basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2012

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages, 4 diagram sheets and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.

4. Answers only will not necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Diagram sheets for QUESTION 2.1, QUESTION 2.3, QUESTION 2.4, QUESTION 3.2, QUESTION 3.3, QUESTION 7.1 and QUESTION 10.4 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert the sheets inside the back cover of your ANSWER BOOK.

9. An information sheet, with formulae, is included at the end of this question paper.

10. Number the answers correctly according to the numbering system used in this question paper.

11. Write neatly and legibly.

QUESTION 1

A large company employs 9 salespersons. The commission that each salesperson earned (in rand) in a certain month is shown below.

3 900	5 700	7 300	10 600	13 000	13 600	15 100	15 800	17 100

1.1 Calculate the mean of the above data. (2)

1.2 Calculate the standard deviation for the data. (2)

1.3 The company rates the sales staff according to the amount of commission earned. A salesperson whose commission is more than one standard deviation above the mean receives a rating of 'good'. How many salespersons will receive a rating of 'good' for that month? (2)
[6]

QUESTION 2

The table shows the approximate number of people using the Internet from 1995 to 2001.

YEAR	1995	1996	1997	1998	1999	2000	2001
N (Number in millions)	8	17	34	67	135	281	552

2.1 Draw a scatter plot of this data on the grid provided on DIAGRAM SHEET 1. (2)

2.2 Which of the curves, linear, quadratic or exponential, will fit the data? (1)

2.3 The table below, is also shown on DIAGRAM SHEET 2. Complete the values for log N in the table on DIAGRAM SHEET 2. (2)

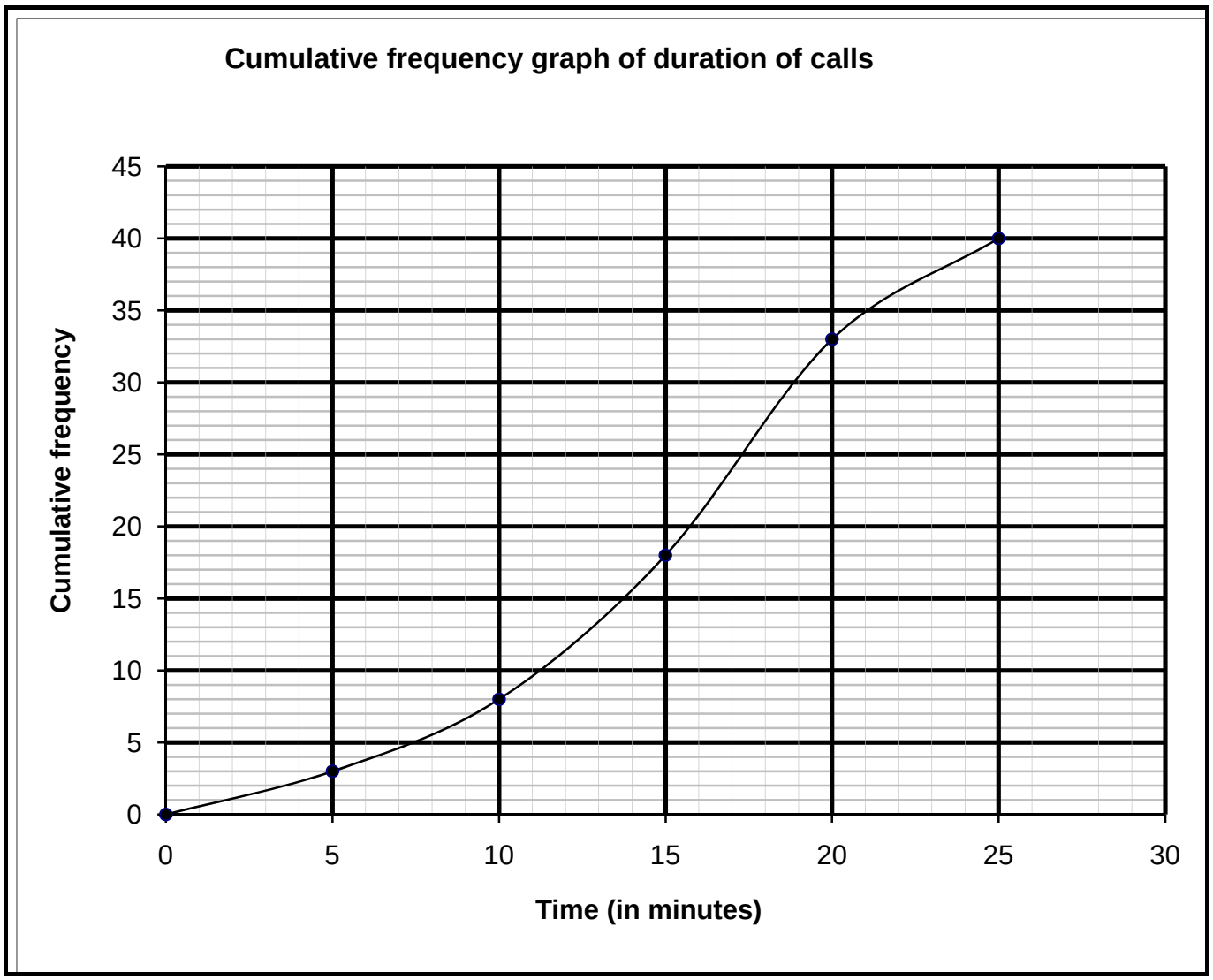
YEAR	1995	1996	1997	1998	1999	2000	2001
N (Number in millions)	8	17	34	67	135	281	552
Log N (Correct to ONE decimal place)							

2.4 Plot the points representing log N on the grid provided on DIAGRAM SHEET 2. (2)

2.5 Use your diagram in QUESTION 2.4 to predict the answer to QUESTION 2.2. (2)
[9]

QUESTION 3

The length of time, in minutes, of a certain number of telephone calls was recorded. No call lasted 25 minutes or longer. A cumulative frequency diagram of this data is shown below.



3.1 Determine the total number of calls recorded. (1)

3.2 Complete the frequency table for the data on DIAGRAM SHEET 3. (3)

3.3 Hence, draw a histogram on the grid on DIAGRAM SHEET 3. (3)
[7]

QUESTION 4

In the grid below a, b, c, d, e, f and g represent values in a data set written in an increasing order. No value in the data set is repeated.

a	b	c	d	e	f	g

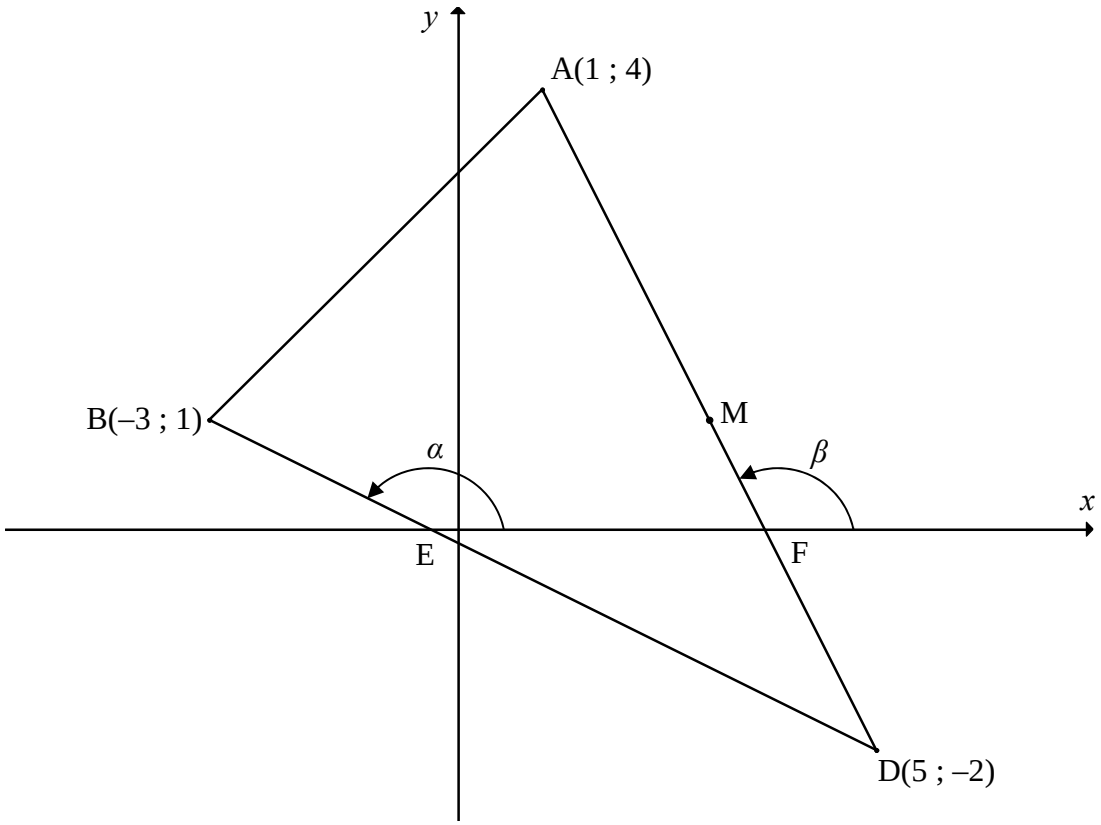
Determine the value of a, b, c, d, e, f and g if:

- The maximum value is 42
- The range is 35
- The median is 23
- The difference between the median and the upper quartile is 14
- The interquartile range is 22
- $e = 2c$
- The mean is 25 **[7]**

QUESTION 5

In the figure below, A(1 ; 4), B(–3 ; 1) and D(5 ; –2) are the coordinates of the vertices of ΔABD.

- BD and AD intersect the *x*-axis at E and F respectively.
- The angle of inclination of BD with the *x*-axis at E is α.
- The angle of inclination of AD with the *x*-axis at F is β.



5.1 Calculate the gradient of AD. (2)

5.2 Determine the length of the line segment AD.
(Leave your answer in surd form, if necessary.) (2)

5.3 Determine the coordinates of M, the midpoint of AD. (2)

5.4 C is a point such that line BC is parallel to AD. Determine the equation of line BC in the form $ax + by + c = 0$. (3)

5.5 5.5.1 Calculate the size of β. (2)

5.5.2 Calculate ALL the angles of ΔDEF. (5)

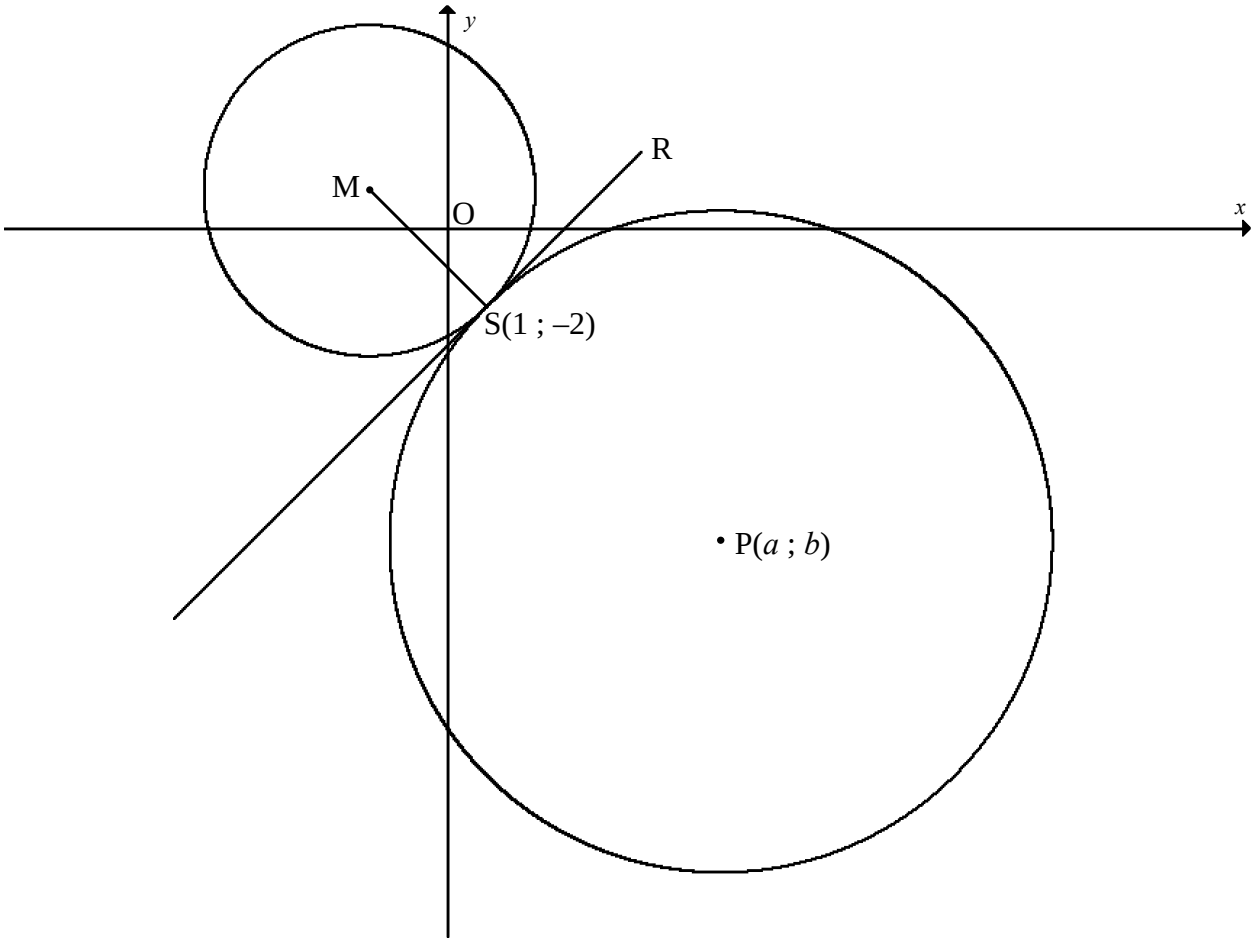
5.6 Determine the equation of a circle, with centre M, which passes through the points A and D. Give your answer in the form: $(x - a)^2 + (y - b)^2 = r^2$. (2)

5.7 Does the point B lie inside, outside or on the circle in QUESTION 5.6? Show ALL calculations to justify your answer. (2)

[20]

QUESTION 6

In the figure below, a circle with centre M is drawn. The equation of the circle is $(x + 2)^2 + (y – 1)^2 = r^2$.
S(1 ; –2) is a point on the circle.
SR is a tangent to the circle.



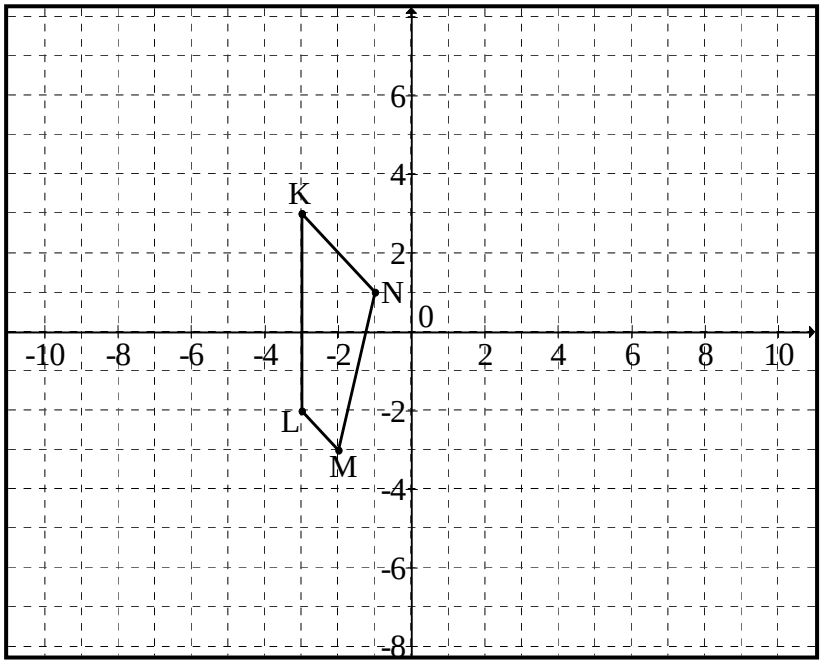
6.1 Write down the coordinates of M and the radius of the circle centre M. (4)

6.2 Determine the equation of the tangent RS in the form $y = mx + c$. (4)

6.3 The circles having centres P and M touch externally at point S. SR is a tangent to both these circles. If MS : MP = 1 : 3, determine the coordinates $(a ; b)$ of point P. (8)
[16]

QUESTION 7

The coordinates of the vertices of a quadrilateral KLMN are K(–3 ; 3); L(–3 ; –2); M(–2 ; –3) and N(–1 ; 1).



7.1 The quadrilateral is rotated through 90° in an anticlockwise direction about the origin to form quadrilateral $K'L'M'N'$. Draw and label quadrilateral $K'L'M'N'$ on the grid on DIAGRAM SHEET 4. (4)

7.2 Quadrilateral $K'L'M'N'$ is enlarged by a scale factor of 2 to form quadrilateral $K''L''M''N''$.

7.2.1 State whether the transformation is rigid or not. Give a reason for your answer. (2)

7.2.2 Determine the coordinates of N''. (2)

7.3 A point $P(x\ ;\ y)$ is transformed to $P''(x''\ ;\ y'')$ after applying the two transformations described in QUESTION 7.1 and QUESTION 7.2 respectively. Write down the general rule for the transformations in the form: $(x\ ;\ y) \rightarrow (\ldots\ ;\ \ldots) \rightarrow (\ldots\ ;\ \ldots)$ (4)

7.4 Determine the ratio: area of KLMN : area of $K''L''M''N''$ (2)

7.5 Quadrilateral KLMN is enlarged by a factor of p. Find the largest value of p for which the image of KLMN is contained in the circle centred at (0 ; 0) with radius 1. (3)

[17]

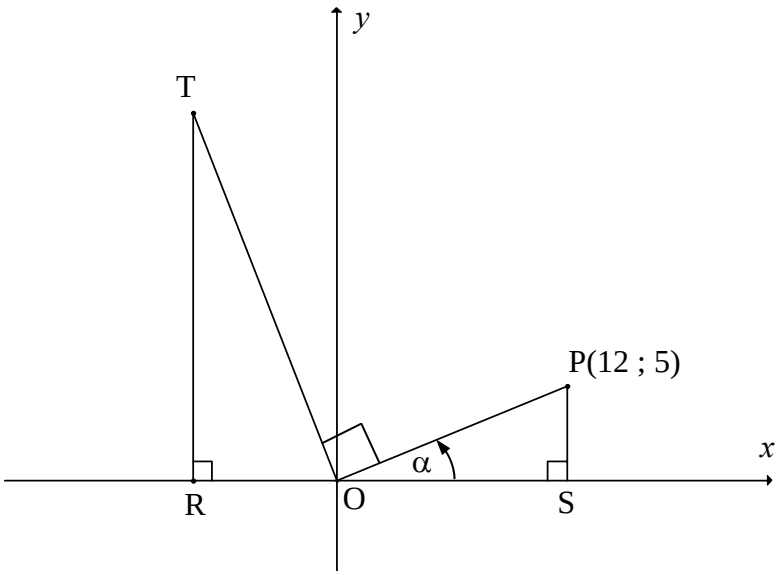
QUESTION 8

If $Q'(-2 ; -3)$ is the image of Q after rotation of 135° in an anticlockwise direction about the origin, calculate the coordinates of Q. (Leave your answer in surd form.) **[5]**

QUESTION 9

9.1 **Answer this question without using a calculator.**

In the diagram, P is the point (12 ; 5). OT ⊥ OP. PS and TR are perpendicular to the x-axis. $P\hat{O}S = \alpha$ and OR = 7,5 units.



Determine:

9.1.1 $\cos\alpha$ (2)

9.1.2 $T\hat{O}R$, in terms of α (2)

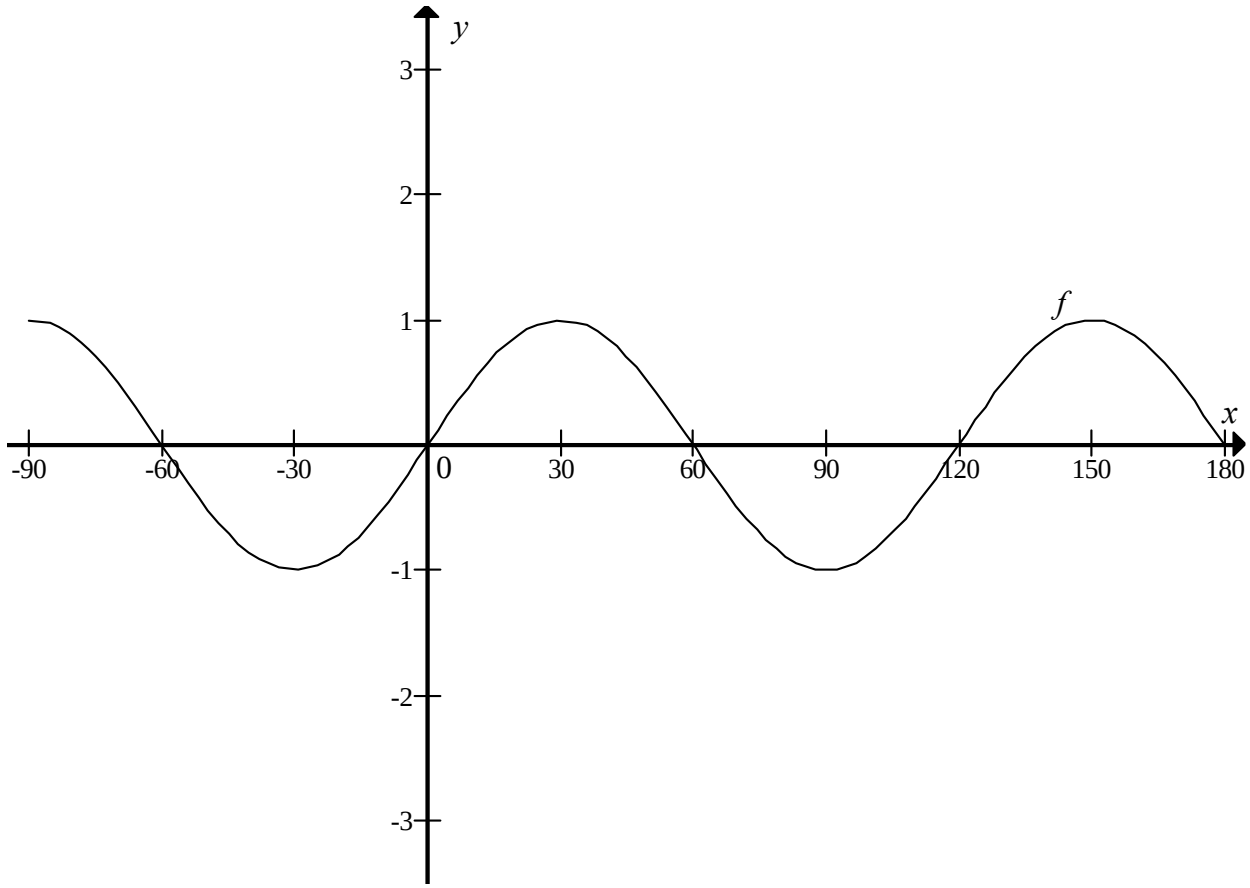
9.1.3 The length of OT (4)

9.2 Show that $\dfrac{\sin(90° + x).\cos x.\tan(-x)}{\cos(180° + x)} = \sin x$. (4)

[12]

QUESTION 10

The graph of $f(x) = \sin 3x$ is drawn below for $x \in [-90° ; 180°]$.



10.1 Write down the period of f. (1)

10.2 Write down the solutions for $\sin 3x = -1$ on the interval $[-90° ; 180°]$. (2)

10.3 Give the maximum value of h if $h(x) = f(x) - 1$. (2)

10.4 Draw the graph of $g(x) = 3\cos x$ for $x \in [-90° ; 180°]$ on the grid on DIAGRAM SHEET 4. (3)

10.5 Use the graphs to determine how many solutions there are to the equation $\frac{\sin 3x}{3} - \cos x = 0$ on the interval $[-90° ; 180°]$. (2)

10.6 Use the graphs to solve: $f(x).g(x) < 0$. (4)
[14]

QUESTION 11

11.1 If $\sin 61° = \sqrt{p}$, determine the following in terms of p:

11.1.1 $\sin 241°$ (2)

11.1.2 $\cos 61°$ (2)

11.1.3 $\cos 122°$ (3)

11.1.4 $\cos 73° \cos 15° + \sin 73° \sin 15°$ (3)

11.2 11.2.1 Prove the identity:

$$\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$$ (6)

11.2.2 Determine a value of x in the interval [0° ; 180°] for which the identity is not valid. (2)

11.3 11.3.1 Given: $\sin x = \cos 2x - 1$. Show that $2\sin^2 x + \sin x = 0$. (1)

11.3.2 Determine the general solution of the equation: $\sin x = \cos 2x - 1$. (6)

11.4 Determine the value of:

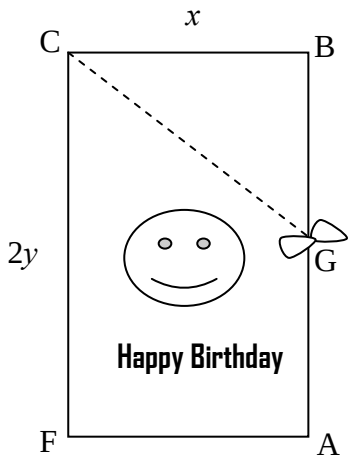
$$\tan 1° \times \tan 2° \times \tan 3° \times \tan 4° \times ... \times \tan 87° \times \tan 88° \times \tan 89°.$$ (4)

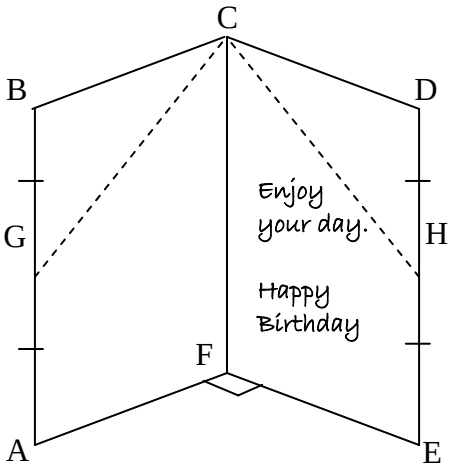
[29]

QUESTION 12

A rectangular birthday card is tied with a ribbon at the midpoints, G and H, of the longer sides. The card is opened to read the message inside and then placed on a table in such a way that the angle $A\hat{F}E$ between the front cover and the back cover of the card is $90°$. The points G and H are joined by straight lines to the point C inside the card, as shown in the sketch.

Let the shorter side of the card, BC = x, and the longer side, CF = $2y$.





Prove that $\cos G\hat{C}H = \dfrac{y^2}{x^2 + y^2}$. **[8]**

TOTAL: 150

CENTRE NUMBER: | | | | | | | | |

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DIAGRAM SHEET 1

QUESTION 2.1

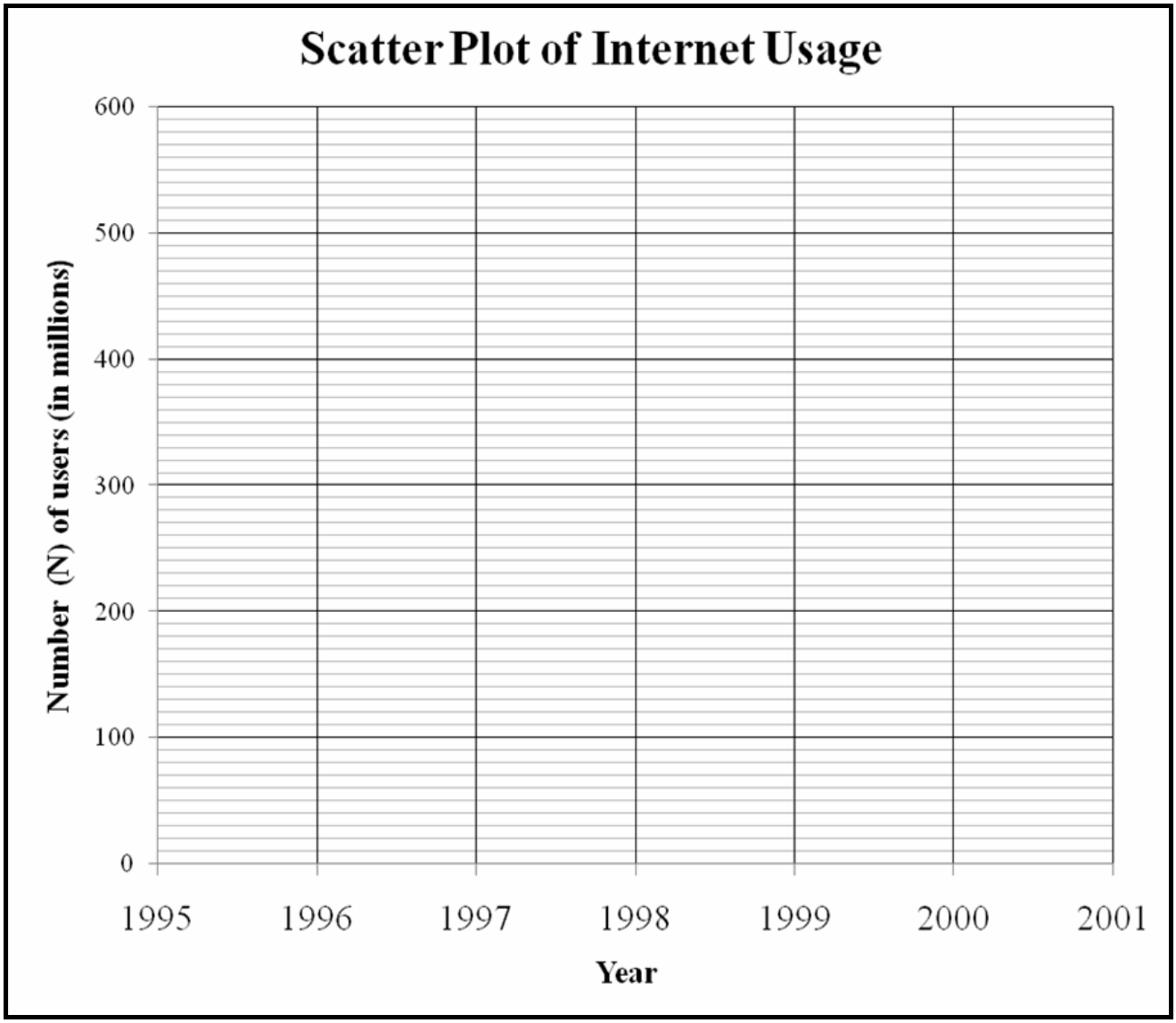

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DIAGRAM SHEET 2

QUESTION 2.3

YEAR	1995	1996	1997	1998	1999	2000	2001
N (Number in millions)	8	17	34	67	135	281	552
Log N (Correct to ONE decimal)							

QUESTION 2.4

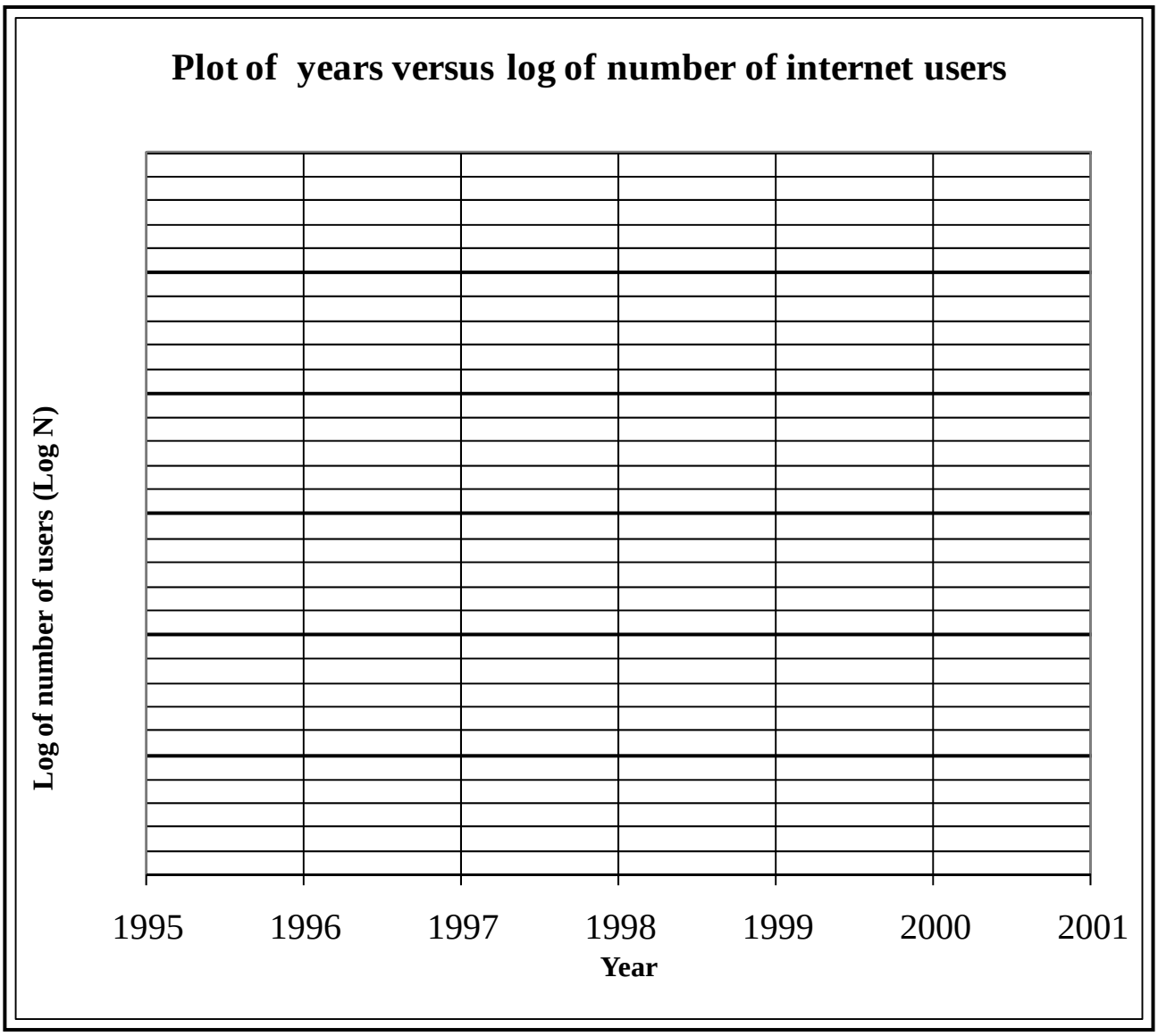

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DIAGRAM SHEET 3

QUESTION 3.2

Time, t, in minutes	Frequency
$0 \leq t < 5$	
$5 \leq t < 10$	
$10 \leq t < 15$	
$15 \leq t < 20$	
$20 \leq t < 25$	

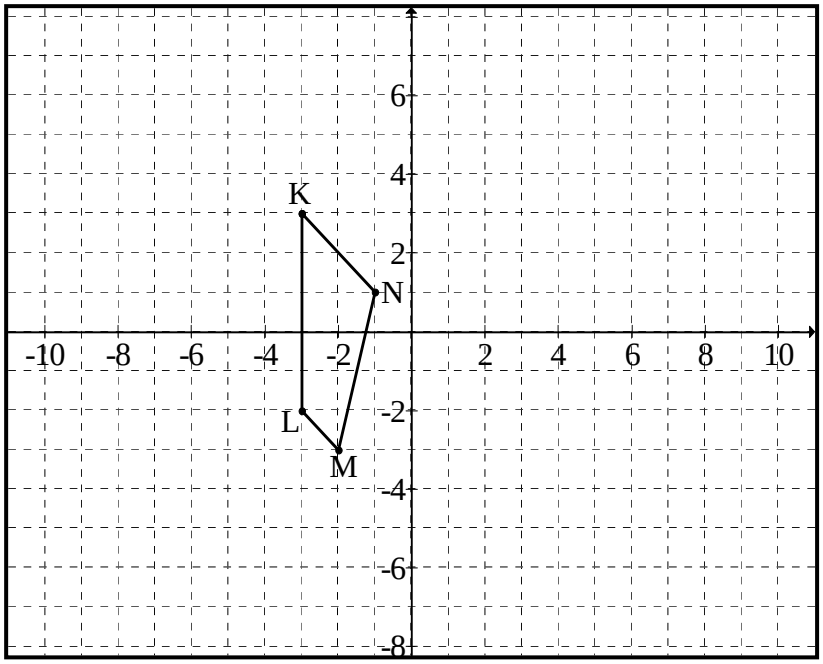
QUESTION 3.3

CENTRE NUMBER:

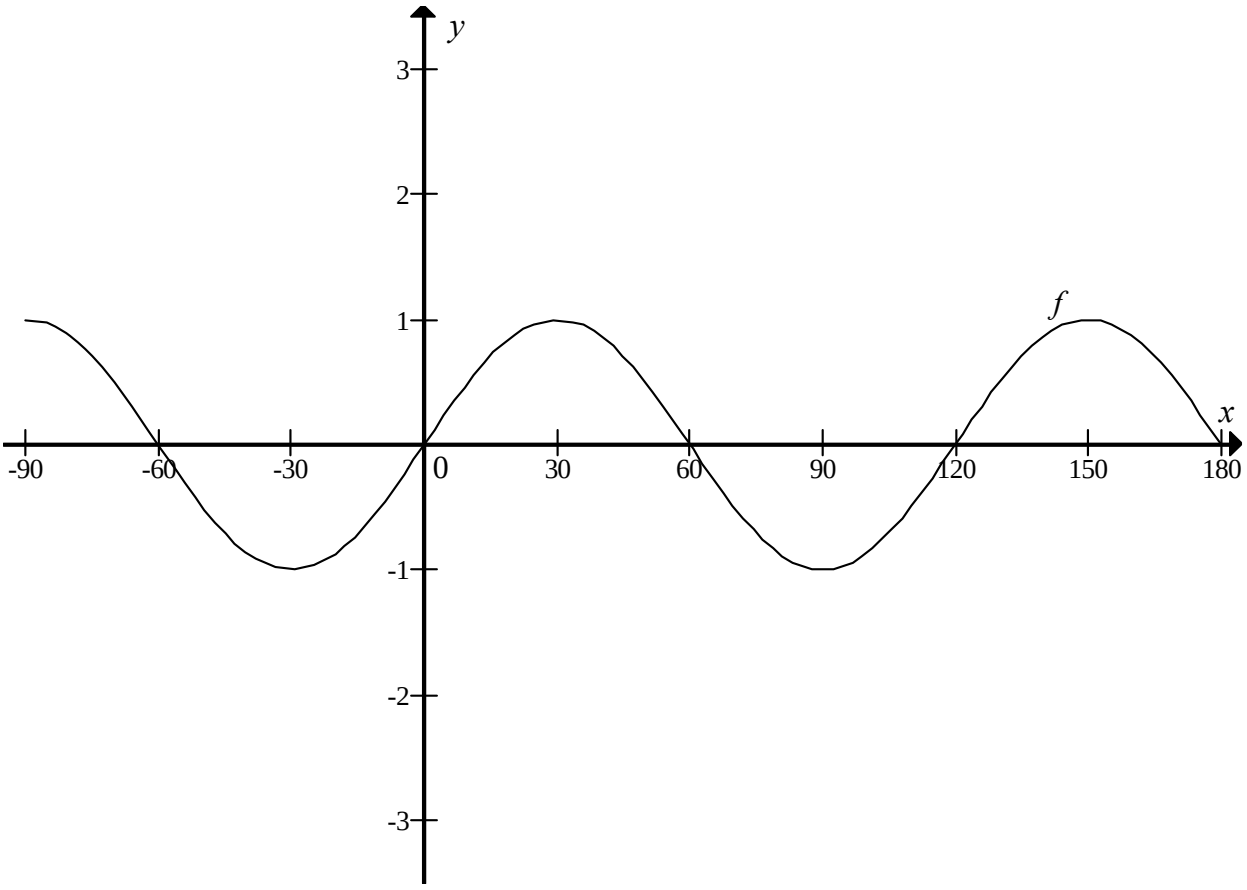
EXAMINATION NUMBER:

DIAGRAM SHEET 4

QUESTION 7.1



QUESTION 10.4



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x\ ;\ y) \to (x\cos\theta + y\sin\theta\ ;\ y\cos\theta - x\sin\theta) \qquad (x\ ;\ y) \to (x\cos\theta - y\sin\theta\ ;\ y\cos\theta + x\sin\theta)$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$