basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

FEBRUARY/MARCH 2012

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages, 1 diagram sheet and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. ONE diagram sheet for answering QUESTION 12.2 is attached at the end of this question paper. Write your centre number and examination number on this sheet in the spaces provided and insert the page inside the back cover of your ANSWER BOOK.

9. An information sheet, with formulae, is included at the end of the question paper.

10. Number the answers correctly according to the numbering system used in this question paper.

11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 - 5x = 2$ (3)

1.1.2 $x - \frac{2}{x} = 5$ (4)

1.1.3 $(x+1)(x-3) > 12$ (4)

1.2 Solve simultaneously for r and p in the following set of equations:

$$6r + 5rp - 5p = 8$$
$$r + p = 2$$ (7)

1.3 The volume of a box with a rectangular base is 3 072 cm^3. The lengths of the sides are in the ratio 1 : 2 : 3. Calculate the length of the shortest side. (4)

[22]

QUESTION 2

Given the arithmetic series: $-7 - 3 + 1 + \ldots + 173$

2.1 How many terms are there in the series? (3)

2.2 Calculate the sum of the series. (3)

2.3 Write the series in sigma notation. (3)

[9]

QUESTION 3

3.1 Consider the geometric sequence: $4\ ;\ -2\ ;\ 1 \ldots$

3.1.1 Determine the next term of the sequence. (2)

3.1.2 Determine n if the n^{th} term is $\frac{1}{64}$. (4)

3.1.3 Calculate the sum to infinity of the series $4 - 2 + 1 \ldots$ (2)

3.2 If x is a REAL number, show that the following sequence can NOT be geometric:

$$1\ ;\ x+1\ ;\ x-3 \ldots$$ (4)

[12]

QUESTION 4

An athlete runs along a straight road. His distance d from a fixed point P on the road is measured at different times, n, and has the form $d(n) = an^2 + bn + c$. The distances are recorded in the table below.

Time (in seconds)	1	2	3	4	5	6
Distance (in metres)	17	10	5	2	r	s

4.1 Determine the values of r and s. (3)

4.2 Determine the values of a, b and c. (4)

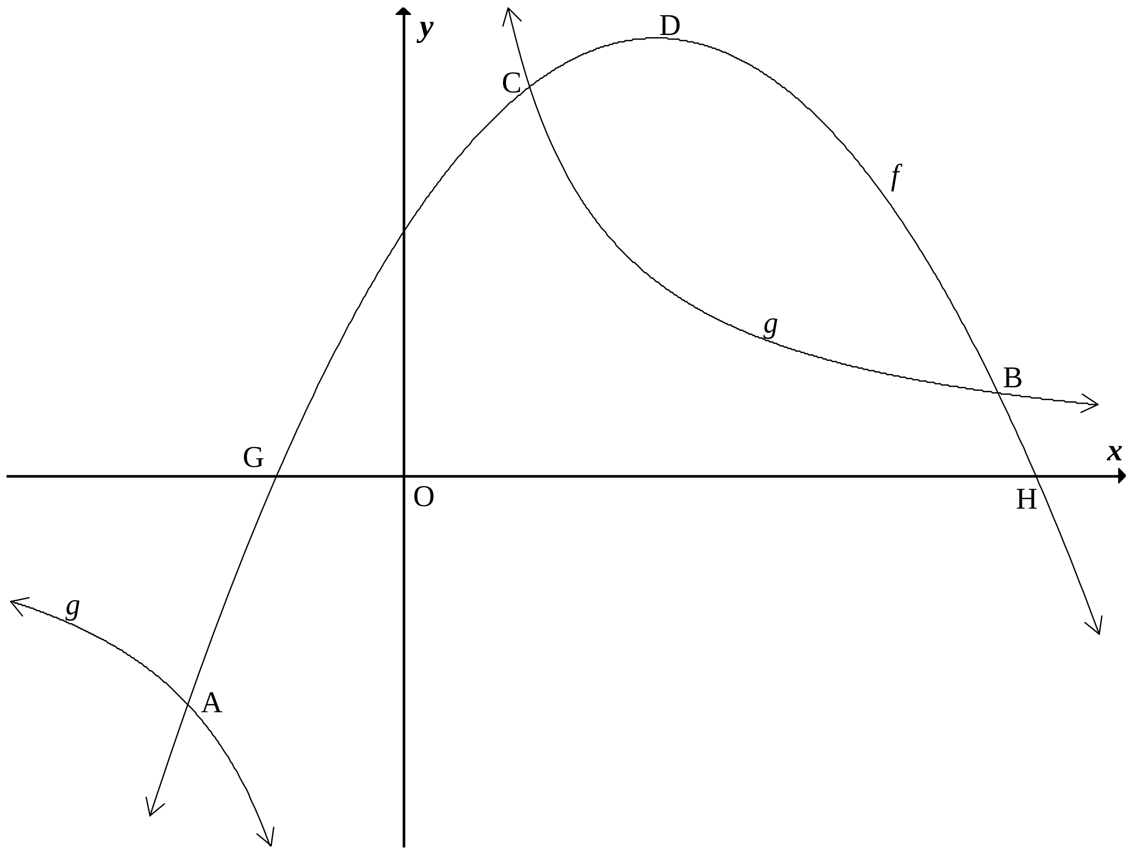
4.3 How far is the athlete from P when $n = 8$? (2)

4.4 Show that the athlete is moving towards P when $n < 5$, and away from P when $n > 5$. (4)
[13]

QUESTION 5

The graphs of the functions $f(x) = -2x^2 + 8x + 10$ and $g(x) = \dfrac{16}{x}$ are sketched below.

G and H are the x-intercepts of f.
D is the turning point of f.
Points A, B and C are the points of intersection of f and g.



5.1 Write down the equations of the asymptotes of the graph of g. (2)

5.2 Determine the coordinates of H. (4)

5.3 Determine the range of f. (4)

5.4 Verify that C is the point (1 ; 16). (2)

5.5 Determine the coordinates of the turning point of p if $p(x) = f(3x)$. (3)
[15]

QUESTION 6

Given: $f(x) = 3^x$

6.1 Determine an equation for f^{-1} in the form $f^{-1}(x) = ...$ (1)

6.2 Sketch, in your ANSWER BOOK, the graphs of f and f^{-1}, showing clearly ALL intercepts with the axes. (4)

6.3 Write down the domain of f^{-1}. (2)

6.4 For which values of x will $f(x).f^{-1}(x) \leq 0$? (2)

6.5 Write down the range of $h(x) = 3^{-x} - 4$ (2)

6.6 Write down an equation for g if the graph of g is the image of the graph of f after f has been translated two units to the right and reflected about the x-axis. (2)

[13]

QUESTION 7

7.1 Lerato wants to purchase a house that costs R850 000. She is required to pay a 12% deposit and she will borrow the balance from a bank. Calculate the amount that Lerato must borrow from the bank. (2)

7.2 The bank charges interest at 9% per annum, compounded monthly on the loan amount. Lerato works out that the loan will carry an effective interest rate of 9,6% per annum. Is her calculation correct or not? Justify your answer with appropriate calculations. (4)

7.3 Lerato takes out a loan from the bank for the balance of the purchase price and agrees to pay it back over 20 years. Her repayments start one month after her loan is granted. Determine her monthly instalment if interest is charged at 9% per annum compounded monthly. (4)

7.4 Lerato can afford to repay R7 000 per month. How long will it take her to repay the loan amount if she chooses to pay R7 000 as a repayment every month? (4)

[14]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 9 - x^2$. (5)

8.2 Evaluate:

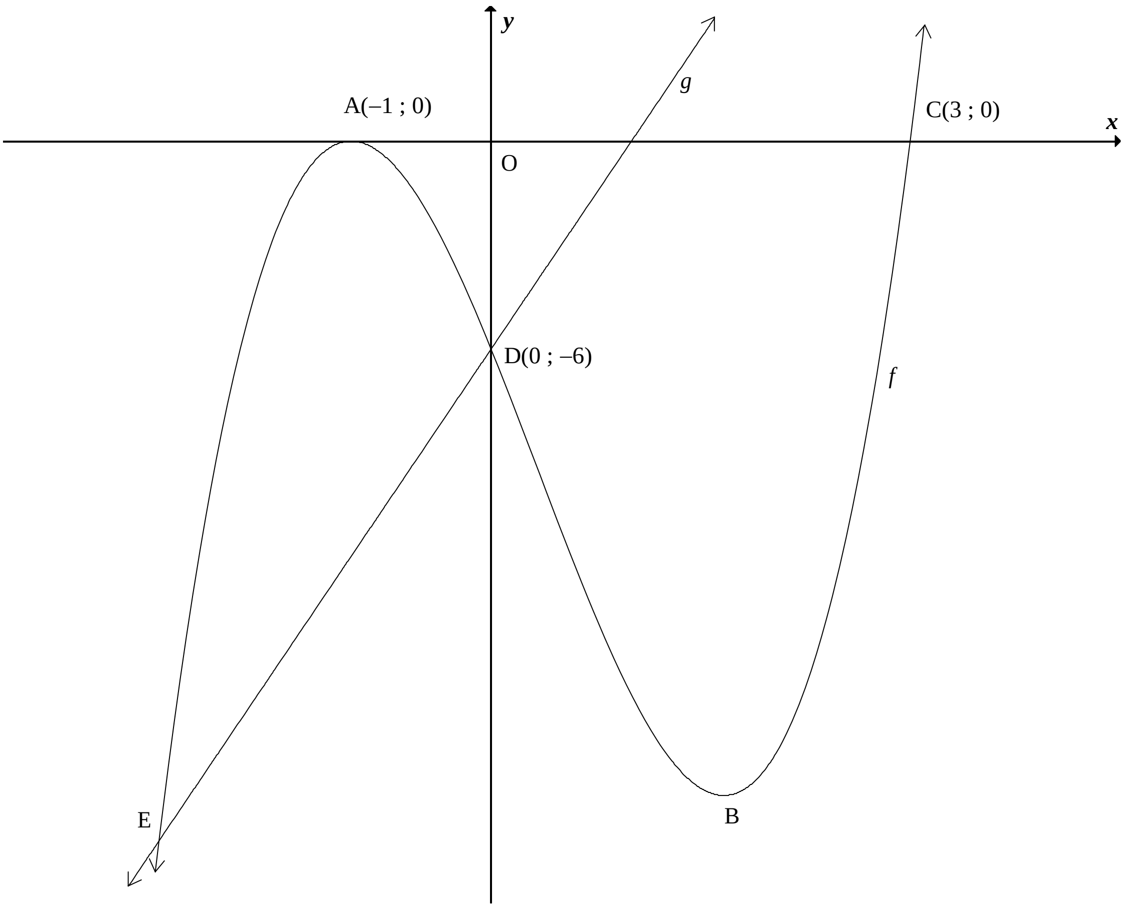
8.2.1 $D_x[1 + 6\sqrt{x}]$ (2)

8.2.2 $\dfrac{dy}{dx}$ if $y = \dfrac{8 - 3x^6}{8x^5}$ (4)

[11]

QUESTION 9

The graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = 6x - 6$ are sketched below.
A(– 1 ; 0) and C(3 ; 0) are the x-intercepts of f.
The graph of f has turning points at A and B.
D(0 ; – 6) is the y-intercept of f.
E and D are points of intersection of the graphs of f and g.



9.1 Show that $a = 2$; $b = -2$; $c = -10$ and $d = -6$. (5)

9.2 Calculate the coordinates of the turning point B. (5)

9.3 $h(x)$ is the vertical distance between $f(x)$ and $g(x)$, that is $h(x) = f(x) - g(x)$. Calculate x such that $h(x)$ is a maximum, where $x < 0$. (5)

[15]

QUESTION 10

The tangent to the curve of $g(x) = 2x^3 + px^2 + qx - 7$ at $x = 1$ has the equation $y = 5x - 8$.

10.1 Show that (1 ; – 3) is the point of contact of the tangent to the graph. (1)

10.2 Hence or otherwise, calculate the values of p and q. (6)
[7]

QUESTION 11

A cubic function f has the following properties:

- $f\left(\frac{1}{2}\right) = f(3) = f(-1) = 0$
- $f'(2) = f'\left(-\frac{1}{3}\right) = 0$
- f decreases for $x \in \left[-\frac{1}{3}\,;\,2\right]$ only

Draw a possible sketch graph of f, clearly indicating the x-coordinates of the turning points and ALL the x-intercepts. **[4]**

QUESTION 12

A furniture factory produces small tables and large tables. The tables undergo sanding and/or painting processes.

- The factory can produce at most 100 tables in total per week.
- At most 50 hours are available for painting the tables per week and at most 180 hours are available for sanding the tables per week.
- A small table requires 1 hour for painting and 1 hour for sanding.
- A large table requires NO painting and 2 hours for sanding.

Let the number of small tables produced per week be x, and let the number of large tables produced per week be y.

12.1 Write down the constraints, in terms of x and y, to represent the above information. (5)

12.2 Represent the constraints graphically on the attached DIAGRAM SHEET. Clearly indicate the feasible region. (4)

12.3 What is the maximum number of large tables that can be produced in a week? (1)

12.4 The profit on a small table is R300 and the profit on a large table is R400. Write down an expression for the total profit made per week. (1)

12.5 Determine the number of each type of table the factory needs to produce in a week in order to ensure a maximum total profit. Indicate this point using the letter A. (2)

12.6 The profit on a small table tends to fluctuate to q rands per table. The profit on a large table is constant at R400. Determine the values of q for which the total profit will be a maximum at point A. (2)
[15]

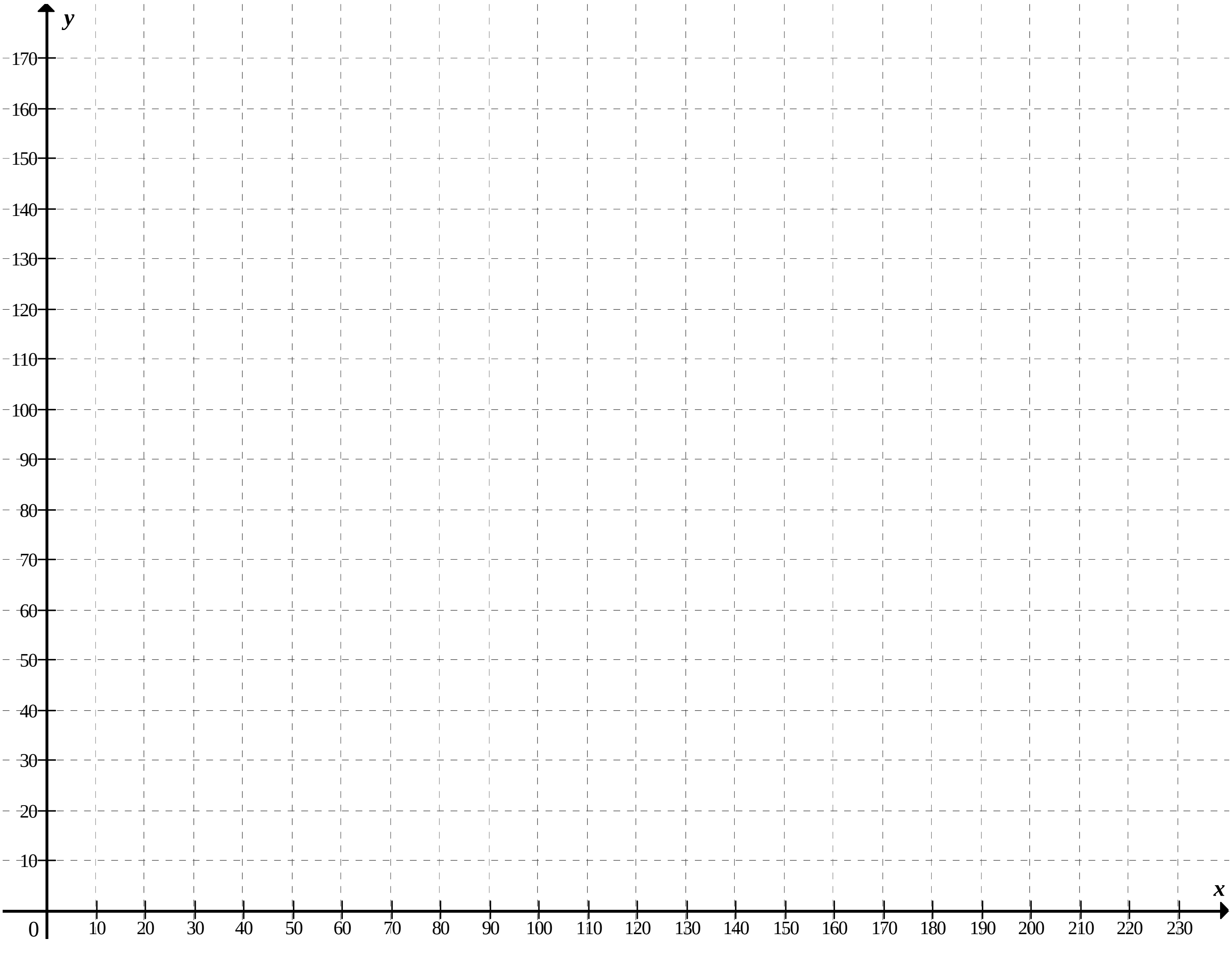
TOTAL: 150

CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 12.2



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$\left(x - a\right)^2 + \left(y - b\right)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2} ab.\sin C$$

$\sin\left(\alpha + \beta\right) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin\left(\alpha - \beta\right) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos\left(\alpha + \beta\right) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos\left(\alpha - \beta\right) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x\ ;\ y) \to (x\cos\theta + y\sin\theta\ ;\ y\cos\theta - x\sin\theta)$ $\quad (x\ ;\ y) \to (x\cos\theta - y\sin\theta\ ;\ y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n\left(S\right)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum\left(x - \bar{x}\right)\left(y - \bar{y}\right)}{\sum\left(x - \bar{x}\right)^2}$