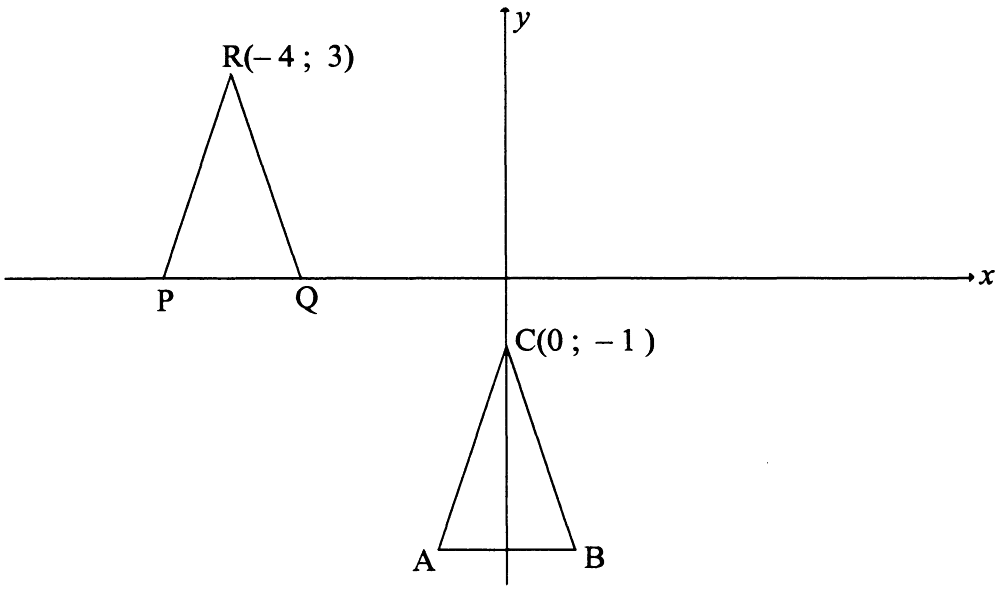
Mo claims that the order in which the transformations are performed will affect the final position of the image. Ziya argues that the final position of the image will be the same, irrespective of the order in which the transformations are performed.

Which of the two, Mo or Ziya, is correct in this case? Explain. (2)

[6]

QUESTION 8

In the diagram, ABC is an isosceles triangle such that vertex C lies at $(0 ; -1)$. AB is parallel to the x-axis and AC = $\sqrt{10}$.



8.1 A rigid transformation is applied to ΔABC to obtain ΔPQR as shown. R(– 4 ; 3) is the image of C. Describe fully, in words, the transformation from ΔABC to ΔPQR. (2)

8.2 ΔPQR is reflected about the line $y = x$. Determine the coordinates of R', the image of R. (2)

8.3 ΔABC is enlarged through the origin to obtain $\Delta A'B'C'$ such that:

$$\frac{\text{area of } \Delta A'B'C'}{\text{area of } \Delta ABC} = 16$$

8.3.1 Determine the scale factor of the enlargement. (1)

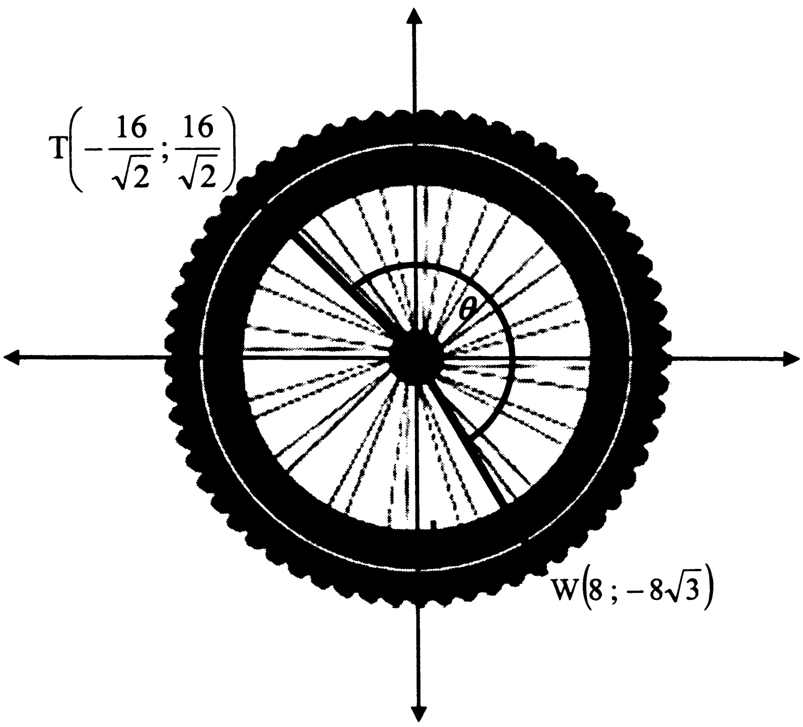
8.3.2 If AC = $\sqrt{10}$ units, write down the length of $A'C'$. (1)

8.4 After a rigid transformation is applied to ΔABC to obtain ΔDEF, F(0 ; 1) is the image of C. If E is the point $(s ; t)$, write down an equation in terms of s and t. (4)

[10]

QUESTION 9

A wheel is positioned so that its centre is directly on the origin in the Cartesian plane.
$T\left(-\frac{16}{\sqrt{2}};\frac{16}{\sqrt{2}}\right)$ is a point on the outer edge of the wheel.
When the wheel is rotated in a clockwise direction about the origin through an angle of θ, T is directly on $W\left(8;-8\sqrt{3}\right)$.



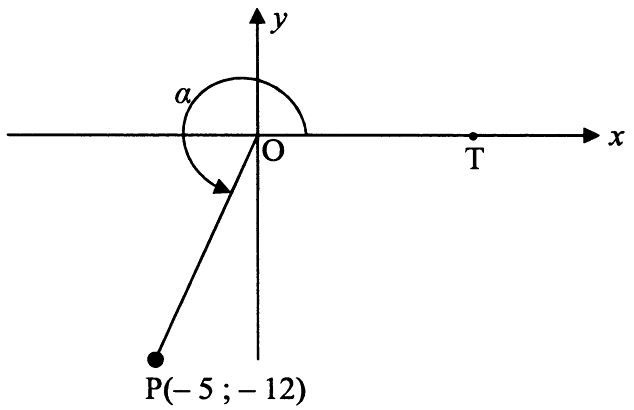
9.1 Show that $\theta = 195°$. (5)

9.2 When the wheel is rotated at a uniform speed in a clockwise direction, it takes 1,3 seconds for T to travel to W. Calculate the speed, in revolutions per minute, at which the wheel is rotated. (5)

[10]

QUESTION 10

In the diagram below, reflex $T\hat{O}P = \alpha$ and P has coordinates $(-5\,;-12)$.



Determine the value of each of the following trigonometric ratios WITHOUT using a calculator:

10.1 $\cos\alpha$ (3)

10.2 $\tan(180° - \alpha)$ (2)

10.3 $\sin(30° - \alpha)$ (3)
[8]

QUESTION 11

11.1 Prove the following identity: $\dfrac{\cos^2(90°+\theta)}{\cos(-\theta)+\sin(90°-\theta)\cos\theta}=\dfrac{1}{\cos\theta}-1$ (6)

11.2 Determine the general solution of: $\tan x\sin x+\cos x\tan x=0$. (7)

11.3 Consider the following expression: $2\sin^2 3x-\sin^2 x-\cos^2 x$

11.3.1 Simplify the expression to a single trigonometric ratio of x. (3)

11.3.2 Write down the maximum value of the expression. (1)

11.4 It is given that $p=\cos\alpha+\sin\alpha$ and $q=\cos\alpha-\sin\alpha$

11.4.1 Determine the following trigonometric ratios in terms of p and/or q:

(a) $\cos 2\alpha$ (3)

(b) $\tan\alpha$ (4)

11.4.2 Simplify $\dfrac{p}{2q}-\dfrac{q}{2p}$ to a single trigonometric ratio of α. (6)

[30]

QUESTION 12

12.1 Draw the graphs of $f(x) = \tan x + 1$ and $g(x) = \cos 2x$ for $x \in [-180° ; 180°]$ on the same system of axes provided on DIAGRAM SHEET 2. Clearly show all intercepts with the axes, turning points and asymptotes. (6)

12.2 Write down the period of g. (1)

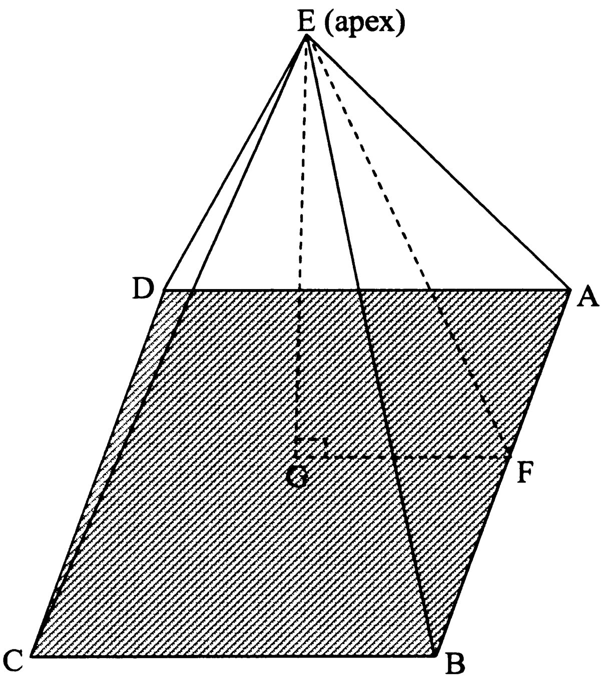
12.3 If $h(x) = -\cos 2(x + 10°)$, describe fully, in words, the transformation from g to h. (2)

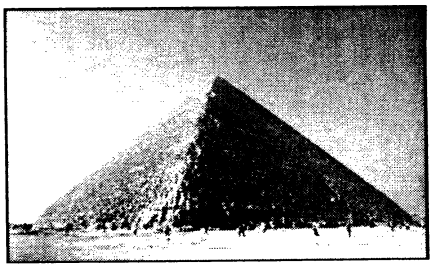
12.4 For which values of x, where $x > 0$, will $f'(x)g(x) > 0$? (4)

[13]

QUESTION 13

The Great Pyramid at Giza in Egypt was built around 2 500 BC. The pyramid has a square base (ABCD) with sides 232,6 metres long. The distance from each corner of the base to the apex (E) was originally 221,2 metres.





Great Pyramid at Giza in Egypt

13.1 Calculate the size of the angle at the apex of a face of the pyramid (for example $C\hat{E}B$). (3)

13.2 Calculate the angle each face makes with the base (for example $E\hat{F}G$, where $EF \perp AB$ in ΔAEB). (6)

[9]

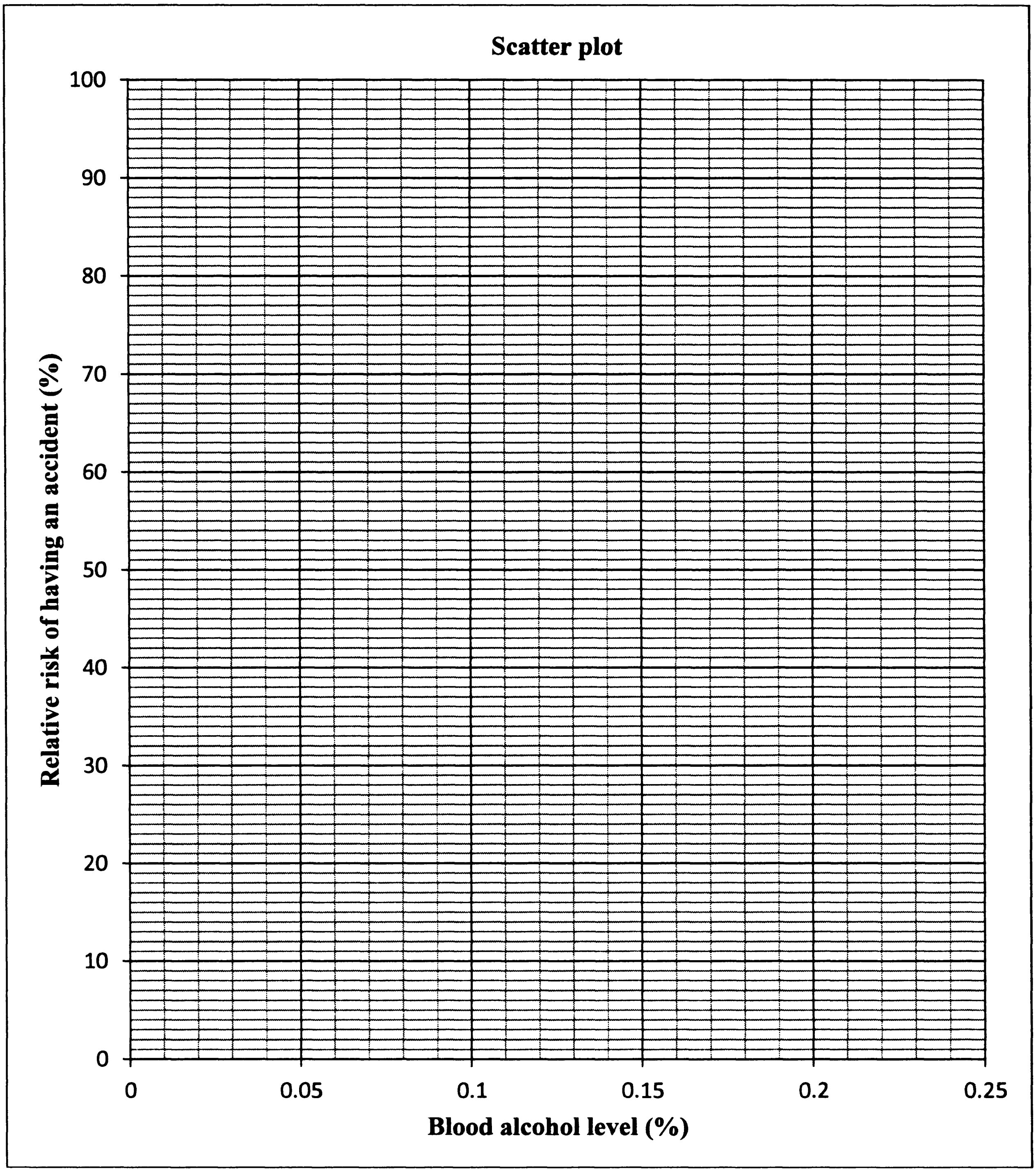
TOTAL: 150

CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTIONS 2.1 and 2.2

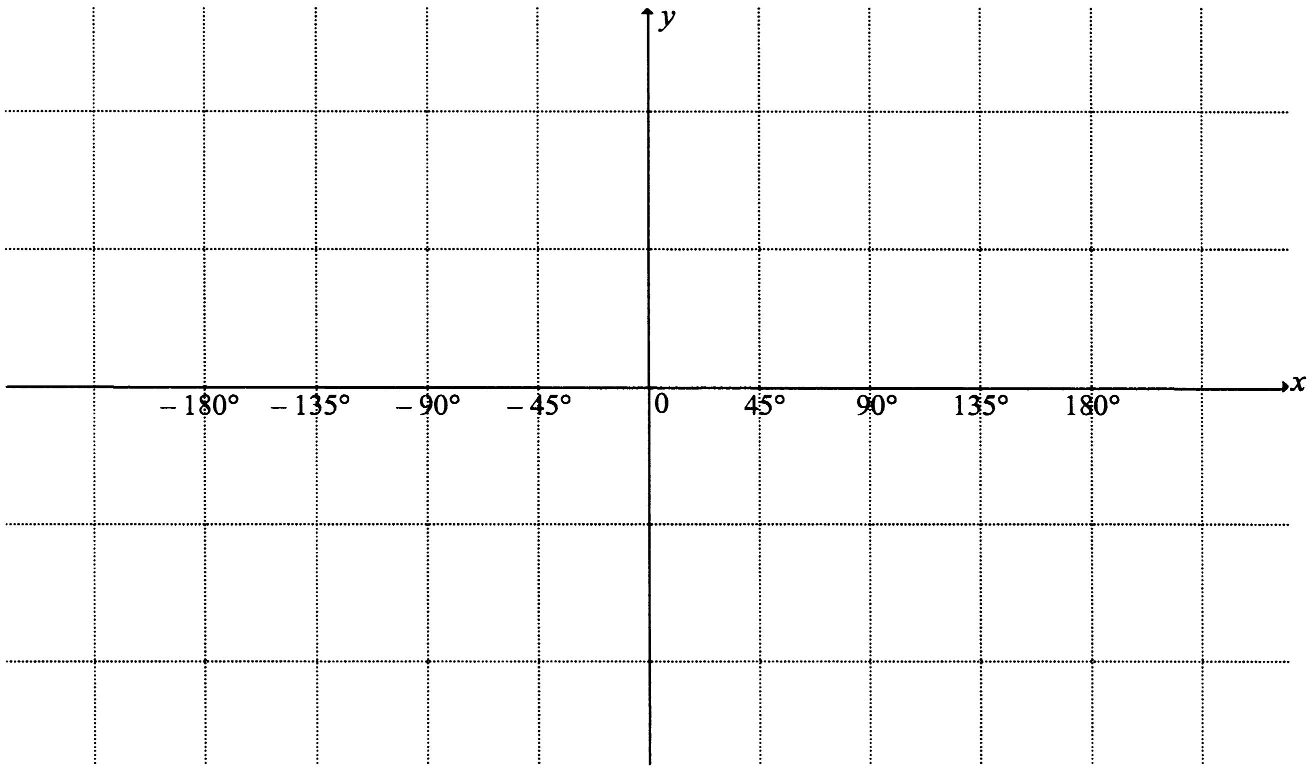


CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTION 12



INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha \, . \cos\alpha$

$$(x ; y) \rightarrow (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$