



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12/*GRAAD 12*

MATHEMATICS P1/*WISKUNDE V1*

NOVEMBER 2015

MEMORANDUM

MARKS: 150

PUNTE: 150

**This memorandum consists of 25 pages.
*Hierdie memorandum bestaan uit 25 bladsye.***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent accuracy applies in ALL aspects of the marking memorandum.

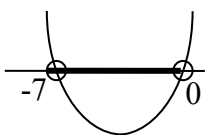
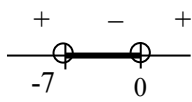
LET WEL:

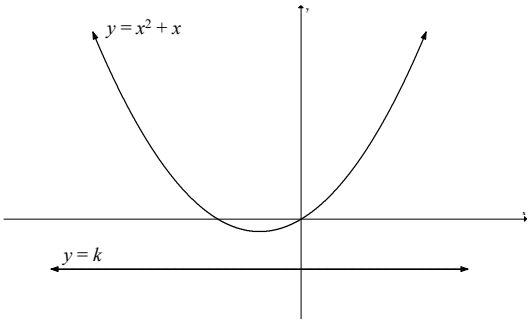
- Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging.
- Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing.

QUESTION/VRAAG 1

1.1.1	$x^2 - 9x + 20 = 0$ $(x - 4)(x - 5) = 0$ $x = 4 \text{ or } x = 5$	✓ factors ✓ $x = 4$ ✓ $x = 5$ (3)
1.1.2	$3x^2 + 5x - 4 = 0$ $x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-4)}}{2(3)}$ $x = \frac{-5 \pm \sqrt{73}}{6}$ $x = -2,26 \text{ or } x = 0,59$ <p>OR/OF</p> $x^2 + \frac{5}{3}x + \frac{25}{36} = \frac{4}{3} + \frac{25}{36}$ $\left(x + \frac{5}{6}\right)^2 = \frac{73}{36}$ $x + \frac{5}{6} = \pm \frac{\sqrt{73}}{6}$ $x = \frac{-5 \pm \sqrt{73}}{6}$ $x = -2,26 \text{ or } x = 0,59$	✓ standard form ✓ substitution into correct formula ✓ ✓ answers (4) ✓ for adding $\frac{25}{36}$ on both sides ✓ $x = \frac{-5 \pm \sqrt{73}}{6}$ ✓ ✓ answers (4)
1.1.3	$2x^{\frac{-5}{3}} = 64$ $x^{\frac{-5}{3}} = 32$ $x = (2^5)^{\frac{-3}{5}}$ $x = 2^{-3} \text{ or } \frac{1}{8} \text{ or } 0,125$ <p>OR/OF</p>	✓ dividing both sides by 2 ✓ $32 = 2^5$ or $64 = 2^6$ ✓ raising RHS to $\frac{-3}{5}$ ✓ answer (4)

	$2x^{\frac{-5}{3}} = 64$ $x^{\frac{-5}{3}} = 32$ $x = (32)^{\frac{-3}{5}}$ $x = \sqrt[5]{32^{-3}}$ $x = 2^{-3} \text{ or } \frac{1}{8} \text{ or } 0,125$ <p>OR/OF</p> $\left(2x^{\frac{-5}{3}}\right)^{\frac{-3}{5}} = 64^{\frac{-3}{5}}$ $0,659x = 0,0825$ $x = 0,125$ <p>OR/OF</p> $x^{\frac{-5}{3}} = 32$ $\frac{-5}{3} \log x = \log 32$ $\log x = \frac{3}{-5} \log 32$ $\log x = -0,903$ $x = 10^{-0,903}$ $= 0,125 \text{ or } \frac{1}{8}$	<p>✓ dividing both sides by 2</p> <p>✓ raising RHS to $\frac{-3}{5}$</p> <p>✓ $\sqrt[5]{32^{-3}}$</p> <p>✓ answer (4)</p> <p>✓ raising both sides to $\frac{-3}{5}$</p> <p>✓ 0,659 and 0,0825</p> <p>✓ dividing both sides by 0,659</p> <p>✓ answer (4)</p> <p>✓ dividing both sides by 2</p> <p>✓ logs on both sides</p> <p>✓ $\log x = -0,903$</p> <p>✓ answer (4)</p>
1.1.4	$\sqrt{2-x} = x-2$ $2-x = (x-2)^2$ $2-x = x^2 - 4x + 4$ $x^2 - 3x + 2 = 0$ $(x-1)(x-2) = 0$ $x = 1 \text{ or } x = 2$ <p>if $x = 1$, $\sqrt{2-x} = 1$ and $x-2 = -1$</p> <p>$x = 2$ only</p> <p>OR/OF</p>	<p>✓ squaring both sides</p> <p>✓ factors</p> <p>✓ $x = 1$ or $x = 2$</p> <p>✓ $x = 2$ only (4)</p>

	$\sqrt{2-x} = x-2$ $2-x = (x-2)^2$ $2-x = (2-x)^2$ $2-x = 1 \text{ or } 2-x = 0$ $x = 1 \text{ or } x = 2$ <p>if $x = 1$, $\sqrt{2-x} = 1$ and $x-2 = -1$ $\therefore x = 2$ only</p> <p>OR/OF</p> $\sqrt{2-x} = x-2$ $2-x \geq 0 \text{ and } x-2 \geq 0$ $x \leq 2 \text{ and } x \geq 2$ $\therefore x = 2 \text{ only}$	<p>✓ squaring both sides ✓ $2-x=1$ or $2-x=0$</p> <p>✓ $x = 1$ or $x = 2$</p> <p>✓ $x = 2$ only (4)</p> <p>✓ $2-x \geq 0$ ✓ $x-2 \geq 0$</p> <p>✓ $x \leq 2$ and $x \geq 2$ ✓ $x = 2$ (4)</p>
<p>1.1.5</p>	$x^2 + 7x < 0$ $x(x+7) < 0$ <div style="display: flex; align-items: center; justify-content: center;">  <div style="margin: 0 10px;">OR/OF</div>  </div> $-7 < x < 0 \text{ OR/OF } x \in (-7; 0)$	<p>✓ factors</p> <p>✓✓ inequality or interval (3)</p>
<p>1.2</p>	<p>The square of any number is always positive or zero So for the sum of two squares to be zero, both squares must be zero, i.e. <i>Die kwadraat van enige getal is altyd positief of nul. Vir die som van twee kwadrate om nul te wees, moet beide die kwadrate nul wees, d.i.</i></p> $(3x-y)^2 = 0 \text{ and/en } (x-5)^2 = 0$ $3x-y = 0 \text{ and/en } x-5 = 0$ $x = 5$ $3(5)-y = 0$ $y = 15$	<p>✓ $3x-y = 0$ ✓ $x-5 = 0$</p> <p>✓ $x = 5$ ✓ $y = 15$ (4)</p>

<p>1.3</p>	$x^2 + x = k$ $x^2 + x - k = 0$ $\Delta < 0$ $b^2 - 4ac < 0$ $1^2 - 4(1)(-k) < 0$ $1 + 4k < 0$ $k < \frac{-1}{4}$ <p>OR/OF</p> $x^2 + x = k$ $x^2 + x + \frac{1}{4} = k + \frac{1}{4}$ $\left(x + \frac{1}{2}\right)^2 = k + \frac{1}{4}$ <p>for nonreal roots $k + \frac{1}{4} < 0$</p> $k < \frac{-1}{4}$ <p>OR/OF</p> <p>Consider the functions $y = x^2 + x$ and $y = k$ <i>Beskou die funksies $y = x^2 + x$ en $y = k$</i></p>  <p>Turning point of/<i>Draaipunt van</i> $y = x^2 + x$ is $\left(\frac{-1}{2}; \frac{-1}{4}\right)$</p> <p>$x^2 + x = k$ does not have real roots when the line $y = k$ does not intersect $y = x^2 + x$.</p> <p>$x^2 + x = k$ <i>het geen reële wortels as die lyn $y = k$ nie met $y = x^2 + x$ sny nie.</i></p> <p>Therefore $k < \frac{-1}{4}$</p>	<p>✓ standard form</p> <p>✓ $\Delta < 0$</p> <p>✓ $1^2 - 4(1)(-k)$</p> <p>✓ $k < \frac{-1}{4}$</p> <p>(4)</p> <p>✓ adds $\frac{1}{4}$ to both sides</p> <p>✓ $\left(x + \frac{1}{2}\right)^2 = k + \frac{1}{4}$</p> <p>✓ $k + \frac{1}{4} < 0$</p> <p>✓ $k < \frac{-1}{4}$</p> <p>(4)</p> <p>✓ sketch or explanation</p> <p>✓ $x = -\frac{1}{2}$</p> <p>✓ $y = -\frac{1}{4}$</p> <p>✓ $k < \frac{-1}{4}$</p> <p>(4)</p> <p>[26]</p>
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QUESTION/VRAAG 2

2.1	$r = \frac{T_2}{T_1}$ $= \frac{5}{10}$ $= \frac{1}{2}$ $T_5 = 1,25 \left(\frac{1}{2} \right)$ $= \frac{5}{8} \text{ or } 0,625$ <p style="text-align: center;">OR/OF</p> $T_5 = 10 \left(\frac{1}{2} \right)^4$ $= \frac{5}{8} \text{ or } 0,625$	$\checkmark r = \frac{1}{2}$ \checkmark answer (2)
2.2	$T_n = 10 \left(\frac{1}{2} \right)^{n-1}$	\checkmark substitutes $a = 10$ into GP formula \checkmark substitutes $r = \frac{1}{2}$ into GP formula (2)
2.3	For convergence/ <i>Om te konvergeer</i> $-1 < r < 1$ Since/ <i>Aangesien</i> $r = \frac{1}{2}$ and/ <i>en</i> $-1 < \frac{1}{2} < 1$ the sequence converges/ <i>die ry konvergeer</i>	$\checkmark -1 < r < 1$ \checkmark show that $r = \frac{1}{2}$ is $-1 < r < 1$ (2)
2.4	$S_\infty - S_n = \frac{a}{1-r} - \frac{a(1-r^n)}{1-r}$ $= \frac{10}{1-\frac{1}{2}} - \frac{10 \left(1 - \frac{1^n}{2} \right)}{1-\frac{1}{2}}$ $= 20 - 20 \left(1 - \frac{1^n}{2} \right)$ $= 20 - 20 + 20 \left(\frac{1}{2} \right)^n$ $= 20 \left(\frac{1}{2} \right)^n$ <p style="text-align: center;">OR/OF</p>	$\checkmark \frac{10}{1-\frac{1}{2}}$ $\checkmark \frac{10 \left(1 - \frac{1^n}{2} \right)}{1-\frac{1}{2}}$ $\checkmark 20 \left(1 - \frac{1^n}{2} \right)$ \checkmark answer (4) \checkmark constructing the series

	$S_{\infty} - S_n = T_{n+1} + T_{n+2} + T_{n+3} + \dots$ $= 10\left(\frac{1}{2}\right)^n \left[1 + \frac{1}{2} + \frac{1}{4} + \dots \right]$ $= 10\left(\frac{1}{2}\right)^n \left[\frac{1}{1 - \frac{1}{2}} \right]$ $= 20\left(\frac{1}{2}\right)^n$ <p>OR/OF</p> $S_{\infty} - S_n = \frac{a}{1-r} - \frac{a(1-r^n)}{1-r}$ $= \frac{a - a + ar^n}{1-r}$ $= \frac{ar^n}{1-r}$ $= \frac{10\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$ $= 20\left(\frac{1}{2}\right)^n$	<p>✓</p> $10\left(\frac{1}{2}\right)^n \left[1 + \frac{1}{2} + \frac{1}{4} + \dots \right]$ <p>✓ $\frac{1}{1 - \frac{1}{2}}$</p> <p>✓ answer (4)</p> <p>✓ $\frac{a - a + ar^n}{1-r}$</p> <p>✓ $\frac{ar^n}{1-r}$</p> <p>✓ $\frac{10\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$</p> <p>✓ answer (4)</p> <p>[10]</p>
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QUESTION/VRAAG 3

3.1	$d = 8$ $T_k = a + (k - 1)d$ $= -3 + (k - 1)(8)$ $= -3 + 8k - 8$ $= 8k - 11$	✓ d value ✓ answer (2)
3.2	$\sum_{k=1}^n (8k - 11) \quad \text{OR/OF} \quad \sum_{k=0}^{n-1} (8(k+1) - 11) = \sum_{k=0}^{n-1} (8k - 3)$	✓ for general term ✓ lower and upper values in sigma notation (2)
3.3	$S_n = \frac{n}{2} [2a + (n - 1)d]$ $= \frac{n}{2} [2(-3) + (n - 1)(8)]$ $= \frac{n}{2} [-6 + 8n - 8]$ $= \frac{n}{2} [8n - 14]$ $= n(4n - 7)$ $= 4n^2 - 7n$ <p>OR/OF</p> $S_n = \frac{n}{2} [2a + (n - 1)d]$ $= \frac{n}{2} [2(-3) + (n - 1)(8)]$ $= \frac{n}{2} [-6 + 8n - 8]$ $= \frac{n}{2} [8n - 14]$ $= 4n^2 - 7n$ <p>OR/OF</p> $S_n = \frac{n}{2} [a + l]$ $= \frac{n}{2} [-3 + 8n - 11]$ $= \frac{n}{2} [8n - 14]$ $= 4n^2 - 7n$	✓ formula ✓ substitution ✓ $\frac{n}{2} [8n - 14]$ (3)

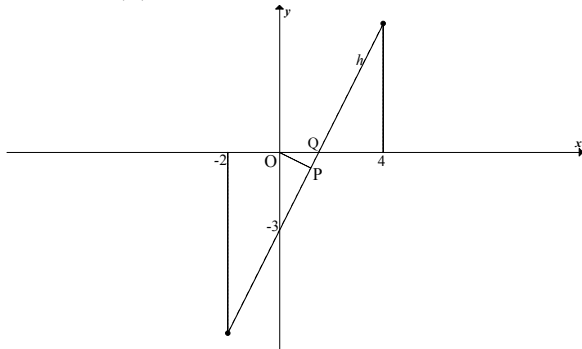
	<p>OR/OF</p> <p> $S_1 = -3$ $S_2 = 2$ $S_3 = 15$ $S_4 = 36$ 5 13 21 8 8 </p> <p> $S_n = an^2 + bn + c$ $a = \frac{8}{2}$ $a = 4$ $S_1 = 4 + b + c = -3$ $b + c = -7$(1) $S_2 = 16 + 2b + c = 2$ $2b + c = -14$.....(2) $b = -7$.....(2)–(1) $c = 0$ Hence $S_n = 4n^2 - 7n$ </p>	<p> $S_2 = -3 + 5 = 2$ $S_3 = 2 + 13 = 15$ $S_4 = 15 + 21$ ✓ calculates S_1, S_2, S_3 and S_4, ✓ $a = 4$ ✓ solves simultaneously for b and c. (3) </p>
<p>3.4.1</p>	<p>$Q_6 = -6 - 3 + 5 + 13 + 21 + 29$</p>	<p>✓✓ answer (2)</p>
<p>3.4.2</p>	<p> $Q_{129} = -6 + S_{128}$ $= -6 + 4(128)^2 - 7(128)$ $= 64634$ </p> <p>OR/OF</p> <p> $Q_1 = -6$ $Q_2 = -9$ $Q_3 = -4$ $Q_4 = 9$ -3 5 13 8 8 8 </p> <p> $Q_n = an^2 + bn + c$ $a = 4$ $Q_1 = 4 + b + c = -6$ $b + c = -10$(1) $Q_2 = 16 + 2b + c = -9$ $2b + c = -25$.....(2) $b = -15$.....(2)–(1) $c = 5$ Hence $Q_n = 4n^2 - 15n + 5$ $Q_{129} = 4(129)^2 - 15(129) + 5$ $= 64\ 634$ </p>	<p> ✓✓ $-6 + 4(128)^2 - 7(128)$ ✓ answer (3) ✓ $a = 4$ ✓ $Q_n = 4n^2 - 15n + 5$ ✓ answer (3) [12] </p>

QUESTION/VRAAG 4

Given: $f(x) = 2^{x+1} - 8$		
4.1	$y = -8$	✓ $y = -8$ (1)
4.2		✓ x-intercept ✓ y-intercept ✓ shape ✓ asymptote (only if the graph does not cut the asymptote) (4)
4.3	$g(x) = 2^{-x+1} - 8$ OR/OF $g(x) = \left(\frac{1}{2}\right)^{x-1} - 8$	✓ answer (1) ✓ answer (1) [6]

QUESTION/VRAAG 5

Given $h(x) = 2x - 3$ for $-2 \leq x \leq 4$.

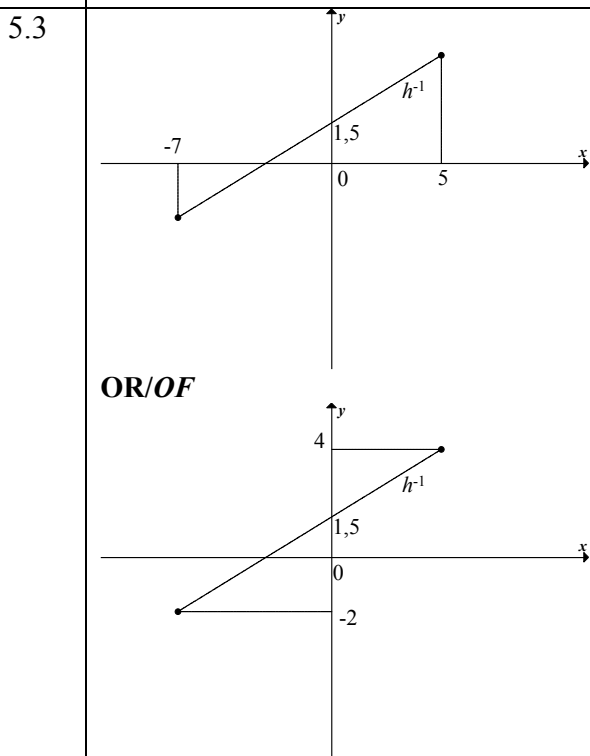


5.1 For x -intercepts, $y = 0$
 $2x - 3 = 0$
 $x = 1,5$
 $Q(1,5 ; 0)$

✓ $x = 1,5$
 ✓ $y = 0$
 (2)

5.2 h :
 $x = -2$: $y = 2(-2) - 3 = -7$
 $x = 4$: $y = 2(4) - 3 = 5$
 Domain of h^{-1} : $-7 \leq x \leq 5$ **OR/OF** $[-7; 5]$

✓ $h(-2) = -7$
 ✓ $h(4) = 5$
 ✓ $-7 \leq x \leq 5$
 (3)



✓ y -intercept on a straight line
 ✓ line segment
 ✓ accurate endpoints (x or y or both)
 (3)

5.4	$h(x) = 2x - 3$ <p>For the inverse of h,</p> $x = 2y - 3$ $y = \frac{x+3}{2}$ $h^{-1}(x) = \frac{x+3}{2}$ $h(x) = h^{-1}(x)$ $2x - 3 = \frac{x+3}{2}$ $4x - 6 = x + 3$ $x = 3$ <p>OR/OF</p> $h(x) = 2x - 3$ <p>h and h^{-1} intersect when $y = x$</p> $h(x) = x$ $2x - 3 = x$ $x = 3$ <p>OR/OF</p> $h(x) = 2x - 3$ <p>For the inverse of h,</p> $x = 2y - 3$ $y = \frac{x+3}{2}$ $h^{-1}(x) = x$ $\frac{x+3}{2} = x$ $x + 3 = 2x$ $x = 3$	$\checkmark y = \frac{x+3}{2}$ $\checkmark 2x - 3 = \frac{x+3}{2}$ $\checkmark x = 3$ <p style="text-align: right;">(3)</p> $\checkmark h(x) = x$ $\checkmark 2x - 3 = x$ $\checkmark x = 3$ <p style="text-align: right;">(3)</p> $\checkmark y = \frac{x+3}{2}$ $\checkmark \frac{x+3}{2} = x$ $\checkmark x = 3$ <p style="text-align: right;">(3)</p>
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	<p>OR/OF For minimum distance $OP \perp$ the line $O(0;0) \quad P(x; 2x-3) \quad Q\left(\frac{3}{2}; 0\right)$ $OP^2 + PQ^2 = OQ^2$ (pythag) $(x-0)^2 + (2x-3-0)^2 + \left(x-\frac{3}{2}\right)^2 + (2x-3-0)^2 = \left(\frac{3}{2}\right)^2$ $x^2 + 4x^2 - 12x + 9 + x^2 - 3x + \frac{9}{4} + 4x^2 - 12x + 9 = \frac{9}{4}$ $10x^2 - 27x + 18 = 0$ $(5x-6)(2x-3) = 0$ $x = \frac{6}{5}$ or $\frac{3}{2}$</p> <p>Hence, $x = \frac{6}{5}$ at P $OP^2 = x^2 + (2x-3)^2$ $= \left(\frac{6}{5}\right)^2 + \left(2\left(\frac{6}{5}\right) - 3\right)^2$ $= \frac{36}{25} + \frac{9}{25}$ $= \frac{9}{5}$ $OP = 1,34$</p> <p>OR/OF For minimum distance $OP \perp$ the line $\tan \hat{Q} = 2$ $\hat{Q} = 63,43^\circ$ $\sin 63,43^\circ = \frac{OP}{1,5}$ $OP = 1,34$</p>	<p>✓ $OP^2 = x^2 + y^2$ ✓ substitute $y = 2x - 3$</p> <p>✓ $10x^2 - 27x + 18$</p> <p>✓ x-value</p> <p>✓ answer (5)</p> <p>✓ $\tan \hat{Q} = 2$ ✓ $\hat{Q} = 63,43^\circ$ ✓ $\sin 63,43^\circ$ ✓ $\frac{OP}{1,5}$ ✓ answer (5)</p>
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OR/OF

$$\begin{aligned}
 OP &= \sqrt{(x-0)^2 + (y-0)^2} \\
 &= \sqrt{(x-0)^2 + (2x-3-0)^2} \\
 &= \sqrt{x^2 + 4x^2 - 12x + 9} \\
 &= \sqrt{5x^2 - 12x + 9}
 \end{aligned}$$

By using the chain rule (which is not in the CAPS):

$$\frac{dOP}{dx} = \frac{1}{2}(5x^2 - 12x + 9)^{-\frac{1}{2}} \cdot (10x - 12)$$

$$0 = \frac{1}{2}(5x^2 - 12x + 9)^{-\frac{1}{2}} \cdot (10x - 12)$$

$$0 = \frac{1}{2}(10x - 12)$$

$$0 = 5x - 6$$

$$x = \frac{6}{5}$$

$$\begin{aligned}
 OP &= \sqrt{5\left(\frac{6}{5}\right)^2 - 12\left(\frac{6}{5}\right) + 9} \\
 &= 1,34
 \end{aligned}$$

OR/OF

For minimum distance $OP \perp$ the line
Let the y -intercept be R

$$OR = 3 \text{ units}$$

$$OQ = \frac{3}{2} \text{ units}$$

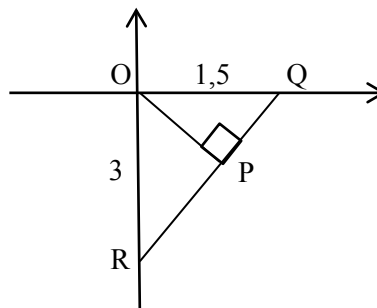
$$RQ = \frac{3}{2}\sqrt{5} \text{ (Pythagoras)}$$

$$\text{Area OQR} = \frac{1}{2} \times \text{base} \times \perp\text{height}$$

$$\frac{1}{2} \cdot OR \cdot OQ = \frac{1}{2} \cdot \left(\frac{3}{2}\sqrt{5}\right) \cdot OP$$

$$\frac{1}{2} \cdot 3 \cdot \left(\frac{3}{2}\right) = \frac{1}{2} \cdot \left(\frac{3}{2}\sqrt{5}\right) \cdot OP$$

$$OP = \frac{3}{\sqrt{5}} = 1,34$$



✓

$$OP = \sqrt{(x-0)^2 + (y-0)^2}$$

✓ substitute

$$y = 2x - 3$$

$$✓ 5x^2 - 12x + 9$$

✓ x -value

✓ answer

(5)

$$✓ RQ = \frac{3}{2}\sqrt{5}$$

$$✓ \frac{1}{2} \cdot \left(\frac{3}{2}\sqrt{5}\right) \cdot OP$$

$$✓ \frac{1}{2} \cdot 3 \cdot \left(\frac{3}{2}\right)$$

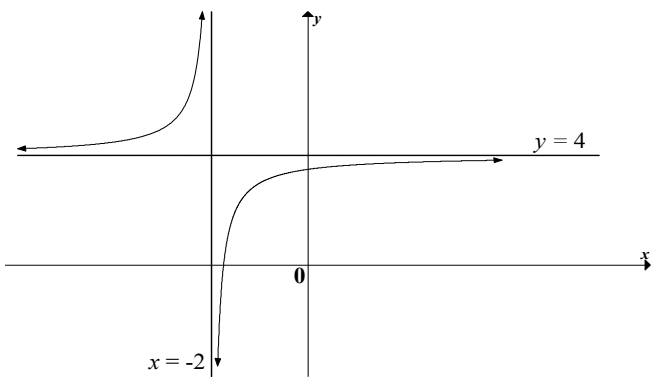
✓ equating

✓ answer

(5)

<p>5.6.1</p>	<p>$f'(x) = 2x - 3$ Turning point at $x = \frac{3}{2}$ $f''(x) = 2 > 0$ or $f''\left(\frac{3}{2}\right) > 0$ f has a local minimum at $x = \frac{3}{2}$ f het 'n lokale minimum by $x = \frac{3}{2}$</p> <p>OR/OF</p> <p>$h(x) = f'(x) < 0$ for $x \in (-2 ; 1,5) \Rightarrow f$ is decreasing on the left of Q / <i>f is dalend links van Q.</i> $h(x) = f'(x) > 0$ for $x \in (1,5 ; 4) \Rightarrow f$ is increasing on the right of Q / <i>f is stygend regs van Q.</i></p> <p>$\therefore f(x)$ has a local minimum when $x = \frac{3}{2}$ / $\therefore f(x)$ het 'n lokaal minimum by $x = \frac{3}{2}$</p> <p>OR/OF</p> <p>$f(x) = x^2 - 3x + c$ f has a minimum value since $a > 0$ f het 'n minimum waarde omdat $a > 0$</p>	<p>✓ Turning point at $x = \frac{3}{2}$ ✓ $f''(x) = 2 > 0$ (2)</p> <p>✓ decreasing left of Q ✓ increasing right of Q (2)</p> <p>✓ $f(x) = x^2 - 3x + c$ ✓ explanation (2)</p>
<p>5.6.2</p>	<p>$m = f'(4) = h(4) = 5$</p>	<p>✓ answer (1) [19]</p>

QUESTION/VRAAG 6

6.1.1	$T(0;18)$	✓ (0;18) (1)
6.1.2	$-2x^2 + 18 = 0$ $(x - 3)(x + 3) = 0$ $Q(3; 0)$ OR/OF $-2x^2 + 18 = 0$ $x^2 = 9$ $Q(3; 0)$	✓ $y = 0$ ✓ factors ✓ $x = 3$ (3) ✓ $y = 0$ ✓ $x^2 = 9$ ✓ $x = 3$ (3)
6.1.3	x -coordinate of S is 4,5/ x -koördinaat van S is 4,5 By symmetry about the line $x = 4,5$ / <i>Deur simmetrie om die lyn $x = 4,5$:</i> $R = (4,5 + 4,5 - 3; 0) = (6; 0)$	✓ $x = 6$ ✓ $y = 0$ (2)
6.1.4	For all $x \in \mathbf{R}$ OR/OF $(-\infty; \infty)$	✓✓ answer (2)
6.2	If $C(x; y)$ is the centre of the hyperbola/ <i>As $C(x; y)$ die middelpunt is van die hiperbool</i> $y = x + 6$ and $x = -2$ $\therefore y = -2 + 6 = 4$ 	✓✓ asymptote $y = 4$ ✓ asymptote $x = -2$ ✓ shape (increasing hyperbolic function) (4) [12]

QUESTION/VRAAG 7

7.1	R450 000	✓ answer (1)
7.2	$A = P(1-i)^n$ $f(x) = 450000(1-i)^x$ $243\,736,90 = 450000(1-i)^4$ $i = 1 - \sqrt[4]{\frac{243\,736,90}{450000}}$ $i = 0,1421$ <p>The rate of depreciation is 14,21% p.a. <i>Die waardeverminderingskoers is 14,21% p.j.</i></p>	✓ substitution of 450 000 into correct formula ✓ substitution of (4; 243 736,90) into correct formula ✓ making i the subject ✓ answer (4)
7.3	At T : $A = P(1+i)^n$ $g(x) = 450000(1+i)^x$ $a = 450000(1+0,081)^4$ $= R614490,66$	✓ $i = 0,081$ & $n = 4$ ✓ correct substitution into formula ✓ answer (3)
7.4	Future Value = R614 490,66 – R243 736,90 = R370 753,76 Let x be the value of monthly payment $F_v = \frac{x[(1+i)^n - 1]}{i}$ $370753,76 = \frac{x \left[\left(1 + \frac{0.062}{12} \right)^{36} - 1 \right]}{\frac{0.062}{12}}$ $x = R9397,11$	✓ R370 753,76 ✓ $i = \frac{0,062}{12}$ ✓ $n = 36$ ✓ substitution into correct formula ✓ answer (5) [13]

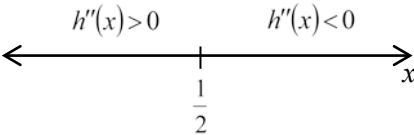
QUESTION/VRAAG 8

8.1	$f(x+h) = (x+h)^2 - 3(x+h)$ $= x^2 + 2xh + h^2 - 3x - 3h$ $f(x+h) - f(x) = x^2 + 2xh + h^2 - 3x - 3h - (x^2 - 3x)$ $= 2xh + h^2 - 3h$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 3h}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h - 3)}{h}$ $= \lim_{h \rightarrow 0} (2x + h - 3)$ $= 2x - 3$ <p>OR/OF</p> $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$ $= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 3h}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h - 3)}{h}$ $= \lim_{h \rightarrow 0} (2x + h - 3)$ $= 2x - 3$	<p>✓ finding $f(x+h)$</p> <p>✓ $2xh + h^2 - 3h$</p> <p>✓ formula</p> <p>✓ factorisation</p> <p>✓ answer (5)</p> <p>✓ formula</p> <p>✓ finding $f(x+h)$</p> <p>✓ $2xh + h^2 - 3h$</p> <p>✓ factorisation</p> <p>✓ answer (5)</p>
8.2.1	$y = \left(x^2 - \frac{1}{x^2}\right)^2$ $y = x^4 - 2 + \frac{1}{x^4}$ $= x^4 - 2 + x^{-4}$ $\frac{dy}{dx} = 4x^3 - 4x^{-5}$ <p>OR/OF</p>	<p>✓ $x^4 - 2 + \frac{1}{x^4}$</p> <p>✓ $4x^3$</p> <p>✓ $-4x^{-5}$</p> <p>(3)</p>

	<p>By using the chain rule (which is not part of CAPS):</p> $y = (x^2 - x^{-2})^2$ $\frac{dy}{dx} = 2(x^2 - x^{-2})(2x + 2x^{-3})$ $= 2(2x^3 + 2x^{-1} - 2x^{-1} - 2x^{-5})$ $= 2(2x^3 - 2x^{-5})$ $= 4x^3 - 4x^{-5}$	<p>✓✓✓</p> $2(x^2 - x^{-2})(2x + 2x^{-3})$ <p>(3)</p>
8.2.2	$D_x \left[\frac{(x-1)(x^2+x+1)}{x-1} \right]$ $= D_x [x^2 + x + 1]$ $= 2x + 1$ <p>OR/OF</p> <p>By using the quotient rule (with is not part of CAPS):</p> $D_x \left[\frac{x^3 - 1}{x - 1} \right]$ $= \frac{3x^2(x-1) - (x^3 - 1)}{(x-1)^2}$	<p>✓ factorisation</p> <p>✓ $x^2 + x + 1$</p> <p>✓ $2x + 1$</p> <p>(3)</p> <p>✓✓✓</p> $\frac{3x^2(x-1) - (x^3 - 1)}{(x-1)^2}$ <p>(3)</p> <p>[11]</p>

QUESTION/VRAAG 9

<p>9.1</p>	<p>Substitute Q(2; 10) into $h(x) = -x^3 + ax^2 + bx$ $-2^3 + a(2^2) + b(2) = 10$ $-8 + 4a + 2b = 10$ $2a + b = 9$line 1 $h'(x) = -3x^2 + 2ax + b$ At Q: $h'(2) = 0$ $-3(2)^2 + 2a(2) + b = 0$ $-12 + 4a + b = 0$ $4a + b = 12$line 2 line 2 – line 1: $2a = 3$ $a = \frac{3}{2}$ Substitute in line 1: $b = 6$</p>	<p>✓ substitute Q into h ✓ finding derivative ✓ $h'(2)$ ✓ equating derivative to 0 ✓ solving simultaneously for a and b (5)</p>
<p>9.2</p>	<p>$f(-1) = -(-1)^3 + \frac{3}{2}(-1)^2 + 6(-1)$ $= -3,5$ Average gradient/<i>Gemiddelde gradiënt</i> = $\frac{f(x_Q) - f(x_P)}{x_Q - x_P}$ Average gradient/<i>Gemiddelde gradiënt</i> = $\frac{10 - (-3,5)}{2 - (-1)}$ $= 4,5$</p>	<p>✓ $f(-1) = -3,5$ ✓ formula ✓ substitution ✓ answer (4)</p>

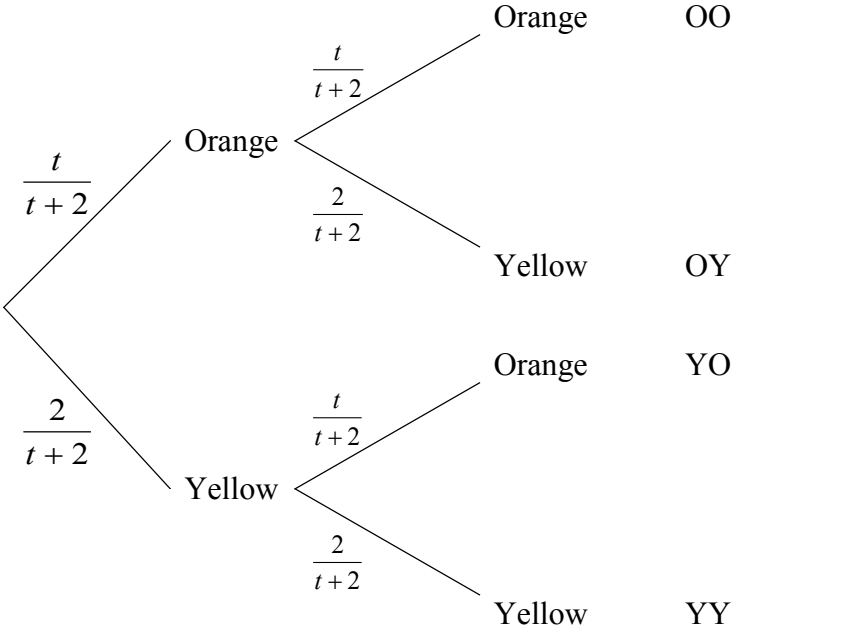
9.3	$h'(x) = -3x^2 + 3x + 6$ $h''(x) = -6x + 3$ $= -3(2x - 1)$  <p>For $x < \frac{1}{2}$, h is concave up and for $x > \frac{1}{2}$, h is concave down <i>Vir $x < \frac{1}{2}$, is h konkaaf na bo en vir $x > \frac{1}{2}$, is h konkaaf na onder</i></p> <p>\therefore concavity changes at $x = \frac{1}{2}$ / \therefore <i>konkwiteit verander by $x = \frac{1}{2}$</i></p>	$\checkmark h'(x) = -3x^2 + 3x + 6$ $\checkmark h''(x) = -6x + 3$ \checkmark explanation using $h''(x)$ (3)
9.4	<p>The graph of h has a point of inflection at $x = \frac{1}{2}$ / <i>Die grafiek van h het 'n buigpunt by $x = \frac{1}{2}$.</i></p> <p>OR/OF</p> <p>The graph of h changes from concave up to concave down at $x = \frac{1}{2}$ / <i>Die grafiek van h verander by $x = \frac{1}{2}$ van konkaaf op na konkaaf af</i></p>	\checkmark answer (1) \checkmark answer (1)
9.5	<p>Gradient of g is -12 / <i>Gradiënt van g is -12</i> Gradient of tangent is / <i>Gradiënt van die raaklyn is:</i></p> $h'(x) = -3x^2 + 3x + 6$ $h'(x) = -12$ $-3x^2 + 3x + 6 = -12$ $3x^2 - 3x + 18 = 0$ $x^2 - x + 6 = 0$ $(x - 3)(x + 2) = 0$ $x = -2 \text{ only}$	$\checkmark h'(x) = -3x^2 + 3x + 6$ $\checkmark h'(x) = -12$ \checkmark factors \checkmark selection of x -value (4) [17]

QUESTION/VRAAG 10

10.1	$\frac{h}{r} = \tan 60^\circ$ $r = \frac{h}{\tan 60^\circ}$ $\therefore r = \frac{h}{\sqrt{3}}$	$\checkmark \frac{h}{r} = \tan 60^\circ$ $\checkmark \text{answer}$ (2)
10.2	$V_{\text{cone}} = \frac{1}{3} \pi r^2 h$ $= \frac{1}{3} \pi \left(\frac{h}{\sqrt{3}} \right)^2 h$ $= \frac{1}{9} \pi h^3$ $\frac{dV}{dh} = \frac{1}{3} \pi h^2$ $\left. \frac{dV}{dh} \right _{h=9} = \frac{1}{3} \pi (9)^2$ $= 27\pi \text{ or } 84,82 \text{ cm}^3/\text{cm}$	$\checkmark \text{formula}$ $\checkmark \text{substitution of the value of } r \text{ in terms of } h$ $\checkmark \text{simplified volume answer}$ $\checkmark \text{derivative}$ $\checkmark \text{answer}$ (5) [7]

QUESTION/VRAAG 11

11.1	$P(A) \times P(B)$ $= 0,2 \times 0,63$ $= 0,126$ i.e. $P(A) \times P(B) = P(A \text{ and } B)$ Therefore A and B are independent/ <i>Dus is A en B onafhanklik</i>	$\checkmark 0,2 \times 0,63$ $\checkmark P(A) \times P(B) = P(A \text{ and } B)$ \checkmark conclusion (3)
11.2.1	$7^7 = 823\ 543$	$\checkmark \checkmark 7^7$ (2)
11.2.2	$7! = 5040$	$\checkmark \checkmark 7!$ (2)
11.2.3	There are 3 vowels \Rightarrow 3 options for first position There are 4 consonants \Rightarrow 4 options for last position The remaining 5 letters can be arranged in $5 \times 4 \times 3 \times 2 \times 1$ ways $3 \times (5 \times 4 \times 3 \times 2 \times 1) \times 4 = 1440$ <i>Daar is 3 klinkers \Rightarrow 3 opsies vir die eerste posisie</i> <i>Daar is 4 konsonante \Rightarrow 4 opsies vir die laaste posisie</i> <i>Die oorblywende 5 letters kan as volg gerangskik word</i> $5 \times 4 \times 3 \times 2 \times 1$ ways/ <i>maniere</i> $3 \times (5 \times 4 \times 3 \times 2 \times 1) \times 4 = 1440$	$\checkmark \times 3$ $\checkmark \times 4$ $\checkmark 5 \times 4 \times 3 \times 2 \times 1$ \checkmark answer (4)

<p>11.3</p>	 <p> $P(\text{Orange, Orange}) + P(\text{Yellow, Yellow}) = \frac{52}{100}$ $\left(\frac{t}{t+2}\right)\left(\frac{t}{t+2}\right) + \left(\frac{2}{t+2}\right)\left(\frac{2}{t+2}\right) = \frac{52}{100}$ $\frac{t^2}{t^2 + 4t + 4} + \frac{4}{t^2 + 4t + 4} = \frac{13}{25}$ $25(t^2 + 4) = 13(t^2 + 4t + 4)$ $3t^2 - 13t + 12 = 0$ $(3t - 4)(t - 3) = 0$ $t = 3$ </p> <p>There are 3 orange balls in the bag/<i>Daar is 3 oranje balle in die sak</i></p>	<p> $\checkmark P(O) = \left(\frac{t}{t+2}\right)$ $\checkmark P(Y) = \left(\frac{2}{t+2}\right)$ $\checkmark P(O,O) = \left(\frac{t}{t+2}\right)^2$ $\checkmark P(Y,Y) = \left(\frac{2}{t+2}\right)^2$ \checkmark $\left(\frac{t}{t+2}\right)\left(\frac{t}{t+2}\right) + \left(\frac{2}{t+2}\right)\left(\frac{2}{t+2}\right) = \frac{52}{100}$ $\checkmark t = 3$ (no ca) </p> <p style="text-align: right;">(6) [17]</p>
TOTAL/TOTAAL:		150 marks