basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2013

MEMORANDUM

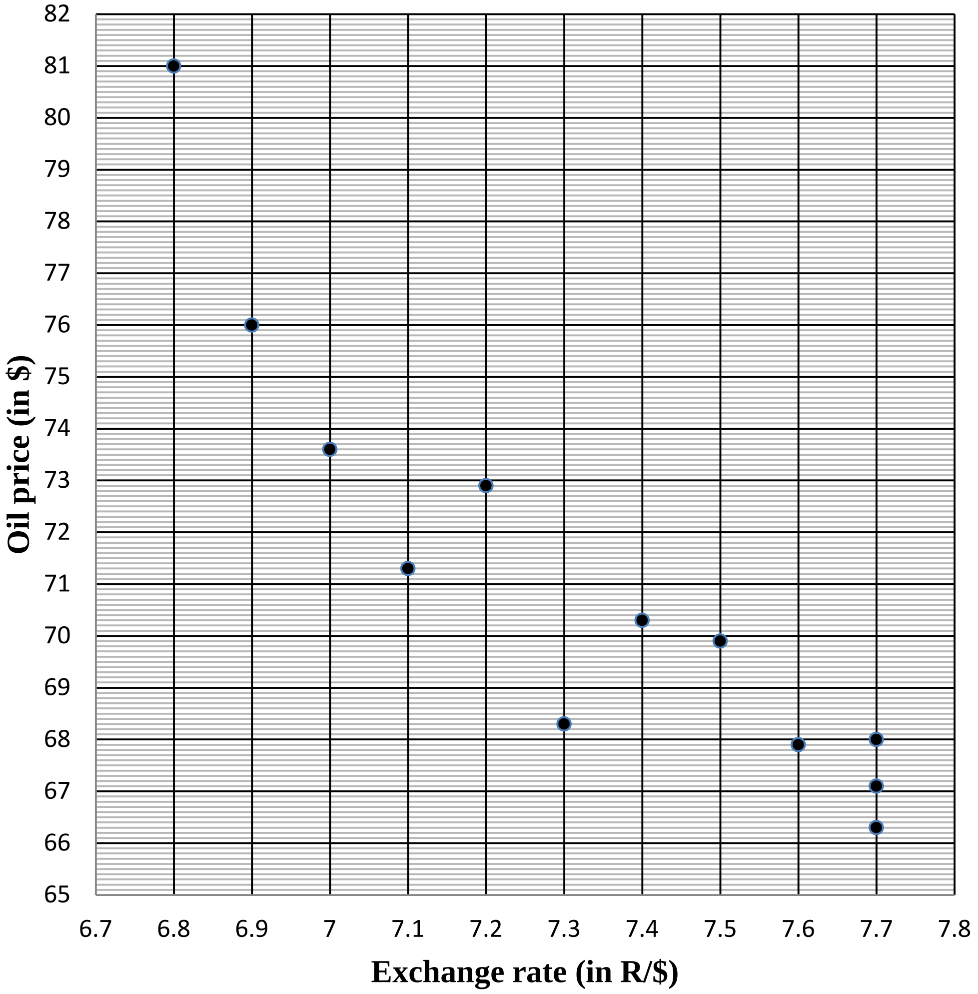
MARKS: 150

This memorandum consists of 21 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in **ALL** aspects of the marking memorandum.

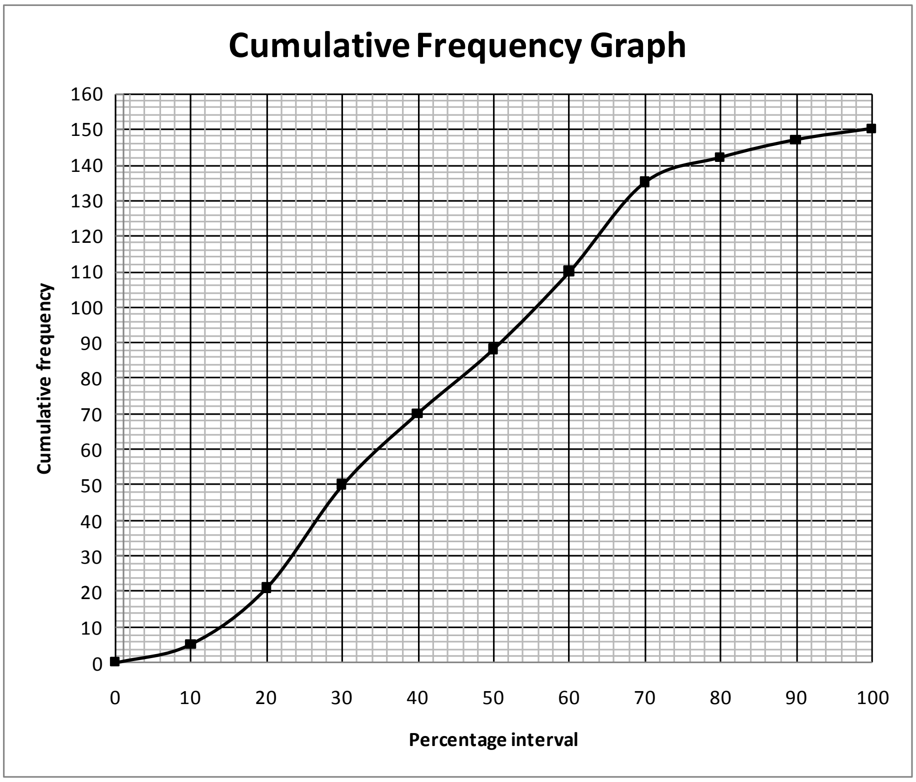
QUESTION 1

1.1	**Scatter plot of exchange rate versus oil price** 	✓ any 4 points correctly plotted ✓ any 9 points correctly plotted ✓ all points correctly plotted (3)
1.2	As the exchange rate (R/$) increases the oil price ($) decreases. **OR** There is a negative correlation between the exchange rate and oil price.	✓✓ reason (2)
1.3	$\text{Mean} = \dfrac{852,6}{12}$ $= 71,05$	✓ 852,6 ✓ 71,05 (2)
1.4	Standard deviation is: $\sigma = 4,09$	✓✓ 4,09 (2)
1.5	2 standard deviations from the mean = 71,05+ 2(4,09) = 79,23 The public will be concerned in December 2010	✓ 79,23 ✓ Dec 2010 (2) **[11]**

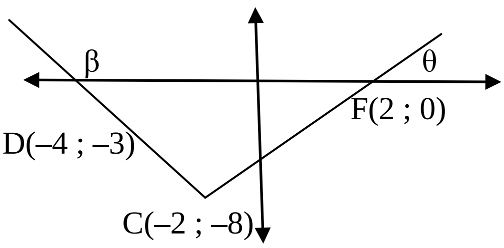
QUESTION 2

2.1	Range of Peter's scores is 94 – 68 = 26	✓ 94 – 68 ✓ 26 (2)
2.2	Vuyani's minimum score is 76	✓ 76 (1)
2.3	Vuyani was more consistent during the year because the range of his scores is more clustered about the median value **OR** the range and inter-quartile range are smaller than Peters.	✓ Vuyani ✓ reason (2) **[5]**

QUESTION 3

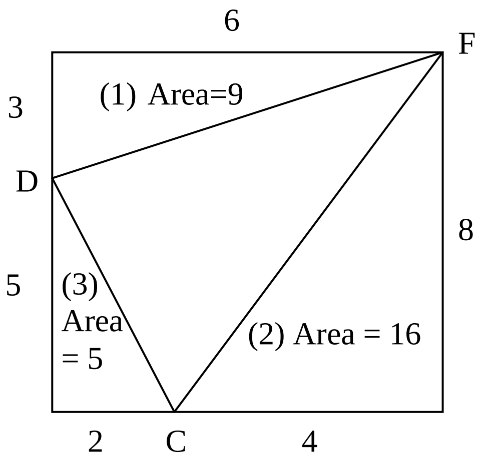
3.1		✓ plotting points at cumulative frequencies ✓ plot against upper limits ✓ grounded at (0 ; 0) ✓ smooth curve (4)
3.2.1	(85 ; ± 144) ± 144 learners that scored below 85% (Accept: 144 to 146)	✓ (85 ; ± 144) ✓ ± 144 learners (2)
3.2.2	Q_1 = 25 or 27 or 26 Q_3 = 61 or 62 or 64 Interquartile range = 36 or 35 or 38	✓ lower quartile ✓ upper quartile ✓ IQR (3) **[9]**

QUESTION 4

4.1	$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{7-(-3)}{1-(-4)}$ $= 2$	✓ substitution ✓ 2 (2)
4.2	AD//BC $m_{AD} = m_{BC} = 2$ $y - y_1 = m(x - x_1)$ $y - (-8) = 2(x - (-2))$ $\therefore y = 2x - 4$	✓ $m_{AD} = 2$ ✓ substitute into formula ✓ $y = 2x - 4$ (3)
4.3	At F: $y = 0$ $0 = 2x - 4$ $x = 2$ F(2 ; 0)	✓ $y = 0$ ✓ $x = 2$ (2)
4.4	D is translated C according to the rule: $D(x;y) \rightarrow C(x+2\ ; y-5)$ A must also be translated according to this rule to B$^{/}$. $\therefore$ A(1 ; 7) $\rightarrow$ B$^{/}$ (3 ; 2) **OR** $x_{B^/} = -2 + (1+4) = 3$ $y_{B^/} = -8 + (7+3) = 5$	✓ $x = 3$ ✓ $y = 2$ (2) ✓ $x = 3$ ✓ $y = 2$ (2)
4.5	$m_{BC} = 2$ $\tan\theta = 2$ $\theta = 63{,}43°$ $m_{DC} = \frac{-8-(-3)}{-2-(-4)} = -\frac{5}{2}$ $\tan\beta = -\frac{5}{2}$ $\beta = 180° - 68{,}20° = 111{,}80°$ $\alpha = 111{,}80° - 63.43° = 48{,}37°$  **OR**	✓ 63,43° ✓ $\tan\beta = -\frac{5}{2}$ ✓ 111,8° ✓ 48,37° (4)

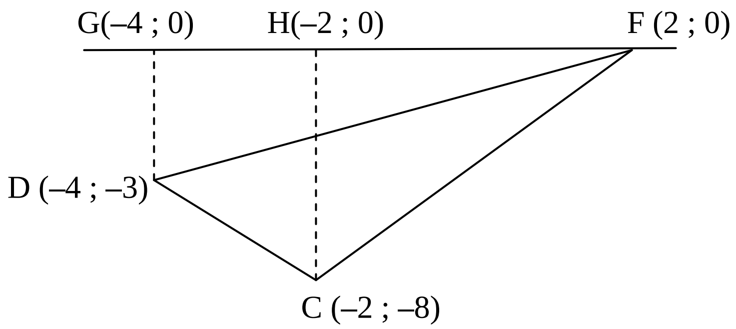
	$DC = \sqrt{(-4+2)^2 + (-3+8)^2}$ $= \sqrt{29}$ $CF = \sqrt{(-2-2)^2 + (-8-0)^2}$ $= \sqrt{80}$ $DF = \sqrt{(2+4)^2 + (0+3)^2}$ $= \sqrt{45}$ $\cos\alpha = \dfrac{29+80-45}{2(\sqrt{29})(\sqrt{80})}$ $= 0,6643...$ $\alpha = 48,37°$	✓Subst in cos-formula ✓cos α subject ✓0,6643... ✓48,37° (4)
	OR $DC = \sqrt{(-4+2)^2 + (-3+8)^2}$ $= \sqrt{29}$ $DB = \sqrt{(3+4)^2 + (2+3)^2}$ $= \sqrt{74}$ $BC = \sqrt{(3+2)^2 + (2+8)^2}$ $= \sqrt{125}$ $\cos\alpha = \dfrac{29+125-74}{2(\sqrt{29})(\sqrt{125})}$ $= 0,6643...$ $\alpha = 48,37°$	✓Subst in cos-formula ✓cos α subject ✓0,6643... ✓48,37° (4)
4.6	$DC = \sqrt{(-4+2)^2 + (-3+8)^2}$ $= \sqrt{29}$ $CF = \sqrt{(-2-2)^2 + (-8-0)^2}$ $= \sqrt{80}$ Area $\Delta DCF = \frac{1}{2}.DC.CF.\sin\alpha$ $= \frac{1}{2}(\sqrt{29})(\sqrt{80})\sin 48,37°$ $= 18 \text{ units}^2$	✓ substitution into formula ✓ $\sqrt{29}$ ✓ substitution into formula ✓ $\sqrt{80}$ ✓ substitution into the area rule ✓ 18 (6)

OR



Area ΔDCF = Area of rectangle – (1) – (2) – (3) = 48 – 9 – 5 – 16 = 18 sq units	✓ establishing rectangle and area ✓ relationship of areas ✓ (1) = 9 ✓(2) = 16 ✓(3) = 5 ✓18 units^2 (6)

OR



Area CDF = Area CHF + Area CDGH - Area DGF $= \frac{1}{2} \times 4 \times 8 + 2 \times \frac{1}{2}(3 \times 8) - \frac{1}{2} \times 6 \times 3$ $= 16 + 11 - 9$ $= 18$	✓ drawing perpendiculars ✓ relationship of areas ✓ 16 ✓ 11 ✓ 9 ✓18 units^2 (6)

[19]

QUESTION 5

5.1.1	$x^2+y^2+2x+6y+2=0$ $x^2+2x+1+y^2+6y+9=-2+10$ $(x+1)^2+(y+3)^2=8$ $\text{M}(-1\,;\,-3)$	✓ $(x+1)^2+(y+3)^2=8$ ✓ -1 ✓ -3 (3)
5.1.2	radius of circle $\text{C}_1=\sqrt{8}$	✓ $\sqrt{8}$ (1)
5.2	$x^2+(x-2)^2+2x+6(x-2)+2=0$ $x^2+x^2-4x+4+2x+6x-12+2=0$ $2x^2+4x-6=0$ $x^2+2x-3=0$ $(x+3)(x-1)=0$ $x=-3$ *or* $x\neq 1$ $y=-3-2=-5$ $\therefore \text{D}(-3;-5)$ **OR** $(x+1)^2+(y+3)^2=8$ *subst.* $y=x-2$ $(x+1)^2+(x-2+3)^2=8$ $(x+1)^2+(x+1)^2=8$ $x^2+2x-3=0$ $(x+3)(x-1)=0$ $x=-3$ *or* $x\neq 1$ $y=-3-2=-5$ **OR** $(x+1)^2+(y+3)^2=8$ *subst.* $y=x-2$ $(x+1)^2+(x-2+3)^2=8$ $(x+1)^2+(x+1)^2=8$ $(x+1)^2=4$ $x+1=\pm 2$ $x=-3$ *or* $x\neq 1$ $y=-3-2=-5$ **OR**	✓ substitution ✓ standard form ✓ factors ✓ value of *x* ✓ value of *y* (5) ✓ substitution ✓ standard form ✓ factors ✓ value of *x* ✓ value of *y* (5) ✓ substitution ✓ simplification ✓ square root of both sides ✓ value of *x* ✓ value of *y*

	PM makes 45° with the x-axis. $\sqrt{8}=\sqrt{2^2+2^2}$ Therefore: $x_D = x_M - 2 = -1-2 = -3$ $y_D = -3-2 = -5$	✓✓ $\sqrt{8}=\sqrt{2^2+2^2}$ ✓ value of x ✓ value of y (5)
5.3	MD $\perp$ DB (tangent $\perp$ radius) $MB^2 = MD^2 + DB^2$ (Pythagoras) $= (\sqrt{8})^2 + (4\sqrt{2})^2$ $= 40$ MB is the radius of C_2 $MB = \sqrt{40}$	✓ tangent $\perp$ radius ✓ substitution into Pythagoras ✓ $\sqrt{40}$ (3)
5.4	$(x+1)^2 + (y+3)^2 = 40$	✓ LHS ✓ RHS (2)
5.5	Distance from $(2\sqrt{5}\,;\,0)$ to centre $=\sqrt{(2\sqrt{5}+1)^2+(0+3)^2}$ $= 6{,}24$ $6{,}24 < 6{,}32\ (\sqrt{40})$ Distance from $(2\sqrt{5}\,;\,0)$ to centre < radius of circle. $(2\sqrt{5}\,;\,0)$ lies inside the circle.	✓substitution into distance formula ✓ 6,24 ✓6,24 < 6,32 ✓ conclusion (4) **[18]**

QUESTION 6

6.1.1	$A(-5\,;3)$ $A^{/}(-5+4\,;\,3-3)=(-1\,;\,0)$	✓ -1 ✓ 0 (2)
6.1.2	$A^{/}\,(-5\,;-3)$	✓ -5 ✓ -3 (2)
6.2.1	Scale factor of enlargement is $\frac{K^{/}M^{/}}{KM}=\frac{15}{10}=\frac{3}{2}$ **OR** $K(-4;2)\rightarrow K^{/}(-6\,;3)=K^{/}\left(\frac{3}{2}\times-4;\,\frac{3}{2}\times2\right)$ Scale factor is $\frac{3}{2}$	✓ $\frac{K^{/}M^{/}}{KM}$ ✓ $\frac{3}{2}$ ✓ $\left(\frac{3}{2}\times-4;\frac{3}{2}\times2\right)$ ✓ $\frac{3}{2}$ (2)
6.2.2	$(x;y)\rightarrow\left(\frac{3}{2}x\,;\,\frac{3}{2}y\right)$	✓ $\frac{3}{2}x$ ✓ $\frac{3}{2}y$ (2)
6.2.3	$P^{/}\left(\frac{3}{2}\times3\,;\,2\times\frac{3}{2}\right)$ $=P^{/}\left(\frac{9}{2}\,;\,3\right)$	✓ $\frac{9}{2}$ ✓ 3 (2)
6.2.4	$a=1$	✓ ✓ $a=1$ (2)
6.2.5	$K^{//}(4;-2)$	✓ 4 ✓ -2 (2)

6.2.6	$K^{///}K^{/}=5$
	$K^{/}M^{///}=15$
	$\frac{K^{/}K^{///}}{K^{/}M^{///}}=\frac{5}{15}=\frac{1}{3}$
		✓ $K^{///}K^{/}=5$
		✓ $K^{/}M^{///}=15$
		✓ $\frac{1}{3}$
		(3)
		[17]

QUESTION 7

7.1	$K^{/}(b\ ;\ -a)$	✓ b ✓ $-a$ (2)

7.2	$K^{//}(b\cos\theta - a\sin\theta\ ;\ -a\cos\theta - b\sin\theta)$
	
	OR
	$K^{//}(a\cos(90+\theta)+b\sin(90^\circ+\theta)\ ;\ b\cos(90^\circ+\theta)-a\sin(90^\circ+\theta))$
	$= K^{//}(-a\sin\theta + b\cos\theta\ ;\ -b\sin\theta - a\cos\theta)$
		✓
		$b\cos\theta - a\sin\theta$
		✓
		$-a\cos\theta - b\sin\theta$
		(2)

7.3	$T^{//}(-(-4)\sin\theta + (-2)\cos\theta\ ;\ -(-2)\sin\theta - (-4)\cos\theta)$
	$= T^{//}(4\sin\theta - 2\cos\theta\ ;\ 2\sin\theta + 4\cos\theta)$
	OR
	$T^{//}(-2\cos\theta - (-4)\sin\theta\ ;\ -(-4)\cos\theta - (-2)\sin\theta)$
	$= T^{//}(-2\cos\theta + 4\sin\theta\ ;\ 4\cos\theta + 2\sin\theta)$
		✓
		$4\sin\theta - 2\cos\theta$
		✓
		$2\sin\theta + 4\cos\theta$
		(2)
		
		✓
		$4\sin\theta - 2\cos\theta$
		✓
		$2\sin\theta + 4\cos\theta$
		(2)

7.4	$2\sqrt{3}+1 = 4\sin\theta - 2\cos\theta$ ....(1) $\sqrt{3}-2 = 2\sin\theta + 4\cos\theta$ ....(2) $(2)\times 2:\ \ 2\sqrt{3}-4 = 4\sin\theta + 8\cos\theta$ ....(3) $(1)-(3):\ \ 5 = -10\cos\theta$ $-\frac{1}{2} = \cos\theta$ $\therefore\ \theta = 180^\circ - 60^\circ = 120^\circ$ **OR** $2\sqrt{3}+1 = 4\sin\theta - 2\cos\theta$ ....(1) $\sqrt{3}-2 = 2\sin\theta + 4\cos\theta$ ....(2) $(1)\times 2:\ \ 4\sqrt{3}+2 = 8\sin\theta - 4\cos\theta$ ....(3) $(2)+(3):\ \ 5\sqrt{3} = 10\sin\theta$ $\frac{\sqrt{3}}{2} = \sin\theta$ $\therefore\ \theta = 180^\circ - 60^\circ = 120^\circ$ **OR**	✓ substitution to form equation ✓ substitution to form equation ✓ $5 = -10\cos\theta$ ✓ $-\frac{1}{2} = \cos\theta$ ✓ 120° (5) ✓ substitution to form equation ✓ substitution to form equation ✓ $5\sqrt{3} = 10\sin\theta$ ✓ $\frac{\sqrt{3}}{2} = \sin\theta$ ✓ 120° (5)

	$m_{OT} = \frac{1}{2} \Rightarrow \tan X\hat{O}T = \frac{1}{2}$ $X\hat{O}T = 206{,}565...°$ $m_{OT'} = \frac{\sqrt{3}-2}{2\sqrt{3}+1} \Rightarrow \tan X\hat{O}T'' = \frac{\sqrt{3}-2}{2\sqrt{3}+1} = -0{,}06....$ $X\hat{O}T = -3{,}434°$ $90° + \theta = 209{,}99...° \approx 210°$ $\theta = 120°$	✓ $\tan X\hat{O}T = \frac{1}{2}$ ✓ 206.565...° ✓ – 0,06... ✓ – 3.434° ✓ 120° (5)
	OR $(TT')^2 = OT^2 + (OT')^2 - 2(OT)(OT').\cos(90° + \theta)$ $40 + 20\sqrt{3} = 40 - 40.\cos(90° + \theta)$ $\cos(90° + \theta) = -\frac{\sqrt{3}}{2}$ $90° + \theta = 150°$ $\theta = 60°$	✓$(TT')^2 = 40 + 20\sqrt{3}$ ✓substitution in cos-rule ✓simplification ✓150° ✓60° (5) **[11]**

QUESTION 8

8.1	$1-\sin^2\theta+3-\cos^2\theta$ $=4-(\sin^2\theta+\cos^2\theta)$ $=3$ **OR** $\cos^2\theta+3-\cos^2\theta$ $=3$	✓ simplification ✓ 3 (2) ✓ substitution with identity ✓ 3 (2)
8.2	$\sqrt{4^{\sin 150°}.2^{3\tan 225°}}$ $=\sqrt{4^{\sin 30°}.2^{3\tan 45°}}$ $=\sqrt{(2^2)^{\frac{1}{2}}.2^3}$ $=\sqrt{16}$ $=4$ **OR** $\sin 150°=\frac{1}{2}$ $\tan 225°=1$ $\sqrt{4^{\sin 150°}2^{3\tan 225°}}$ $=\sqrt{4^{\frac{1}{2}}2^3}$ $=\sqrt{2.2^3}$ $=\sqrt{16}$ $=4$	✓ rewrite using reduction formula ✓ substituting special angles ✓ simplification ✓ 4 (4) ✓ $\sin 150°=\frac{1}{2}$ ✓ $\tan 225°=1$ ✓ $4^{\frac{1}{2}}=2$ ✓ 4 (4)
8.3	$LHS=\frac{\cos^2 x(\sin^2 x+\cos^2 x)}{1-\sin x}$ $=\frac{\cos^2 x.(1)}{1-\sin x}$ $=\frac{(1-\sin^2 x)}{1-\sin x}$ $=\frac{(1+\sin x)(1-\sin x)}{1-\sin x}$ $=1+\sin x$ $=RHS$	✓ factorisation ✓ 1 ✓ $1-\sin^2 x$ ✓ factors (4)

8.4	$\cos 3\theta$ $= \cos(2\theta + \theta)$ $= \cos 2\theta.\cos\theta - \sin 2\theta.\sin\theta$ $= (2\cos^2\theta - 1).\cos\theta - 2\sin\theta.\cos\theta.\sin\theta$ $= 2\cos^3\theta - \cos\theta - 2\sin^2\theta.\cos\theta$ $= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta).\cos\theta$ $= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$ $= 4\cos^3\theta - 3\cos\theta$	✓expansion ✓ $2\cos^2\theta - 1$ ✓ $2\sin\theta.\cos\theta$ ✓$1-\cos^2\theta$ (4)
8.5	$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ $\cos 3(20°) = 4\cos^3(20°) - 3\cos(20°)$ $\frac{1}{2} = 4x^3 - 3x$ $8x^3 - 6x - 1 = 0$	✓θ = 20° ✓cos 60° = $\frac{1}{2}$ (2) **[16]**

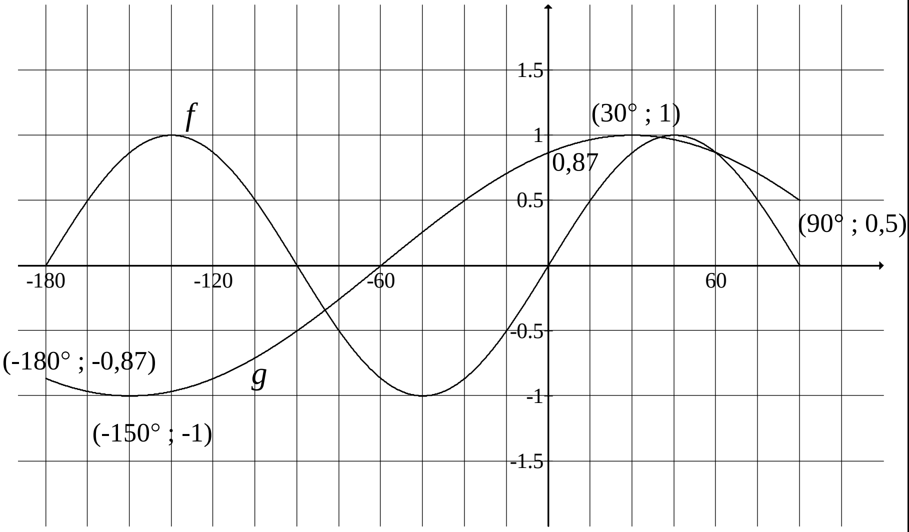
QUESTION 9

9.1	$\frac{\cos 160°.\tan 200°}{2\sin(-10°)}$ $= \frac{(-\cos 20°)(\tan 20°)}{2(-\sin 10°)}$ $= \frac{(-\cos 20°)\left(\frac{\sin 20°}{\cos 20°}\right)}{-2\sin 10°}$ $= \frac{2\sin 10°\cos 10°}{2\sin 10°}$ $= \cos 10°$	✓ $-\cos 20°$ ✓ $\tan 20°$ ✓ $-\sin 10°$ ✓ $\frac{\sin 20°}{\cos 20°}$ ✓ $2\sin 10°\cos 10°$ ✓ $\cos 10°$ (6)
9.2.1	$LHS = \cos(x+45°).\cos(x-45°)$ $= (\cos x.\cos 45° - \sin x\sin 45°)(\cos x\cos 45° + \sin x\sin 45°)$ $= \cos^2 x.\cos^2 45° - \sin^2 x.\sin^2 45°$ $= \cos^2 x\left(\frac{\sqrt{2}}{2}\right)^2 - \sin^2\left(\frac{\sqrt{2}}{2}\right)^2$ or $\left[\cos^2 x\left(\frac{1}{\sqrt{2}}\right)^2 - \sin^2 x\left(\frac{1}{\sqrt{2}}\right)^2\right]$ $= \frac{1}{2}\cos^2 x - \frac{1}{2}\sin^2 x$ $= \frac{1}{2}(\cos^2 x - \sin^2 x)$ $= \frac{1}{2}\cos 2x$	✓ expand $\cos(x+45°)$ ✓ expand $\cos(x-45°)$ ✓ substitute special angles ✓ simplification (4)
	OR	
	$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$ $\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$ Let $\alpha = x + 45°$ and $\beta = x - 45°$ $\therefore \cos(x+45°)\cos(x-45°)$ $= \frac{1}{2}(\cos((x+45°+x-45°) + \cos(x+45°-x+45°))$ $= \frac{1}{2}(\cos 2x + \cos 90°)$ $= \frac{1}{2}\cos 2x$	✓✓ deriving identity ✓ substitution ✓ simplification (4)

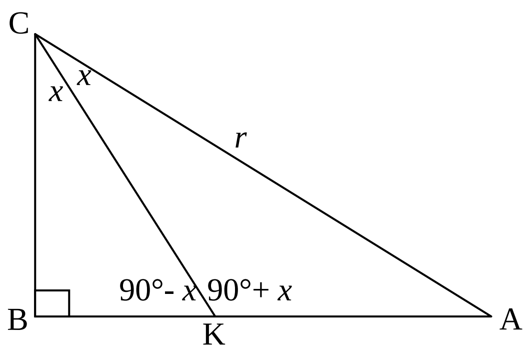
9.2.2	$\cos(x+45°)\cos(x-45°)$ has a minimum when $\frac{1}{2}\cos 2x$ has a minimum. The minimum value of $\cos 2x$ is –1 $\cos 2x = -1$ $2x = 180°$ $x = 90°$	✓ minimum value of –1 ✓$2x$ = 180° ✓ x = 90° (3) **[13]**

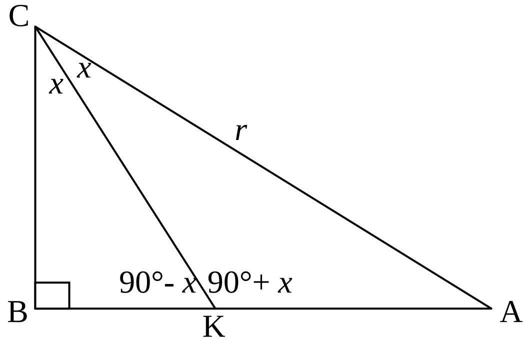
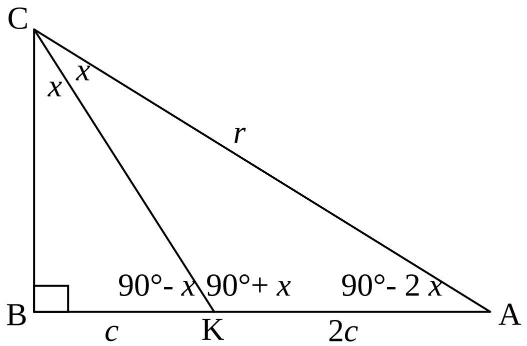
QUESTION 10

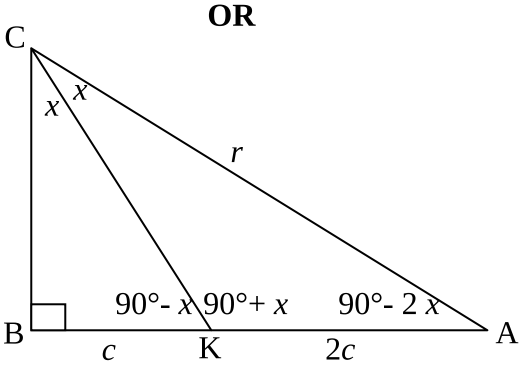
10.1	Range $= [-1\ ;\ 1]$	✓✓ [–1 ; 1] (2)
10.2	$f\left(\frac{3}{2}x\right) = \sin 2\left(\frac{3}{2}x\right)$ $= \sin 3x$ $\therefore\ Period = \frac{360°}{3} = 120°$ **OR** $f\left(\frac{3}{2}x\right) = \sin 2\left(\frac{3}{2}x\right)$ $= \sin 3x$ $= \sin(3x + 360°)$ $= \sin 3(x + 120°)$ $\therefore\ Period = 120°$	✓ sin 3x ✓120° (2) ✓ sin 3x ✓120° (2)

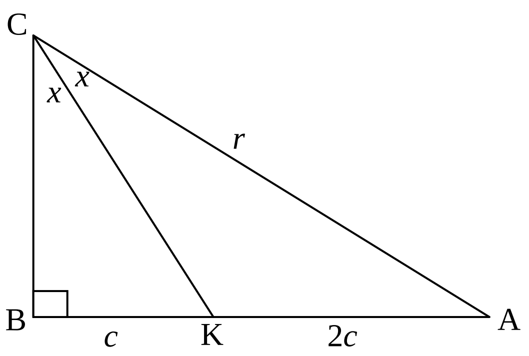
10.3		✓ x intercepts ✓✓ turning points ✓ shape (4)
10.4	$(-180°; -90°)$ *or* $(-60°\ ;\ 0°)$ **OR** $-180° < x < -90°$ *or* $-60° < x < 0°$	✓ $> -180°$ ✓ $< -90°$ ✓ $> -60°$ ✓ $< 0°$ (4)
10.5	$y = \sin 2(x + 30°)$ $\therefore$ translation of 30° to the left	✓ translation 30° ✓ to the left (2)
10.6	$\sin 2x = \cos(x - 30°)$ $\sin 2x = \sin[90° - (x - 30°)]$ $= \sin(120° - x)$ $2x = 120° - x + 360°k; k \in Z$ **or** $2x = 180° - (120° - x) + 360°k$ $3x = 120° + 360°k$ **or** $2x - x = 60° + 360°k$ $x = 40° + 120°k; k \in Z$ **or** $x = 60° + 360°k; k \in Z$ **OR** $\sin 2x = \cos(x - 30°)$ $\cos(90° - 2x) = \cos(x - 30°)$ $90° - 2x = x - 30° + 360°k$ or $90° - 2x = 360° - (x - 30°) + 360°k$ $-3x = -120° + 360°k$ or $-x = 300° + 360°k$ $x = 40° - 120°k; k \in Z$ or $x = -300° - 360°k; k \in Z$ $\therefore x = 40° + 120°k$ or $x = 60° + 360°k\ ;\ k \in Z$	✓ using co-function ✓ $2x = 120° - x + 360°k$ ✓ $x = 40° + 120°k$ ✓ $2x = 180° - (120° - x) + 360°k$ ✓ $x = 60° + 360°k$ ✓ $k \in Z$ (6) ✓ $\cos(90° - x) = \cos(x - 30°)$ ✓ $90° - 2x = x - 30° + 360°k$ ✓ $x = 40° - 120°k$ ✓ $90° - 2x = 360° - (x - 30°) + 360°k$ ✓ $x = -300° - 360°k$ ✓ $k \in Z$ (6) **[20]**

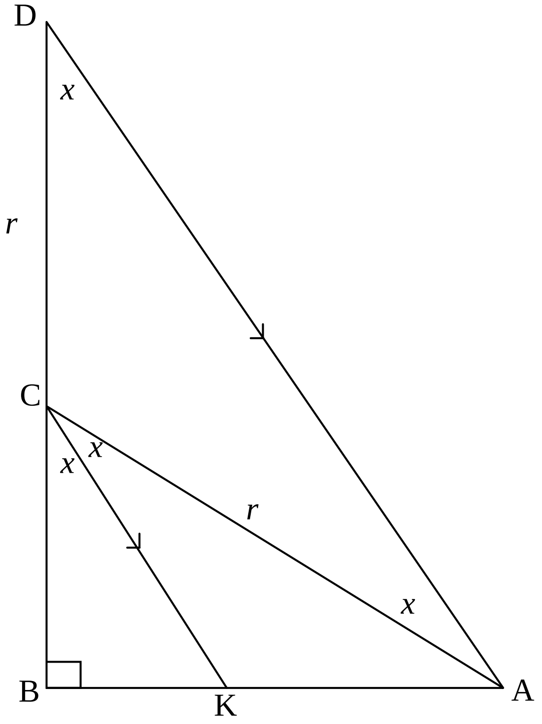
QUESTION 11

11.1	$\frac{AB}{r} = \sin 2x$ $AB = r\sin 2x$	✓ $\frac{AB}{r} = \sin 2x$ ✓ $AB = r\sin 2x$ (2)
11.2	$A\hat{K}C = 90° + x$	✓ $A\hat{K}C = 90° + x$ (1)
11.3	 *In* ΔAKC: $\frac{\sin A\hat{K}C}{AC} = \frac{\sin A\hat{C}K}{AK}$ $\frac{\sin(90° + x)}{r} = \frac{\sin x}{AK}$ $AK = \frac{r\sin x}{\sin(90° + x)} = \frac{r\sin x}{\cos x}$ $\frac{AK}{AB} = \frac{2}{3}$ $\frac{\left(\frac{r\sin x}{\cos x}\right)}{r\sin 2x} = \frac{2}{3}$ $\frac{\frac{\sin x}{\cos x}}{2\sin x\cos x} = \frac{2}{3}$ $\frac{\sin x}{\cos x} \times \frac{1}{2\sin x\cos x} = \frac{2}{3}$ $\frac{1}{2\cos^2 x} = \frac{2}{3}$ $4\cos^2 x = 3$ $\cos x = \frac{\sqrt{3}}{2}$ $x = 30°$ **OR**	✓ sine rule ✓ substitution ✓ making AK subject of the formula ✓ cos x ✓ 2sin$x$.cos$x$ ✓ $\frac{1}{2\cos^2 x}$ ✓ $\cos x = \frac{\sqrt{3}}{2}$ ✓ $x = 30°$ (8)

		
	Using the sine-formula in ΔCBK and ΔCKA:	
	$\frac{\sin x}{BK}=\frac{\sin(90°-x)}{BC}$ *and* $\frac{\sin x}{KA}=\frac{\sin(90°+x)}{AC}$	✓ $\frac{\sin x}{BK}=\frac{\sin(90°-x)}{BC}$ ✓ $\frac{\sin x}{KA}=\frac{\sin(90°+x)}{AC}$
	$\therefore \frac{BK}{BC}=\frac{KA}{AC}$	✓ $\frac{BK}{BC}=\frac{KA}{AC}$
	$\therefore \frac{1}{BC}=\frac{2}{r}$	✓ $\frac{1}{BC}=\frac{2}{r}$
	$\therefore BC=\frac{1}{2}r$	✓ $BC=\frac{1}{2}r$
	$\therefore \cos 2x=\frac{BC}{AC}=\frac{\frac{1}{2}r}{r}=\frac{1}{2}$	✓ $\cos 2x=\frac{1}{2}$
	$\therefore 2x=60°$	✓ $2x=60°$
	$\therefore x=30°$	✓ $x=30°$ (8)
	OR	
		
	ΔCBK: KC = $\frac{c}{\sin x}$	✓ KC = $\frac{c}{\sin x}$
	ΔCKA: $\frac{\sin x}{2c}=\frac{\sin(90°-2x)}{KC}=\frac{\sin(90°-2x).\sin x}{c}$	✓ $\frac{\sin x}{2c}=\frac{\sin(90°-2x)}{KC}$ ✓✓substitution
	$\therefore$ sin (90° - 2x) = $\frac{1}{2}$ = sin 30°	✓✓ sin (90° - 2x) = $\frac{1}{2}$
	$\therefore$ 90° - 2x = 30°	✓90° - 2x = 30°
	x = 30°	✓ x = 30°

	OR	(8)
		
	ΔCBK:	
	$\sin 2x = \frac{3c}{r} = 2\sin x.\cos x$	✓ $\sin 2x = \frac{3c}{r}$ ✓ $2\sin x.\cos x$
	$\therefore r\sin x = \frac{3c}{2\cos x}$ ....(1)	✓ $r\sin x = \frac{3c}{2\cos x}$
	ΔCKA:	
	$\frac{2c}{\sin x} = \frac{r}{\cos x}$	✓ $\frac{2c}{\sin x} = \frac{r}{\cos x}$
	$\therefore r\sin x = 2c\cos x$..........................(2)	✓ $r\sin x = 2c\cos x$
	Equate (1) and (2):	
	$2c.\cos x = \frac{3c}{2\cos x}$	✓equating
	$\therefore \cos^2 x = \frac{3}{4}$	
	$\therefore \cos x = \frac{\sqrt{3}}{2}$	✓ $\cos x = \frac{\sqrt{3}}{2}$
	$\therefore \quad x = 30°$	✓30°
	OR	(8)

	$\frac{AK}{KB}=\frac{2}{1}=2$	
	$2=\frac{\frac{1}{2}AK.BC}{\frac{1}{2}BK.BC}$	✓multiplying by $\frac{1}{2}BC$
	$=\frac{\text{area AKC}}{\text{area ABC}}$	✓ area of triangles
	$=\frac{\frac{1}{2}rCK\sin x}{\frac{1}{2}BC.CK\sin x}$	✓ area formula in triangles
	$=\frac{r}{BC}$	✓ $\frac{r}{BC}=2$
	$\therefore \frac{BC}{r}=\frac{1}{2}$	✓ $\frac{BC}{r}=\frac{1}{2}$
	$\therefore \cos 2x=\frac{1}{2}$	✓ $\cos 2x=\frac{1}{2}$
	$\therefore 2x=60°$	✓ $2x=60°$
	$\therefore x=30°$	✓ $x=30°$
		(8)
	OR	
		
	By the Internal Bisector Theorem:	✓✓ For stating Internal Bisector Theorem
	$\frac{CB}{CA}=\frac{BK}{KA}=\frac{1}{2}$	✓✓✓ $\frac{CB}{CA}=\frac{BK}{KA}=\frac{1}{2}$
	$\cos 2x=\frac{1}{2}$	✓ $\cos 2x=\frac{1}{2}$
	$2x=60°$	✓ $2x=60°$
	$x=30°$	✓ $x=30°$
		(8)

	OR  Produce BC to D and draw CK parallel to DA. $C\hat{A}D = K\hat{C}A$ *and* $B\hat{C}K = \hat{D}$ $\therefore DC = CA = r$ $\therefore \Delta BKC \, \lvert\lvert\lvert \, \Delta BAD$ $\therefore \frac{BK}{BA} = \frac{BC}{BD} = 3$ $\therefore BD = 3BC = BC + r$ $\therefore BC = \frac{1}{2}r$ $\therefore \cos 2x = \frac{\frac{1}{2}r}{r} = \frac{1}{2}$ $\therefore 2x = 60°$ $\therefore x = 30°$	✓ $DC = CA = r$ ✓ $\Delta BKC \, \lvert\lvert\lvert \, \Delta BAD$ ✓ $\frac{BK}{BA} = \frac{BC}{BD} = 3$ ✓BD = BC + r ✓ $BC = \frac{1}{2}r$ ✓ $\cos 2x = \frac{1}{2}$ ✓$2x = 60°$ ✓30° (8) **[11]**

TOTAL: 150