basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

FEBRUARY/MARCH 2011

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages, 3 diagram sheets and 1 information sheet.

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round answers off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. THREE diagram sheets for answering QUESTION 5.3, QUESTION 10.4 and QUESTION 12.2 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.

9. An information sheet, with formulae, is included at the end of the question paper.

10. Number the answers correctly according to the numbering system used in this question paper.

11. Write legibly and present your work neatly.

QUESTION 1

1.1 Solve for *x*, correct to TWO decimal places, where necessary:

1.1.1 $x(x-1)=12$ (3)

1.1.2 $2x^2+3x-7=0$ (4)

1.1.3 $7x^2+18x-9>0$ (4)

1.2 Solve for x and y simultaneously:

$$2x-y=7$$
$$x^2+xy=21-y^2$$ (7)

1.3 Simplify completely, without the use of a calculator:

$$\left(\sqrt[5]{\sqrt{35}+\sqrt{3}}\right)\left(\sqrt[5]{\sqrt{35}-\sqrt{3}}\right)$$ (3)

[21]

QUESTION 2

The sequence 3 ; 9 ; 17 ; 27 ; … is a quadratic sequence.

2.1 Write down the next term. (1)

2.2 Determine an expression for the n^{th} term of the sequence. (4)

2.3 What is the value of the first term of the sequence that is greater than 269? (4)

[9]

QUESTION 3

3.1 The first two terms of an infinite geometric sequence are 8 and $\frac{8}{\sqrt{2}}$. Prove, without the use of a calculator, that the sum of the series to infinity is $16+8\sqrt{2}$. (4)

3.2 The following geometric series is given: $x = 5 + 15 + 45 + \ldots$ to 20 terms.

3.2.1 Write the series in sigma notation. (2)

3.2.2 Calculate the value of *x*. (3)

[9]

QUESTION 4

4.1 The sum to n terms of a sequence of numbers is given as: $S_n = \frac{n}{2}(5n+9)$

4.1.1 Calculate the sum to 23 terms of the sequence. (2)

4.1.2 Hence calculate the 23rd term of the sequence. (3)

4.2 The first two terms of a geometric sequence and an arithmetic sequence are the same. The first term is 12. The sum of the first three terms of the geometric sequence is 3 more than the sum of the first three terms of the arithmetic sequence.

Determine TWO possible values for the common ratio, r, of the geometric sequence. (6)

[11]

QUESTION 5

Consider the function $f(x) = \frac{3}{x-1} - 2$.

5.1 Write down the equations of the asymptotes of f. (2)

5.2 Calculate the intercepts of the graph of f with the axes. (3)

5.3 Sketch the graph of f on DIAGRAM SHEET 1. (3)

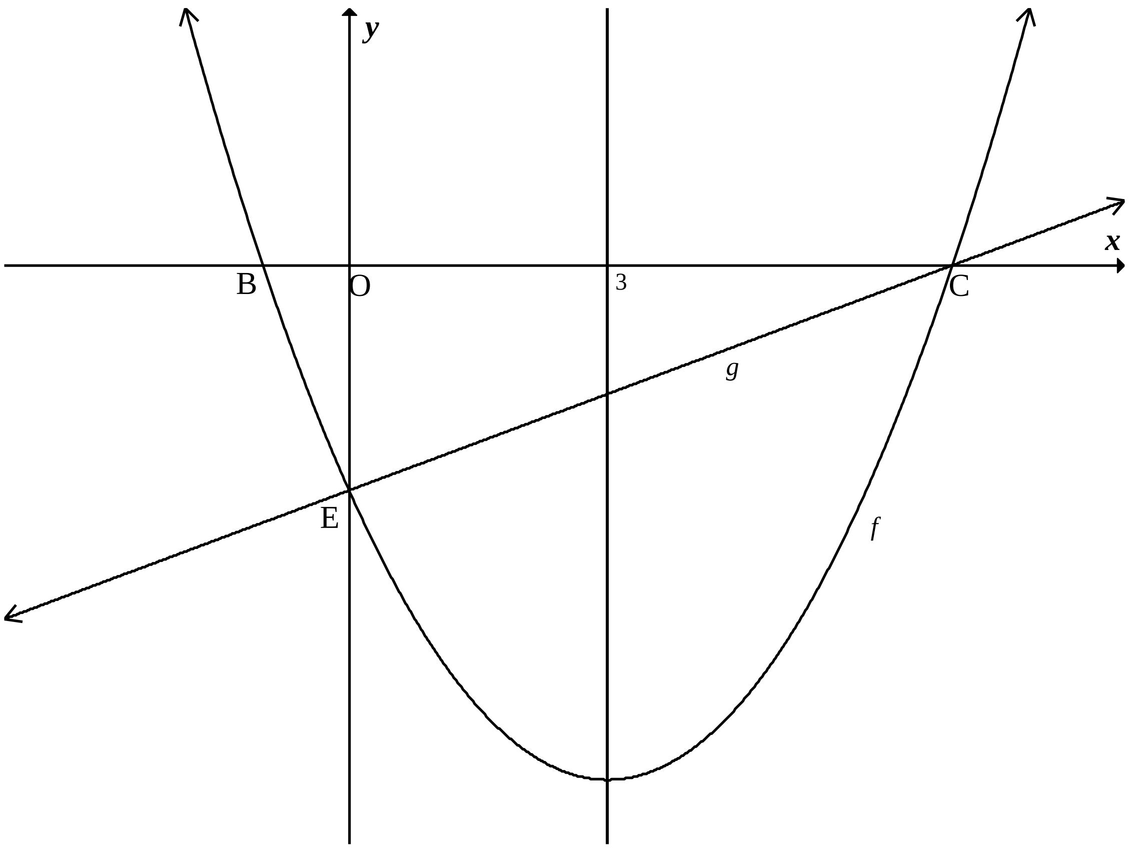
5.4 Write down the range of $y = -f(x)$. (1)

5.5 Describe, in words, the transformation of f to g if $g(x) = \frac{-3}{x+1} - 2$. (2)

[11]

QUESTION 6

A parabola f intersects the x-axis at B and C and the y-axis at E. The axis of symmetry of the parabola has equation $x = 3$. The line through E and C has equation $g(x) = \frac{x}{2} - \frac{7}{2}$.



6.1 Show that the coordinates of C are (7 ; 0). (1)

6.2 Calculate the x-coordinate of B. (1)

6.3 Determine the equation of f in the form $y = a(x - p)^2 + q$. (6)

6.4 Write down the equation of the graph of h, the reflection of f in the x-axis. (1)

6.5 Write down the maximum value of $t(x)$ if $t(x) = 1 - f(x)$. (2)

6.6 Solve for x if $f(x^2 - 2) = 0$. (4)
[15]

QUESTION 7

Consider the function $f(x)=\left(\frac{1}{3}\right)^x$.

7.1 Is f an increasing or decreasing function? Give a reason for your answer. (2)

7.2 Determine $f^{-1}(x)$ in the form $y = \ldots$ (2)

7.3 Write down the equation of the asymptote of $f(x) - 5$. (1)

7.4 Describe the transformation from f to g if $g(x)=\log_3 x$. (2)
[7]

QUESTION 8

8.1 R1 430,77 was invested in a fund paying i% p.a. compounded monthly. After 18 months the fund had a value of R1 711,41. Calculate i. (4)

8.2 A father decided to buy a house for his family for R800 000. He agreed to pay monthly instalments of R10 000 on a loan which incurred interest at a rate of 14% p.a. compounded monthly. The first payment was made at the end of the first month.

8.2.1 Show that the loan would be paid off in 234 months. (4)

8.2.2 Suppose the father encountered unexpected expenses and was unable to pay any instalments at the end of the 120^{th}, 121^{st}, 122^{nd} and 123^{rd} months. At the end of the 124^{th} month he increased his payment so as to still pay off the loan in 234 months by 111 equal monthly payments.
Calculate the value of this new instalment. (7)
[15]

QUESTION 9

9.1 Use the definition to differentiate $f(x)=1-3x^2$. (Use first principles.) (4)

9.2 Calculate $D_x\left[4-\frac{4}{x^3}-\frac{1}{x^4}\right]$. (3)

9.3 Determine $\frac{dy}{dx}$ if $y=\left(1+\sqrt{x}\right)^2$. (3)
[10]

QUESTION 10

Given: $g(x) = (x-6)(x-3)(x+2)$

10.1 Calculate the y-intercept of g. (1)

10.2 Write down the x-intercepts of g. (2)

10.3 Determine the turning points of g. (6)

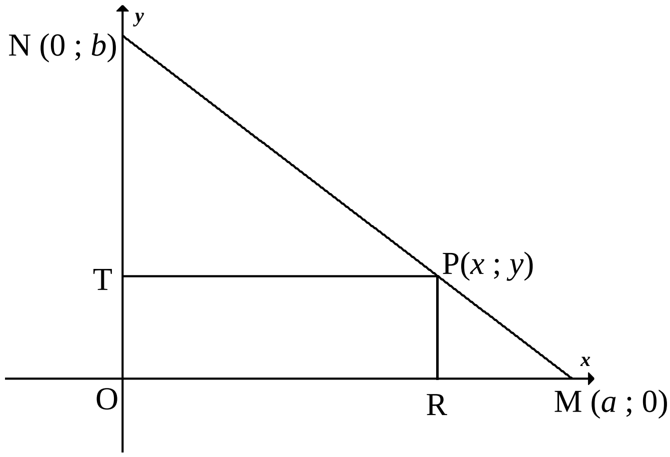
10.4 Sketch the graph of g on DIAGRAM SHEET 2. (4)

10.5 For which values of x is $g(x).g^{/}(x) < 0$? (3)

[16]

QUESTION 11

A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land PTOR to his daughter, giving her the freedom to choose P anywhere along the boundary MN. Let OM = a, ON = b and P(x ; y) be any point on MN.



11.1 Determine an equation of MN in terms of a and b. (2)

11.2 Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN. (6)

[8]

QUESTION 12

While preparing for the 2010 Soccer World Cup, a group of investors decided to build a guesthouse with single and double bedrooms to hire out to visitors. They came up with the following constraints for the guesthouse:

- There must be at least one single bedroom.
- They intend to build at least 10 bedrooms altogether, but not more than 15.
- Furthermore, the number of double bedrooms must be at least twice the number of single bedrooms.
- There should not be more than 12 double bedrooms.

Let the number of single bedrooms be x and the number of double bedrooms be y.

12.1 Write down the constraints as a system of inequalities. (6)

12.2 Represent the system of constraints on the graph paper provided on DIAGRAM SHEET 3. Indicate the feasible region by means of shading. (7)

12.3 According to these constraints, could the guesthouse have 5 single bedrooms and 8 double bedrooms? Motivate your answer. (2)

12.4 The rental for a single bedroom is R600 per night and R900 per night for a double bedroom. How many rooms of each type of bedroom should the contractors build so that the guesthouse produces the largest income per night? Use a search line to determine your answer and assume that all bedrooms in the guesthouse are fully occupied. (3)

[18]

TOTAL: 150

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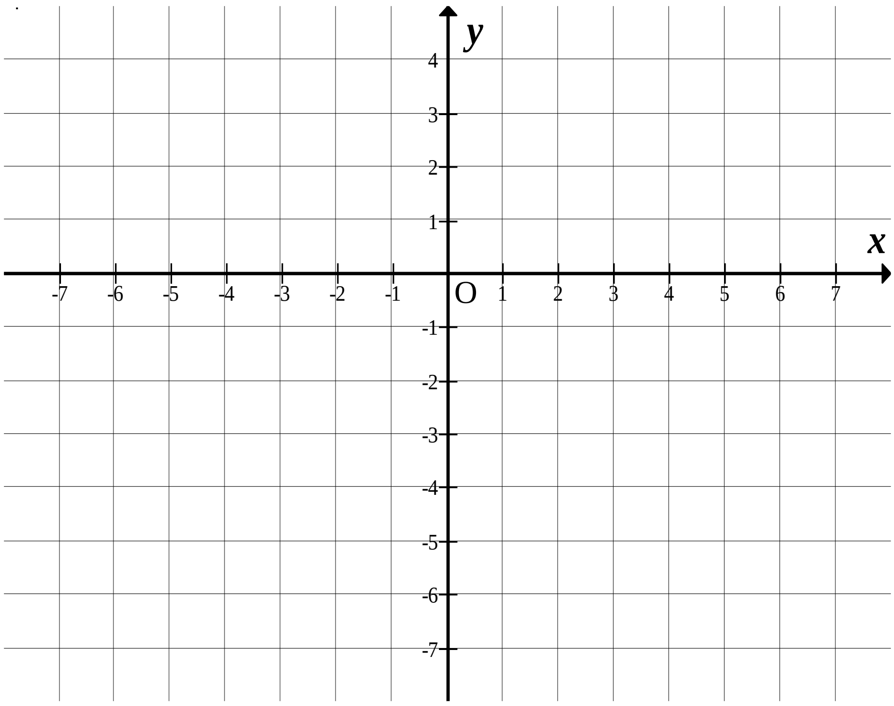
CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 5.3



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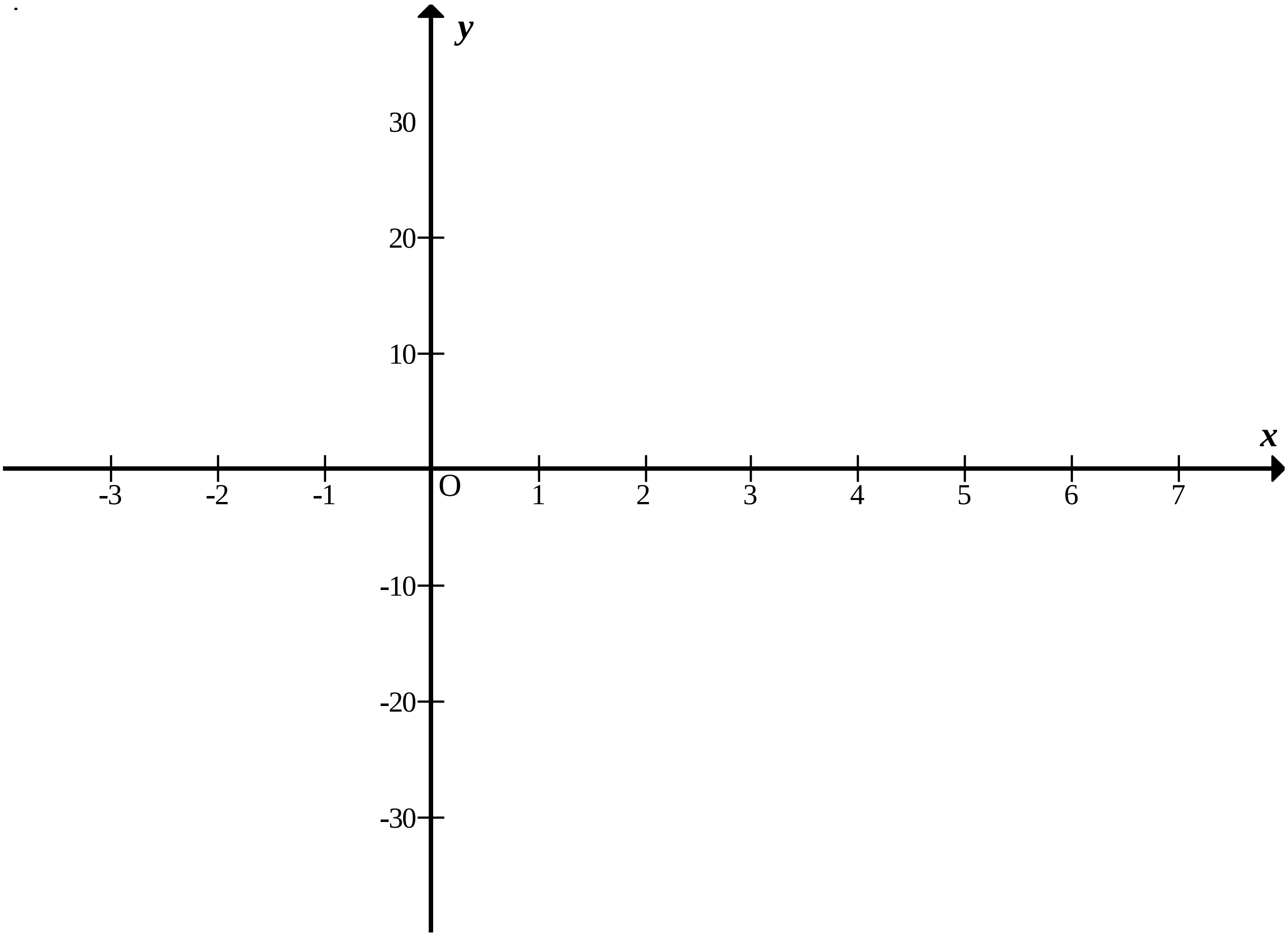
CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTION 10.4

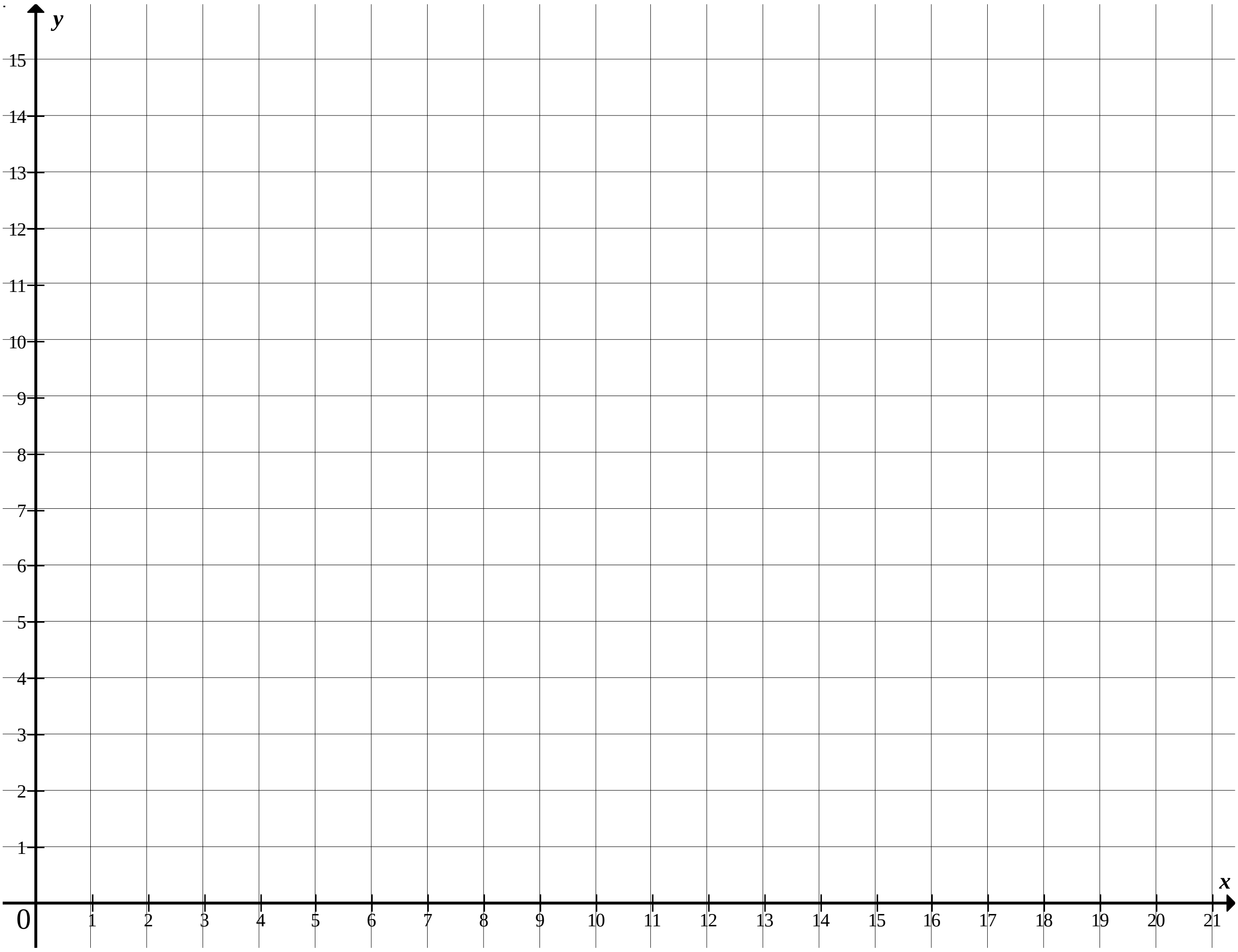


CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 3

QUESTION 12.2



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INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\,\Delta ABC = \frac{1}{2} ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x ; y) \to (x\cos\theta + y\sin\theta ; y\cos\theta - x\sin\theta)$ $\quad (x ; y) \to (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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