MATHEMATICS P3
FEBRUARY/MARCH 2012
MEMORANDUM

MARKS: 100

This memorandum consists of 13 pages.
QUESTION 1

1.1 26; 37

1.2 \( T_1 = 2 \)
\( T_2 = 2 + 3 = 2 + 2(1) + 1 = 5 \)
\( T_3 = 5 + 5 = 5 + 2(2) + 1 = 10 \)
\( T_5 = 10 + 7 = 10 + 2(3) + 1 = 17 \)
\( T_{k+1} = T_k + 2 + 1 \); \( T_1 = 2 \) and \( k \geq 1 \)

OR
\( T_k = T_{k-1} + 2k - 1 \); \( T_1 = 2 \) and \( k \geq 2 \)

QUESTION 2

2.1 Total number of employees
\( = 1 + 2 + 2 + 5 + 30 + 40 + 65 + 5 \)
\( = 150 \)

2.2 Total amount needed
\( = (1 \times 150 000) + (2 \times 100 000) + (2 \times 75 000) + (5 \times 15 000) + (30 \times 10 000) \)
\( + (40 \times 7 500) + (65 \times 6 000) + (5 \times 5 000) \)
\( = \text{R} 1 590 000 \)

2.3 Mean monthly salary
\( = \frac{1 590 000}{150} \)
\( = \text{R} 10 600 \)

2.4 No. Only 10 employees in this company earn more than \text{R} 10 600. The majority (140) of the employees earn below this amount. It is therefore not a good indicator of the average monthly amount earned by an employee.

OR
110 of the 150 employees earn \text{R} 7 500 or less.
### QUESTION 3

#### 3.1
140 – 12 = 128

128 is 1 standard deviation to the left of the mean

∴ percentage of teenagers who sent less than 128 messages

≈ 50% – 34%

≈ 16%

#### 3.2
116 minutes is 2 standard deviations from the mean ∴ 48%

152 minutes is 1 standard deviation from the mean ∴ 34%

Percentage of the teenagers who sent between 116 and 152 messages

≈ 48% + 34%

≈ 82%

**NOTE:** Answer only: Full marks

---

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>104</td>
<td>116</td>
<td>128</td>
<td>140</td>
<td>152</td>
<td>164</td>
</tr>
<tr>
<td>2%</td>
<td>14%</td>
<td>34%</td>
<td>34%</td>
<td>14%</td>
<td>2%</td>
</tr>
</tbody>
</table>

- ✓ 1 standard deviation
- ✓ 50%–34%
- ✓ 16%

- ✓ 48%
- ✓ 34%
- ✓ 82%

(3)
QUESTION 4

4.1 \( a = 91,27 \) \((91,26785714...)\)  
\( b = -4,91 \) \((-4,910714286...)\)  
\( \hat{y} = 91,27 - 4,91x \)  
\( \checkmark \checkmark \, a \)  
\( \checkmark \checkmark \, b \)  
\( \checkmark \checkmark \, \text{equation} \)  

4.2 \( r = -0,87 \) \((-0,8748915491...)\)  
\( \checkmark \checkmark \, \text{answer} \)  

4.3 The greater the number of Saturdays absent, the lower the mark.  
\( \checkmark \checkmark \, \text{number of Saturdays absent} \)  
\( \checkmark \checkmark \, \text{final mark} \)  

4.5 \( \hat{y} = 91,27 - 4,91(4) \)  
\( \approx 71,63\% \)  
\( \approx 72\% \)  
\( \checkmark \checkmark \, \text{substitution} \)  
\( \checkmark \checkmark \, \text{answer} \)

NOTE: Allow for the range 70%–74% for a student who reads off the graph.
QUESTION 5

<table>
<thead>
<tr>
<th></th>
<th>DO NOT PLAY SPORT</th>
<th>PLAY SPORT</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>51</td>
<td>69</td>
<td>120</td>
</tr>
<tr>
<td>Female</td>
<td>49</td>
<td>67</td>
<td>116</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>136</td>
<td>236</td>
</tr>
</tbody>
</table>

5.1.1 \( P(\text{male}) = \frac{120}{236} = \frac{30}{59} = 0.51 \) (0.508474...)

5.1.2 \( P(\text{female and plays sport}) = \frac{67}{236} = 0.28 \) (0.2838983051...)

5.2 No. From the table, \( P(\text{male and do not play sport}) = \frac{51}{236} \), which is greater than zero. Since the probability of the intersection of these two events is greater than zero, these events are not mutually exclusive.

5.3

\[ P(\text{male}) = \frac{120}{236} \]

\[ P(\text{NS}) = \frac{100}{236} \]

\[ P(\text{male}) \times P(\text{NS}) = \frac{120}{236} \times \frac{100}{236} = \frac{750}{3481} = 0.22 \] (0.215455...)

\[ P(\text{male and NS}) = \frac{51}{236} = 0.22 \] (0.2161016949...)

So, \( P(\text{male}) \times P(\text{NS}) = P(\text{male and NS}) \)

Therefore the events 'male' and 'do not play sport' are independent (correct to TWO decimal places).

OR

The events are not independent as there is a discrepancy from the third decimal place.

\[ \text{(4) } [10] \]
QUESTION 6

6.1

\[
\begin{array}{c}
\text{A (0,2)} \\
\text{B (0,3)} \\
\text{C (0,5)} \\
\end{array}
\begin{array}{c}
def (0,01) \\
non \text{ def (0,99)} \\
def (0,02) \\
non \text{ def (0,98)} \\
def (0,06) \\
non \text{ def (0,94)} \\
\end{array}
\begin{array}{c}
(A; D) \\
(A; ND) \\
(B; D) \\
(B; ND) \\
(C; D) \\
(C; ND) \\
\end{array}
\]

✓ first tier
✓ second tier
✓ probabilities
✓ outcomes

6.2.1

\[P(B; ND) = 0,3 \times 0,98 = 0,29\]

Accept: 0,294

✓ 0,3
✓ 0,98
✓ 0,29

6.2.2

\[P(\text{defective}) = P(A; D) + P(B; D) + P(C; D) = (0,2 \times 0,01) + (0,3 \times 0,02) + (0,5 \times 0,06) = 0,04\]

Accept: 0,038

✓ multiplying probabilities
✓ adding probabilities
✓ answer

QUESTION 7

7.1

\[12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 12! = 479 001 600 \text{ different ways}\]

✓ 12
✓ answer

7.2

\[9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 9! = 362 880 \text{ different ways}\]

✓ 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1
✓ 9!

7.3

The items from each department can be arranged in 3! ways.
The departments can be arranged in 4! ways.
Advertisements can be arranged in 3!.4!
\[= 6 \times 24 = 144 \text{ different ways.}\]

✓ 3!
✓ 4!
✓ 144

[7]
QUESTION 8

8.1 \( \hat{P}_1 = 90^\circ \) (\( \angle \) in semicircle)  
\( \checkmark \hat{P}_1 = 90^\circ \) (1)

8.2 \( \hat{P}_4 = 90^\circ \) (vert opp \( \angle \)s)  
BN is a diameter (chord subtends 90°)  
\( \checkmark \hat{P}_4 = 90^\circ \)  
\( \checkmark \) chord subtends 90° (2)

8.3.1 \( \hat{A} = 20^\circ \) (\( \angle \) sum \( \Delta \))  
\( \checkmark \) answer (1)

8.3.2 \( \hat{P}_6 = 70^\circ \) (tan ch th)  
\( \checkmark \) answer (1)

8.3.3 \( \hat{P}_3 = 70^\circ \) (vert opp \( \angle \)s)  
\( \hat{N} = 70^\circ \) (tan ch th)  
\( \hat{P}_4 = 90^\circ \) (proven)  
\( \hat{B} = 20^\circ \) (\( \angle \) sum \( \Delta \))  
\( \checkmark \hat{N} = 70^\circ \)  
\( \checkmark \hat{B} = 20^\circ \) (2)

OR

\( \hat{P}_5 = 20^\circ \) (\( \angle \)s on str line)  
\( \hat{B} = 20^\circ \) (tan ch th)  
\( \checkmark \hat{P}_5 = 20^\circ \)  
\( \checkmark \hat{B} = 20^\circ \) (2) [7]
### QUESTION 9

![Diagram of a circle with labeled points R, S, T, K, and P.]

9.1 \( ST = PT = 2x \) (line from circ centre \( \perp \) ch bis ch)  
\( \checkmark \) S/R 

9.2 In \( \triangle RST \) and \( \triangle PKT \)  
1. \( \hat{R} = \hat{P} \) \( (\angle \text{ in same seg}) \)  
2. \( \hat{S} = \hat{K} \) \( (\angle \text{ in same seg}) \)  
3. \( \hat{R} \hat{T} \hat{K} = \hat{P} \hat{T} \hat{K} = 90^\circ \) (given)  
\( \triangle RST \parallel \triangle PKT \) \( (\angle \angle \angle) \)  
\( \checkmark \) R  

\( \checkmark \) S/R  
\( \checkmark \) S/R  

9.3 \[ \frac{ST}{KT} = \frac{RT}{PT} \] (\( \parallel \) \( \triangle \)s)  
\[ \frac{2x}{x} = \frac{320}{2x} \]  
\[ 4x = 320 \]  
\[ x = 80 \text{ mm} \]  

**OR**

\[ \frac{ST}{RT} = \frac{KT}{PT} \]  
\[ \frac{2x}{x} = \frac{320}{2x} \]  
\[ 2x = 160 \]  
\[ x = 80 \text{ mm} \]  

\( \checkmark \) answer  

\( \checkmark \) substitution  

(3) [7]
**QUESTION 10**

![Diagram of a triangle with points P, Q, R, S, T, and W]

### 10.1

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>WS</td>
<td>SP</td>
<td>3</td>
</tr>
<tr>
<td>SP</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{WS}{SP} = \frac{3}{2}
\]

\[
\frac{WS}{WT} = \frac{3}{2}
\]

\[
WT = \frac{3 \times 6}{2} = 9 \text{ cm}
\]

\[
\begin{align*}
\text{(ST} & \parallel \text{PR; Prop th)} \\
\checkmark \quad WS &= WT \\
\frac{SP}{RT} &= \frac{3}{2} \\
\text{ST} &\parallel \text{PR; Prop th} \\
\checkmark &\quad \text{answer}
\end{align*}
\]

### 10.2

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>WS</td>
<td>SP</td>
<td>RQ</td>
</tr>
<tr>
<td>WR</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

\[
\frac{WS}{SP} = \frac{WR}{RQ} = \frac{3}{2}
\]

\[
9 + 6 = \frac{3}{2}
\]

\[
RQ = 10 \text{ cm}
\]

\[
WQ = 10 + 9 + 6 = 25 \text{ cm}
\]

\[
\begin{align*}
\text{(SR} &\parallel \text{PQ; Prop th)} \\
\checkmark \quad WS &= WR \\
\frac{SP}{RQ} &= \frac{3}{2} \\
\checkmark 9 + 6 &= \frac{3}{2} \\
\checkmark RQ &= 10 \\
\checkmark WQ &= 25
\end{align*}
\]

(4)
QUESTION 11

11.1

Join RO and OP
Let $\hat{O}_1 = 2x$
$\hat{O}_2 = 360^\circ - 2x$ (\$s in a rev)
$\hat{T} = x$ (\$ circ centre = 2 \angle$ circumference)
$\hat{S} = 180^\circ - x$ (\$ circ centre = 2 \angle$ circumference)
$\hat{S} + \hat{T} = x + 180^\circ - x$
$= 180^\circ$

OR

Draw radii OP, OS, OR and OT
Let $\hat{P}_1 = x$, $\hat{S}_2 = w$, $\hat{S}_1 = z$ and $\hat{R}_1 = y$
$\therefore \hat{T}_2 = x$, $\hat{P}_2 = w$, $\hat{R}_2 = z$ and $\hat{T}_1 = y$ (\$s opp = radii)$
$\hat{O}_1 = 180^\circ - 2w$ (\$ sum \Delta$)
Similarly
$\hat{O}_2 = 180^\circ - 2z$, $\hat{O}_3 = 180^\circ - 2y$, $\hat{O}_4 = 180^\circ - 2x$
By angles in a revolution
$\hat{O}_1 + \hat{O}_2 + \hat{O}_3 + \hat{O}_4 = 180^\circ - 2w + 180^\circ - 2z + 180^\circ - 2y + 180^\circ - 2x$
$360^\circ = 720^\circ - 2(w + z + y + x)$
$2(w + z + y + x) = 360^\circ$
$w + z + y + x = 180^\circ$
Draw radii OP, OS, OR and OT

Let \( \hat{P}_1 = x \) and \( \hat{R}_1 = y \)

\[ \therefore \hat{T}_2 = x \text{ and } \hat{T}_1 = y \quad (\angle \text{ opp = radii}) \]

\[ \hat{O}_3 = 180^\circ - 2y \quad (\angle \text{ sum } \Delta) \]

Similarly

\[ \hat{O}_4 = 180^\circ - 2x \]

\[ \hat{S}_2 = \hat{P}_2 \text{ and } \hat{S}_1 = \hat{R}_1 \quad (\angle \text{ opp = radii}) \]

\[ \hat{T}_1 + \hat{T}_2 + \hat{S}_1 + \hat{S}_2 = x + y + 90^\circ - x + 90^\circ - y = 180^\circ \]

\( \checkmark \) construction

\( \checkmark \) Let \( \hat{P}_1 = x \) and \( \hat{R}_1 = y \)

\( \checkmark \hat{O}_3 = 180^\circ - 2y \)

\( \checkmark \hat{S}_2 = \hat{P}_2 \text{ and } \hat{S}_1 = \hat{R}_1 \)

\( \checkmark \hat{T}_1 + \hat{T}_2 + \hat{S}_1 + \hat{S}_2 \)

\( \checkmark \text{ conclusion} \)
11.2.1
\[ \hat{A} = x \quad (\angle \text{circ centre} = 2 \angle \text{circumference}) \]
\[ \hat{F}_1 = 180^\circ - x \quad (\text{opp } \angle \text{s of cyclic quad} = 180^\circ) \]

OR

reflex \( \hat{B} \hat{O} \hat{C} = 360^\circ - 2x \quad (\angle \text{ in revolution}) \)
\[ \hat{F}_1 = 180^\circ - x \quad (\angle \text{circ centre} = 2 \angle \text{circumference}) \]

11.2.2
\[ \hat{F}_2 = x \quad (\text{ext } \angle = \text{ int opp } \angle ) \]
\[ \hat{B}_1 = x = \hat{E} \quad (\text{corres } \angle \text{s}; \text{ EC } \parallel \text{ BF}) \]
\[ \hat{F}_2 = \hat{B}_1 \]
\[ \text{DF} = \text{BD} \quad (\text{sides opp } = \angle \text{s}) \]

OR

\[ \hat{E} = x \quad (\angle \text{circ centre} = 2 \angle \text{circumference}) \]
\[ \hat{B}_1 = \hat{E} = x \quad (\text{corres } \angle \text{s}; \text{ EC } \parallel \text{ BF}) \]
\[ \hat{F}_2 = x \quad (\text{ext } \angle \text{ cyclic quad}) \]
\[ \hat{F}_2 = \hat{B}_1 \]
\[ \text{DF} = \text{BD} \quad (\text{sides opp } = \angle \text{s}) \]
### 11.2.3

\[ \hat{\angle} \angle \text{ opp equal sides in } \triangle \]

\[ \hat{\angle} \text{ CE } \parallel \text{ BF} \]

\[ \hat{\angle} \angle \text{ same seg} \]

\[ \hat{\angle} \text{ opp } \angle \text{ suppl} \]

\[ \hat{\angle} \text{ ext } \angle \text{ cyclic quad} \]

\[ \hat{\angle} \text{ ext } \angle \text{ cyclic quad} \]

\[ \frac{1}{2} \text{DF.BD.sin} \hat{\angle} \]

\[ \frac{1}{2} \text{AO.OC.sin} \hat{\angle} \]

\[ \frac{5.5}{6.6} \]

\[ \frac{25}{36} \]

**TOTAL:** 100