basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

FEBRUARY/MARCH 2013

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, 1 diagram sheet and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. ONE diagram sheet is attached to answer QUESTION 12.1. Write your centre number and examination number on the sheet in the spaces provided and insert it inside the back cover of your ANSWER BOOK.

8. Diagrams are NOT necessarily drawn to scale.

9. An information sheet with formulae is included at the end of the question paper.

10. Number the answers correctly according to the numbering system used in this question paper.

11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(x^2-9)(2x+1)=0$ (3)

1.1.2 $x^2+x-13=0$ (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $2.3^x=81-3^x$ (4)

1.1.4 $(x+1)(4-x)>0$ (3)

1.2 Given: $2^x+2^{x+2}=-5y+20$

1.2.1 Express 2^x in terms of y. (2)

1.2.2 How many solutions for x will the equation have if $y=-4$? (2)

1.2.3 Solve for x if y is the largest possible integer value for which $2^x+2^{x+2}=-5y+20$ will have solutions. (3)

[21]

QUESTION 2

2.1 Given the geometric series: 256 + p + 64 – 32 + ...

2.1.1 Determine the value of p. (3)

2.1.2 Calculate the sum of the first 8 terms of the series. (3)

2.1.3 Why does the sum to infinity for this series exist? (1)

2.1.4 Calculate S_∞ (3)

2.2 Consider the arithmetic sequence: $-8 ; -2 ; 4 ; 10 ; ...$

2.2.1 Write down the next term of the sequence. (1)

2.2.2 If the n^{th} term of the sequence is 148, determine the value of n. (3)

2.2.3 Calculate the smallest value of n for which the sum of the first n terms of the sequence will be greater than 10 140. (5)

2.3 Calculate $\sum_{k=1}^{30}(3k+5)$ (3)

[22]

QUESTION 3

Consider the sequence: 3 ; 9 ; 27 ; ...

Jacob says that the fourth term of the sequence is 81.
Vusi disagrees and says that the fourth term of the sequence is 57.

3.1 Explain why Jacob and Vusi could both be correct. (2)

3.2 Jacob and Vusi continue with their number patterns.

Determine a formula for the n^{th} term of:

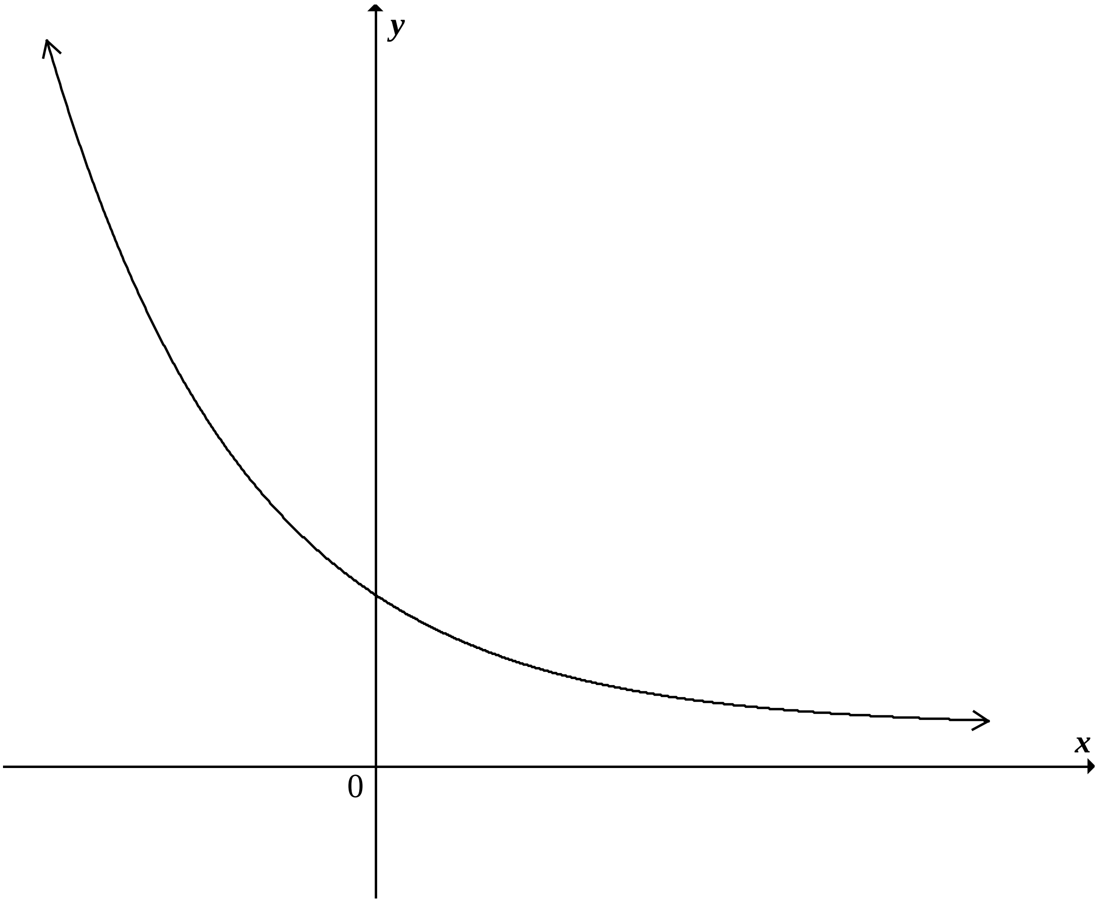
3.2.1 Jacob's sequence (1)

3.2.2 Vusi's sequence (4)

[7]

QUESTION 4

The graph of $f(x)=\left(\frac{1}{3}\right)^x$ is sketched below.



4.1 Write down the domain of f. (1)

4.2 Write down the equation of the asymptote of f. (1)

4.3 Write down the equation of f^{-1} in the form $y = ...$ (2)

4.4 Sketch the graph of f^{-1} in your ANSWER BOOK. Indicate the x-intercept and ONE other point. (3)

4.5 Write down the equation of the asymptote of $f^{-1}(x+2)$. (2)

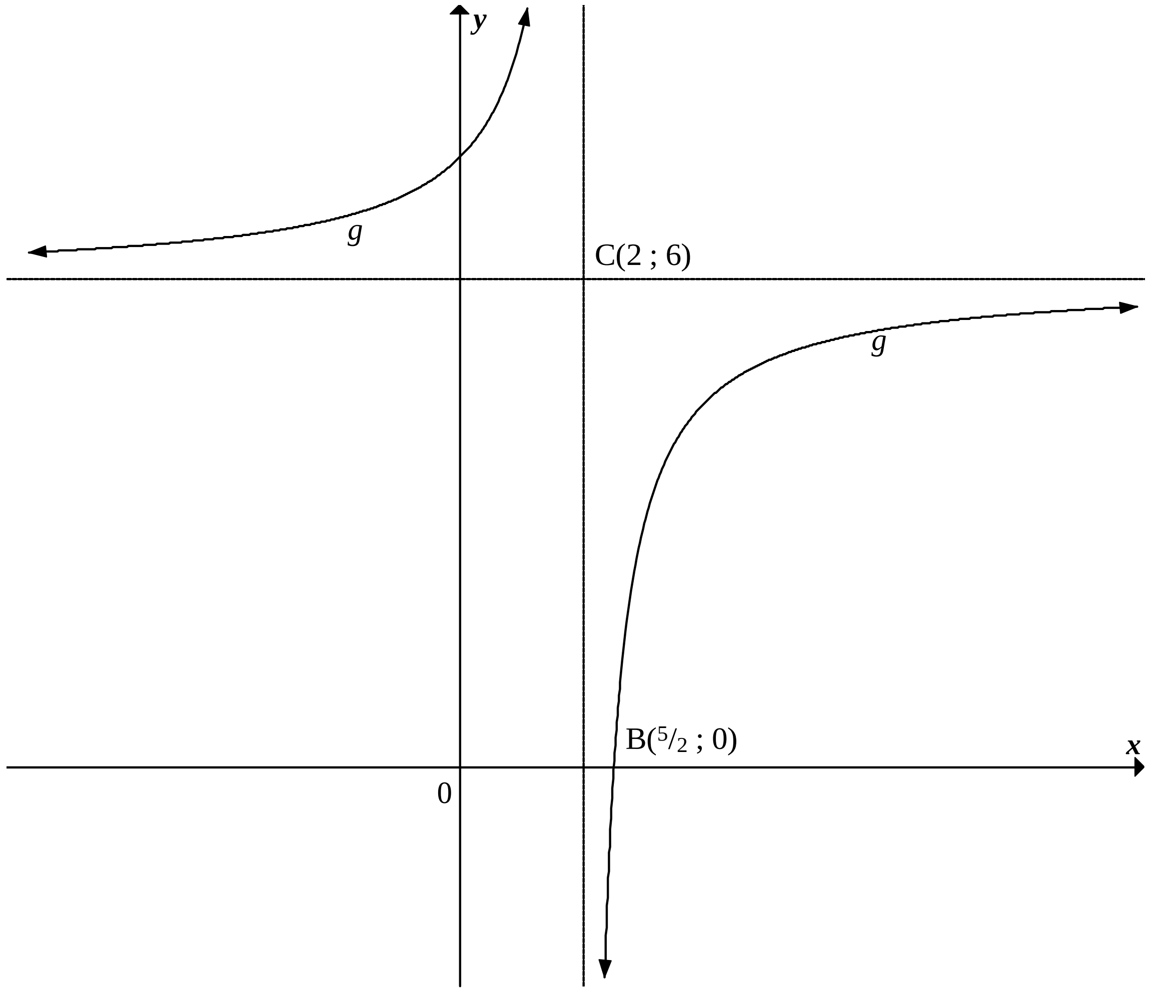
4.6 Prove that: $[f(x)]^2 - [f(-x)]^2 = f(2x) - f(-2x)$ for all values of x. (3)
[12]

QUESTION 5

Sketched below is the graph of $g(x)=\frac{a}{x-p}+q$.

C(2 ; 6) is the point of intersection of the asymptotes of g.

B$\left(\frac{5}{2}\ ;\ 0\right)$ is the x-intercept of g.

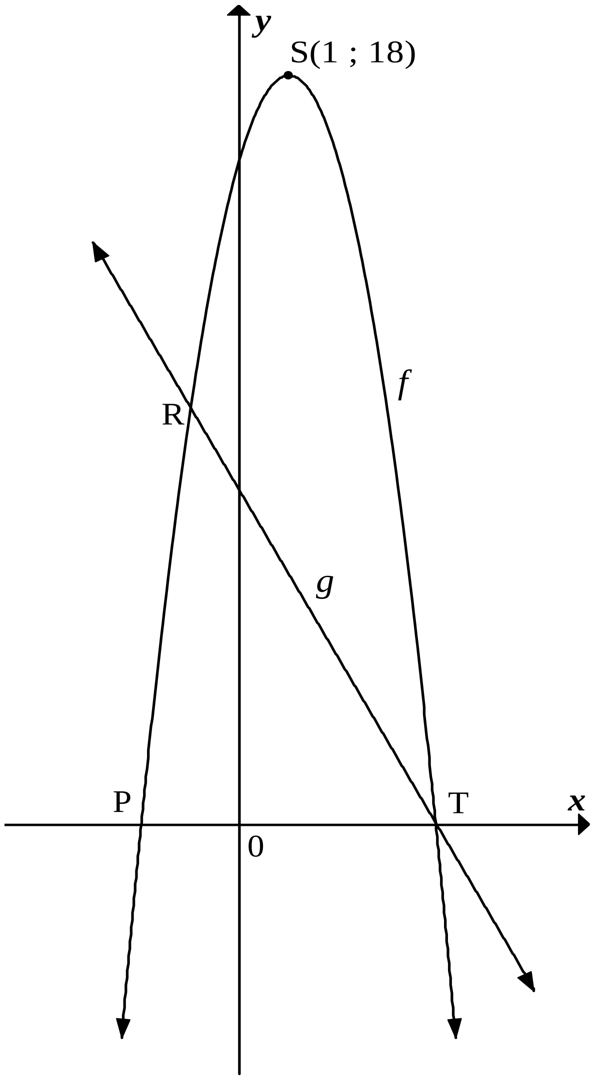


5.1 Determine the equation for g in the form $g(x)=\frac{a}{x-p}+q$ (4)

5.2 F is the reflection of B across C. Determine the coordinates of F. (2)
[6]

QUESTION 6

S(1 ; 18) is the turning point of the graph of $f(x) = ax^2 + bx + c$. P and T are x-intercepts of f. The graph of $g(x) = -2x + 8$ has an x-intercept at T. R is a point of intersection of f and g.



6.1 Calculate the coordinates of T. (2)

6.2 Determine the equation for f in the form $f(x) = ax^2 + bx + c$. Show ALL your working. (4)

6.3 If $f(x) = -2x^2 + 4x + 16$, calculate the coordinates of R. (4)

6.4 Use your graphs to solve for x where:

6.4.1 $f(x) \geq g(x)$ (2)

6.4.2 $-2x^2 + 4x - 2 < 0$ (4)

[16]

QUESTION 7

7.1 Raeesa invests R4 million into an account earning interest of 6% per annum, compounded annually. How much will her investment be worth at the end of 3 years? (3)

7.2 Joanne invests R4 million into an account earning interest of 6% per annum, compounded monthly.

7.2.1 She withdraws an allowance of R30 000 per month. The first withdrawal is exactly one month after she has deposited the R4 million. How many such withdrawals will Joanne be able to make? (6)

7.2.2 If Joanne withdraws R20 000 per month, how many withdrawals will she be able to make? (3)

[12]

QUESTION 8

Jeffrey invests R700 per month into an account earning interest at a rate of 8% per annum, compounded monthly. His friend also invests R700 per month and earns interest compounded semi-annually (that is every six months) at r% per annum. Jeffrey and his friend's investments are worth the same at the end of 12 months. Calculate r. **[3]**

QUESTION 9

9.1 Use the definition of the derivative (first principles) to determine $f'(x)$ if $f(x) = 2x^3$ (5)

9.2 Determine $\frac{dy}{dx}$ if $y = \frac{2\sqrt{x}+1}{x^2}$ (4)

9.3 Calculate the values of a and b if $f(x) = ax^2 + bx + 5$ has a tangent at $x = -1$ which is defined by the equation $y = -7x + 3$ (6)

[15]

QUESTION 10

Given: $f(x) = -x^3 - x^2 + x + 10$

10.1 Write down the coordinates of the y-intercept of f. (1)

10.2 Show that (2 ; 0) is the only x-intercept of f. (4)

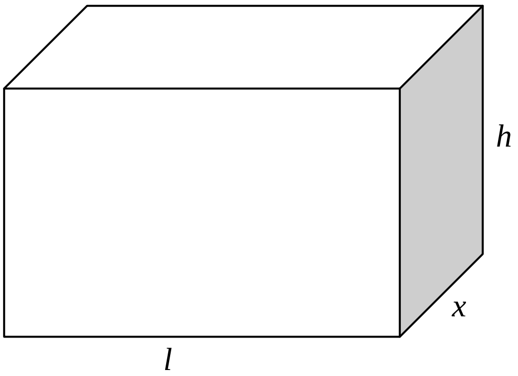
10.3 Calculate the coordinates of the turning points of f. (6)

10.4 Sketch the graph of f in your ANSWER BOOK. Show all intercepts with the axes and all turning points. (3)

[14]

QUESTION 11

A rectangular box is constructed in such a way that the length (l) of the base is three times as long as its width. The material used to construct the top and the bottom of the box costs R100 per square metre. The material used to construct the sides of the box costs R50 per square metre. The box must have a volume of 9 m^3. Let the width of the box be x metres.



11.1 Determine an expression for the height (h) of the box in terms of x. (3)

11.2 Show that the cost to construct the box can be expressed as $C = \frac{1200}{x} + 600x^2$. (3)

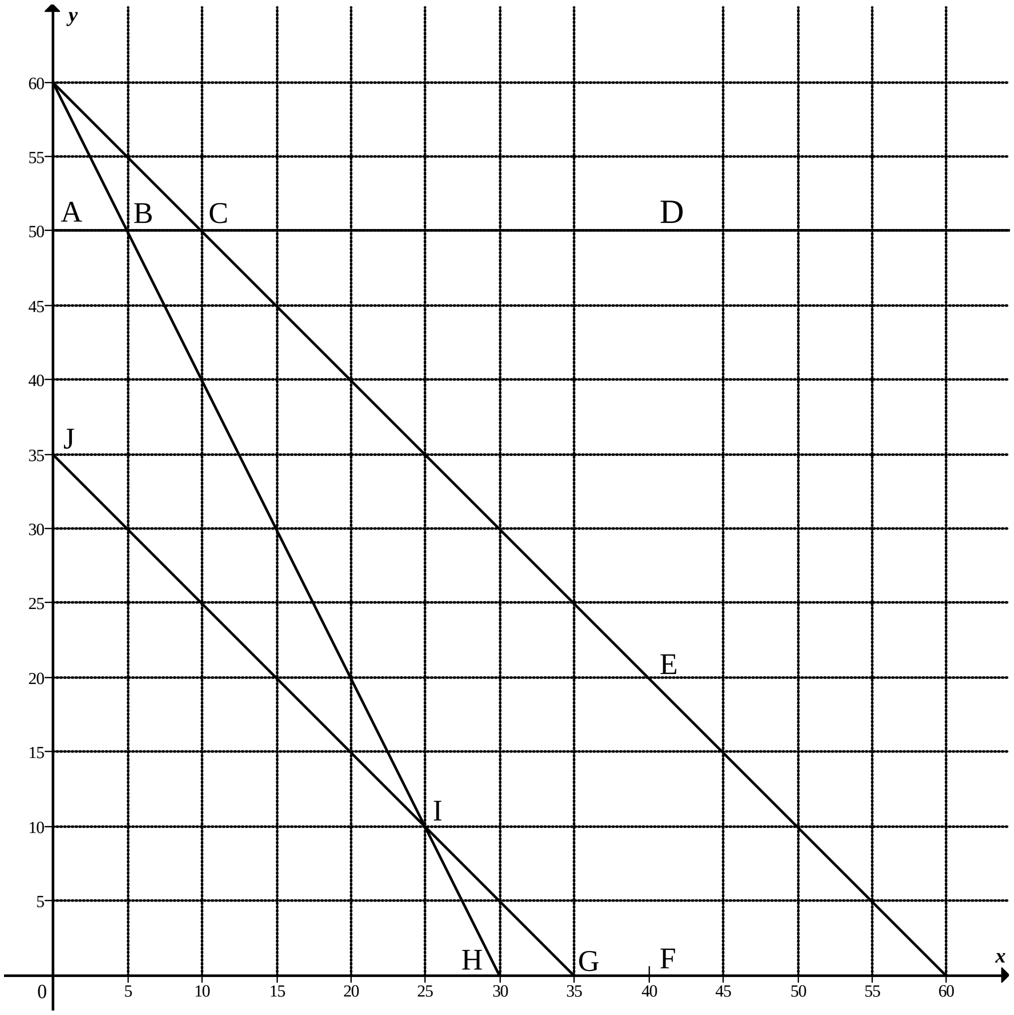
11.3 Calculate the width of the box (that is the value of x) if the cost is to be a minimum. (4)

[10]

QUESTION 12

A system of constraints is given below. Their boundary lines are represented graphically in the sketch below. The diagram is reproduced on DIAGRAM SHEET 1. The constraints are:

$$0 \leq y \leq 50$$
$$0 \leq x \leq 40$$
$$2x + y \leq 60$$
$$35 \leq x + y \leq 60$$



12.1 Shade the feasible region on DIAGRAM SHEET 1. (2)

12.2 Indicate which constraints have no influence on the feasible region. (3)

12.3 What is the maximum value of x allowed by these constraints? (1)

12.4 If $P = 4x + y$ for $(x\,;\,y)$ in the feasible region, determine the maximum value of P. (4)

12.5 If the objective function $C = kx + y$ is minimised at J only, determine ALL possible values of k. (2)

[12]

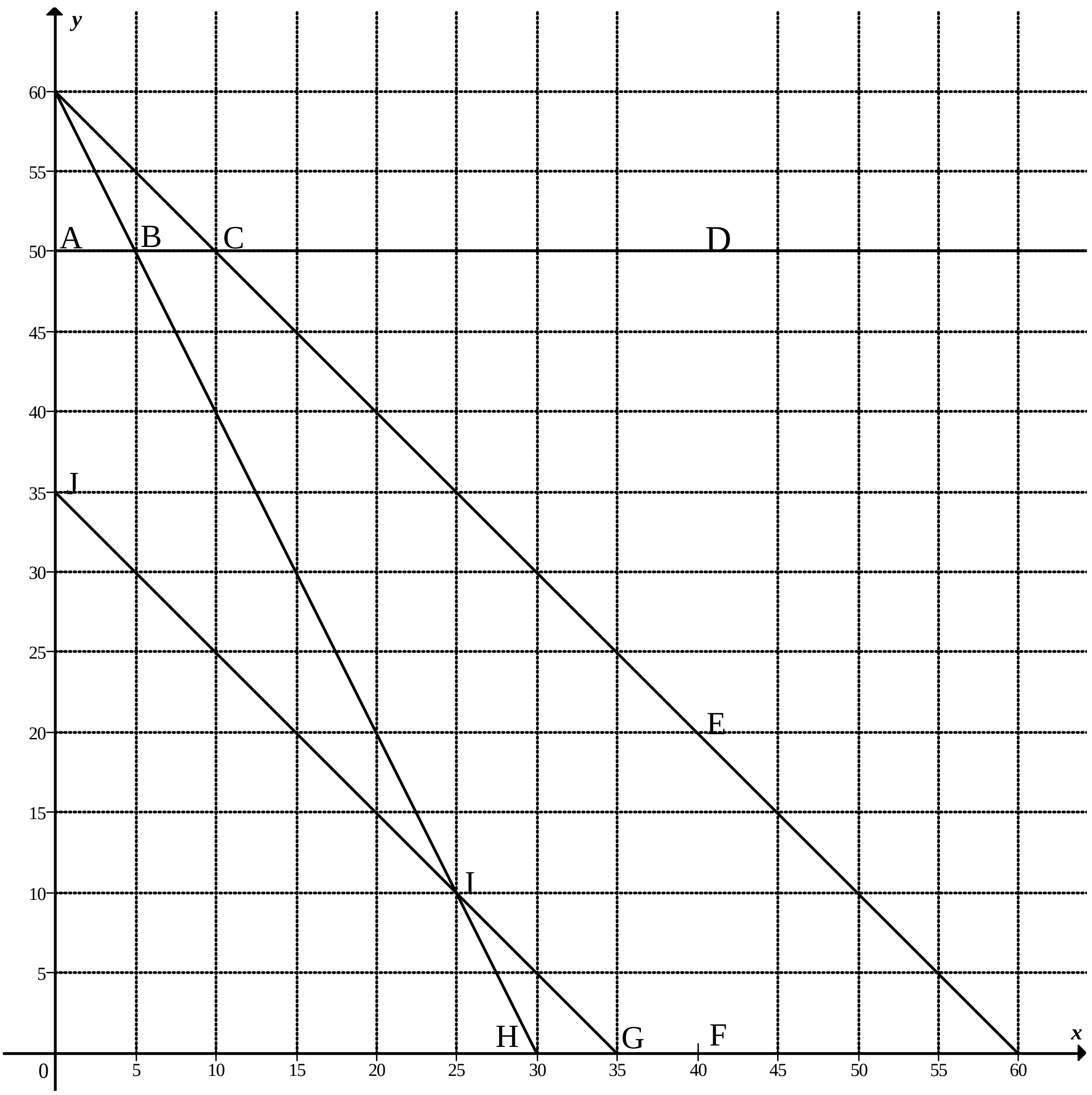
TOTAL: 150

CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 12.1



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$ **;** $\; r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$ **;** $\; -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2} ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$$(x\, ;\, y) \to (x\cos\theta - y\sin\theta\, ;\, y\cos\theta + x\sin\theta)$$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$