# basic education

Department:
Basic Education
**REPUBLIC OF SOUTH AFRICA**

## NATIONAL SENIOR CERTIFICATE

## GRADE 12

**MATHEMATICS P2**

**FEBRUARY/MARCH 2015**

**MARKS: 150**

**TIME: 3 hours**

This question paper consists of 14 pages, 5 diagram sheets and 1 information sheet.

## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.  This question paper consists of 10 questions.

2.  Answer ALL the questions.

3.  Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining your answers.

4.  Answers only will NOT necessarily be awarded full marks.

5.  You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6.  If necessary, round off answers to TWO decimal places, unless stated otherwise.

7.  FIVE diagram sheets for QUESTIONS 1.3, 7, 8, 9.2, 9.3 and 10 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.

8.  Diagrams are NOT necessarily drawn to scale.

9.  Number the answers correctly according to the numbering system used in this question paper.

10. Write neatly and legibly.

**QUESTION 1**

The table below shows the distances (in kilometres) travelled daily by a sales representative for 21 working days in a certain month.

| 131 | 132 | 140 | 140 | 141 | 144 | 146 |
|-----|-----|-----|-----|-----|-----|-----|
| 147 | 149 | 150 | 151 | 159 | 167 | 169 |
| 169 | 172 | 174 | 175 | 178 | 187 | 189 |

1.1     Calculate the mean distance travelled by the sales representative.                          (2)

1.2     Write down the five-number summary for this set of data.                                    (4)

1.3     Use the scaled line on DIAGRAM SHEET 1 to draw a box-and-whisker diagram for this set of data.                                                                                         (2)

1.4     Comment on the skewness of the data.                                                        (1)

1.5     Calculate the standard deviation of the distance travelled.                                 (2)

1.6     The sales representative discovered that his odometer was faulty. The actual reading on each of the 21 days was $p$ km more than that which was indicated. Write down, in terms of $p$ (if applicable), the:

     1.6.1     Actual mean                                                                          (1)

     1.6.2     Actual standard deviation                                                            (1)
                                                                                                   **[13]**

## QUESTION 2

An ice-cream shop recorded the sales of ice cream, in rand, and the maximum temperature, in °C, for 12 days in a certain month. The data that they collected is represented in the table and scatter plot below.

| Temperature in °C | 24,2 | 26,4 | 21,9 | 25,2 | 28,5 | 32,1 | 29,4 | 35,1 | 33,4 | 28,1 | 32,6 | 27,2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sales of ice cream in rand | 215 | 325 | 185 | 332 | 406 | 522 | 412 | 614 | 544 | 421 | 445 | 408 |
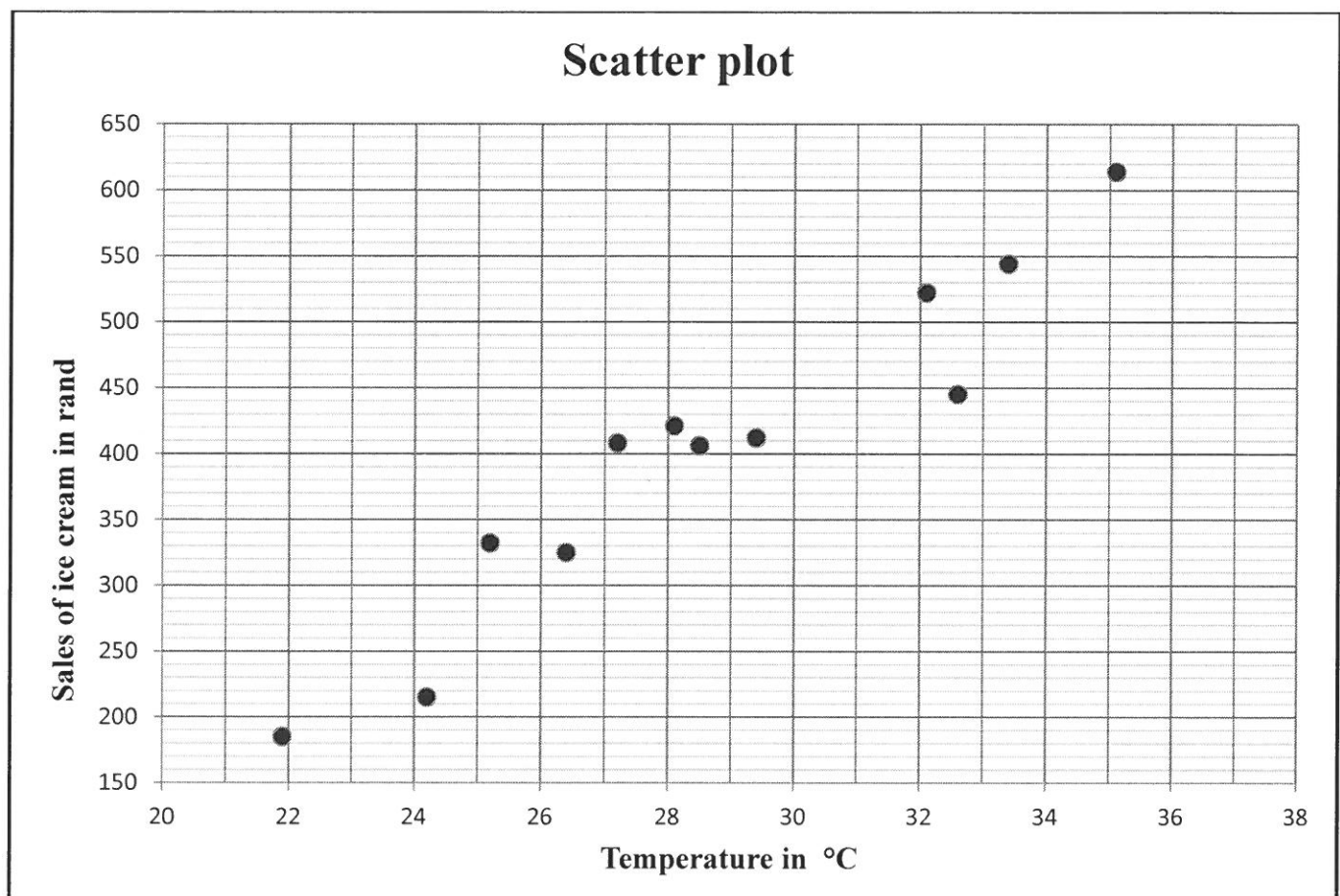


2.1     Describe the influence of temperature on the sales of ice cream in the scatter plot.          (1)

2.2     Give a reason why this trend cannot continue indefinitely.                                      (1)

2.3     Calculate an equation for the least squares regression line (line of best fit).               (4)

2.4     Calculate the correlation coefficient.                                                        (1)

2.5     Comment on the strength of the relationship between the variables.                            (1)
                                                                                                      **[8]**

## QUESTION 3

In the diagram below points  P(5 ; 13),  Q(–1 ; 5)  and  S(7,5 ; 8)  are given.  SR || PQ  where R
is the  y-intercept of  SR.  The  x-intercept of  SR  is  B.  QR  is joined.
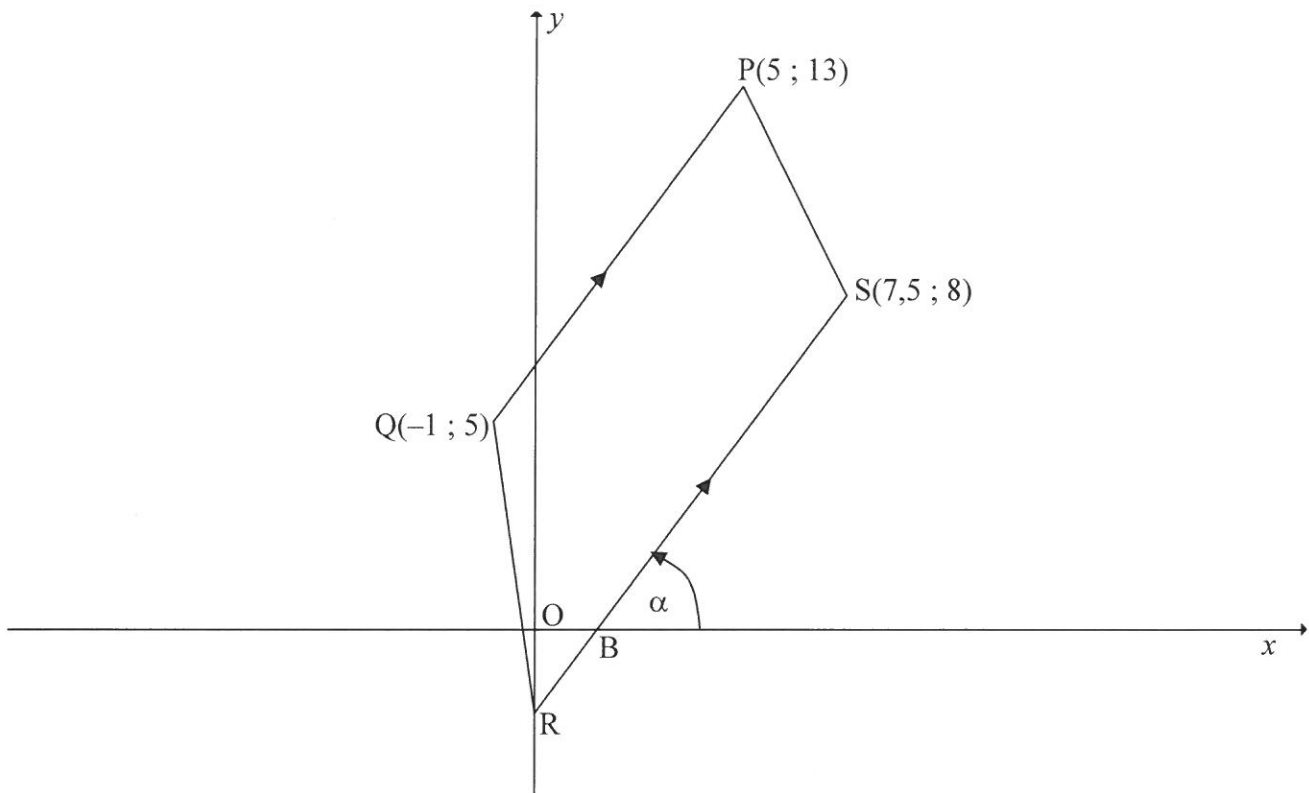


| 3.1 | Calculate the length of  PQ. | (3) |
|---|---|---|
| 3.2 | Calculate the gradient of  PQ. | (2) |
| 3.3 | Determine the equation of line  RS  in the form  $ax + by + c = 0$. | (4) |
| 3.4 | Determine the  x-coordinate of  B. | (2) |
| 3.5 | Calculate the size of  $O\hat{R}B$. | (3) |
| 3.6 | Prove that  QBSP  is a parallelogram. | (4) |
| | | **[18]** |

## QUESTION 4

4.1    In the diagram below, the circle centred at  M(2 ; 4)  passes through  C(–1 ; 2)  and cuts the  $y$-axis at  E.  The diameter  CMD  is drawn and  ACB  is a tangent to the circle.
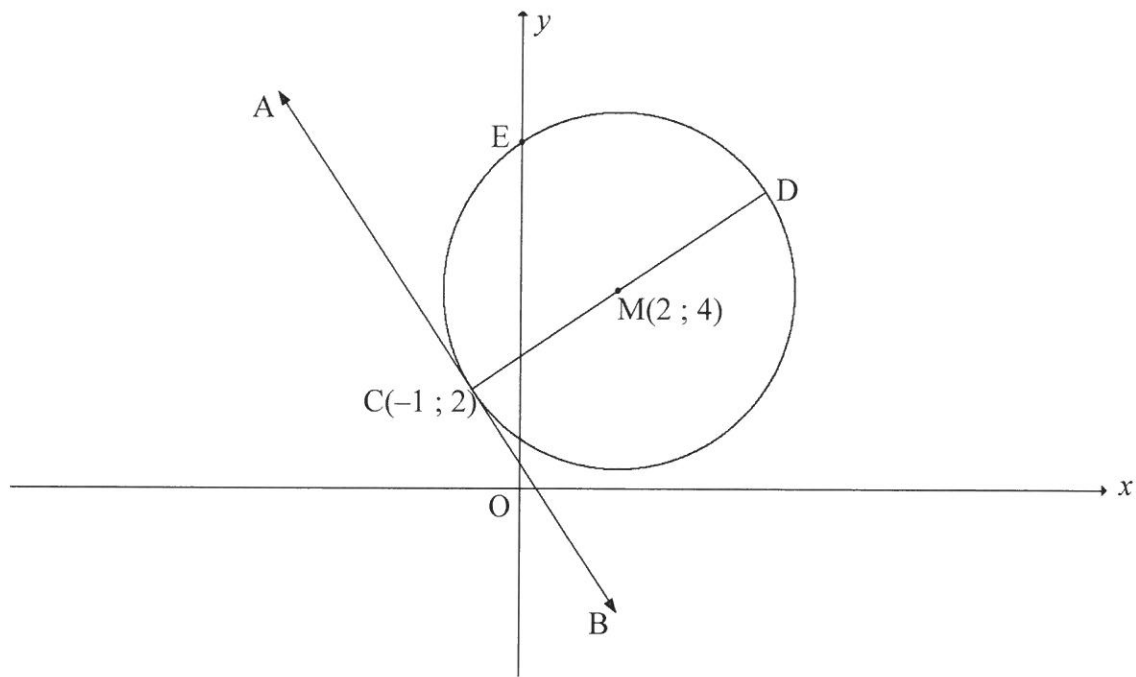


4.1.1    Determine the equation of the circle in the form  $(x - a)^2 + (y - b)^2 = r^2$.    (3)

4.1.2    Write down the coordinates of  D.    (2)

4.1.3    Determine the equation of  AB  in the form  $y = mx + c$.    (5)

4.1.4    Calculate the coordinates of  E.    (4)

4.1.5    Show that  EM  is parallel to  AB.    (2)

4.2    Determine whether or not the circles having equations  $(x + 2)^2 + (y - 4)^2 = 25$  and $(x - 5)^2 + (y + 1)^2 = 9$  will intersect. Show ALL calculations.    (6)
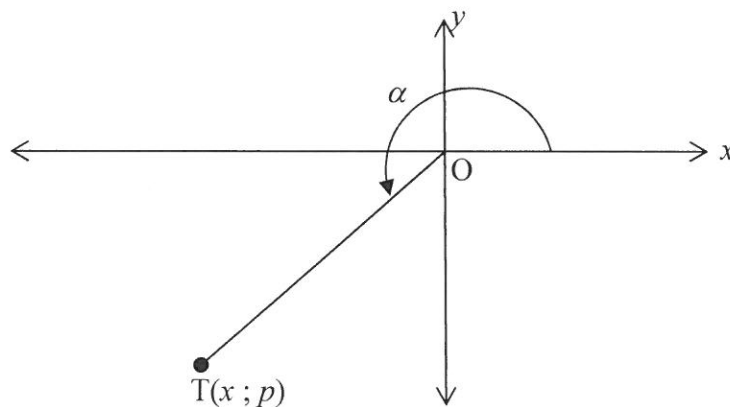
**[22]**

## QUESTION 5

5.1 If $x = 3 \sin \theta$ and $y = 3 \cos \theta$, determine the value of $x^2 + y^2$. (3)

5.2 Simplify to a single term:

$\sin(540° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$ (6)

5.3 In the diagram below, $T(x ; p)$ is a point in the third quadrant and it is given that

$\sin \alpha = \dfrac{p}{\sqrt{1 + p^2}}$ .



5.3.1 Show that $x = -1$. (3)

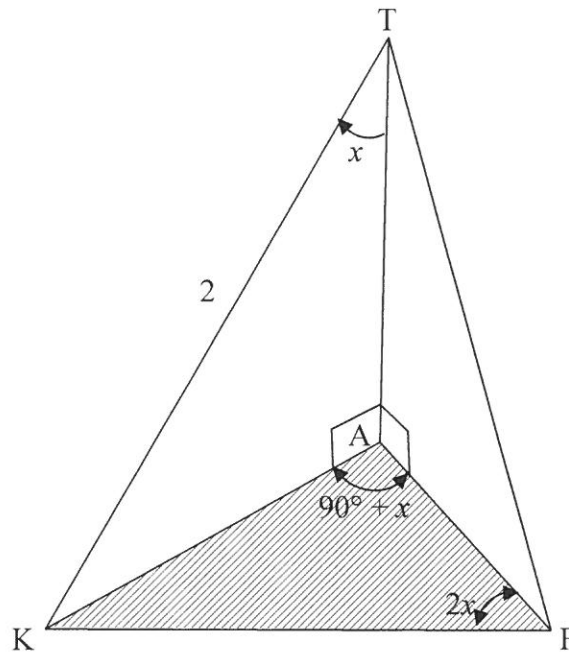5.3.2 Write $\cos(180° + \alpha)$ in terms of $p$ in its simplest form. (2)

5.3.3 Show that $\cos 2\alpha$ can be written as $\dfrac{1 - p^2}{1 + p^2}$. (3)

5.4 5.4.1 For which value(s) of $x$ will $\dfrac{2 \tan x - \sin 2x}{2 \sin^2 x}$ be undefined in the interval $0° \leq x \leq 180°$? (3)

5.4.2 Prove the identity: $\dfrac{2 \tan x - \sin 2x}{2 \sin^2 x} = \tan x$ (6)

**[26]**

## QUESTION 6

6.1 In the figure, points $K$, $A$ and $F$ lie in the same horizontal plane and $TA$ represents a vertical tower. $A\hat{T}K = x$, $K\hat{A}F = 90° + x$ and $K\hat{F}A = 2x$ where $0° < x < 30°$. $TK = 2$ units.



6.1.1 Express $AK$ in terms of $\sin x$. (2)

6.1.2 Calculate the numerical value of $KF$. (5)

6.2 In the diagram below, a circle with centre O passes through A, B and C. BC = AC = 15 units. BO and OC are joined. OB = 10 units and $B\hat{O}C = x$.
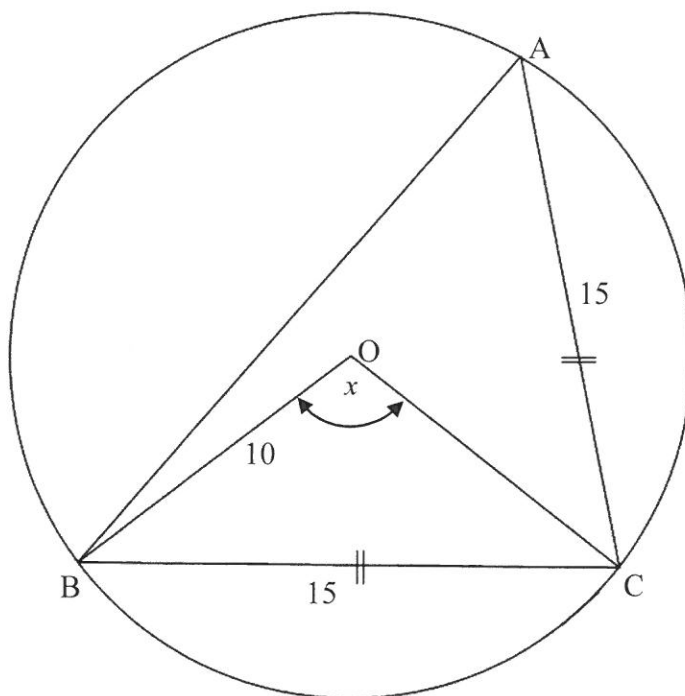


Calculate:

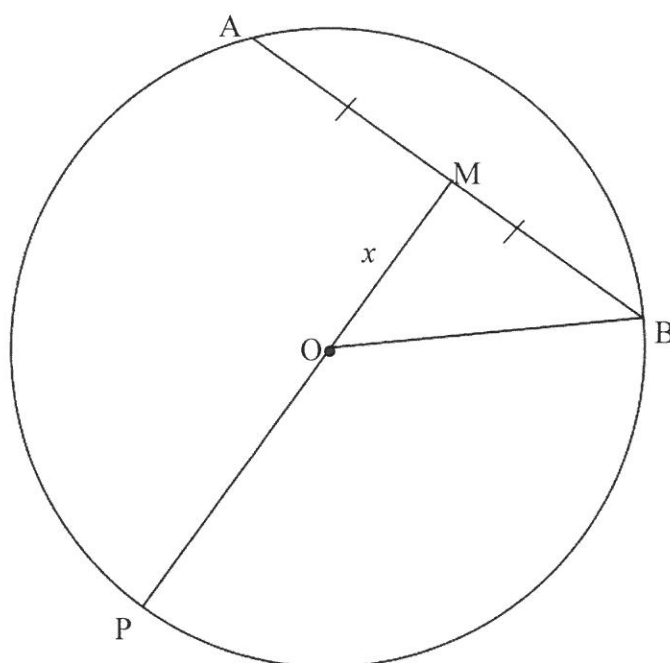6.2.1 The size of $x$ (4)

6.2.2 The size of $A\hat{C}B$ (3)

6.2.3 The area of $\triangle ABC$ (2)

**[16]**

**GIVE REASONS FOR YOUR ANSWERS IN QUESTIONS 7, 8, 9 AND 10.**

**QUESTION 7**

In the diagram, AB is a chord of the circle with centre O. M is the midpoint of AB. MO is produced to P, where P is a point on the circle. OM = $x$ units, AB = 20 units and $\dfrac{PM}{OM} = \dfrac{5}{2}$.



7.1     Write down the length of MB.                                                    (1)

7.2     Give a reason why OM ⊥ AB.                                                    (1)

7.3     Show that OP = $\dfrac{3x}{2}$ units.                                            (2)
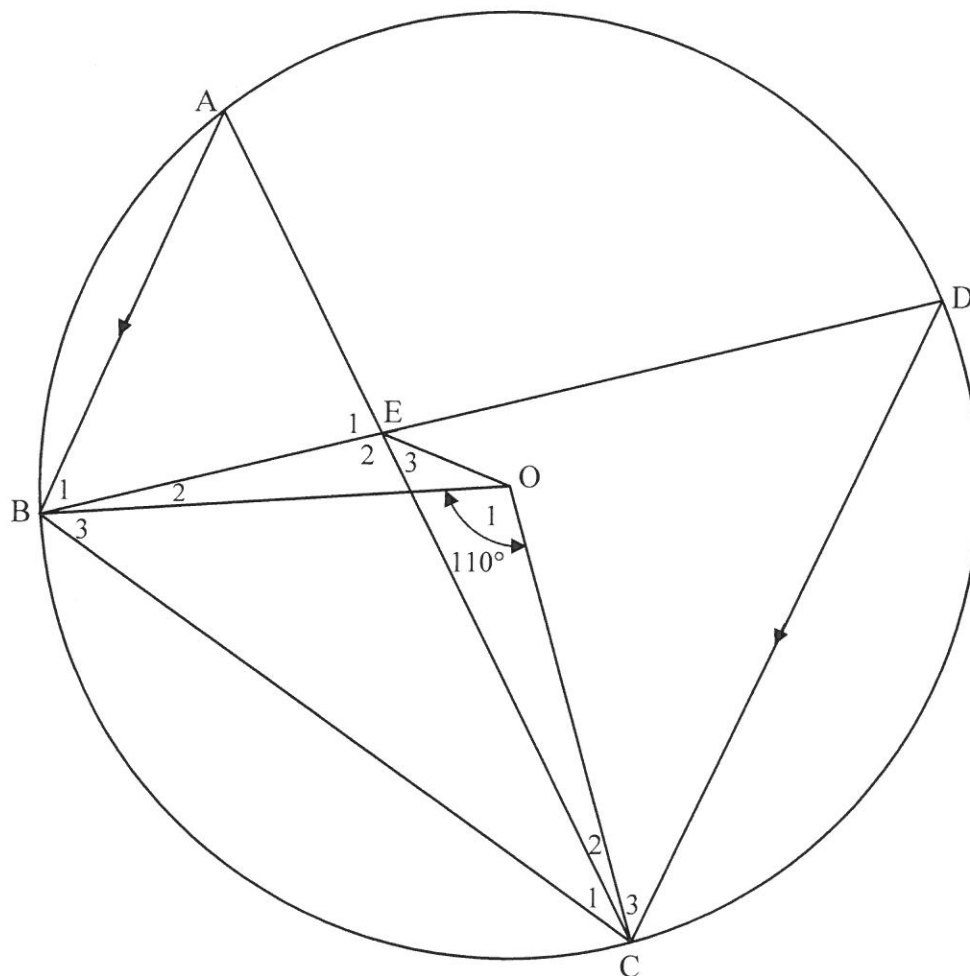
7.4     Calculate the value of $x$.                                                     (3)

[7]

## QUESTION 8

In the diagram below, the circle with centre  O  passes through  A,  B,  C  and  D.
AB ∥ DC  and  BÔC = 110°.
The chords  AC  and  BD  intersect at  E.
EO,  BO,  CO  and  BC  are joined.



8.1     Calculate the size of the following angles, giving reasons for your answers:

8.1.1     $\hat{D}$ (2)

8.1.2     $\hat{A}$ (2)

8.1.3     $\hat{E}_2$ (4)

8.2     Prove that  BEOC  is a cyclic quadrilateral. (2)

**[10]**

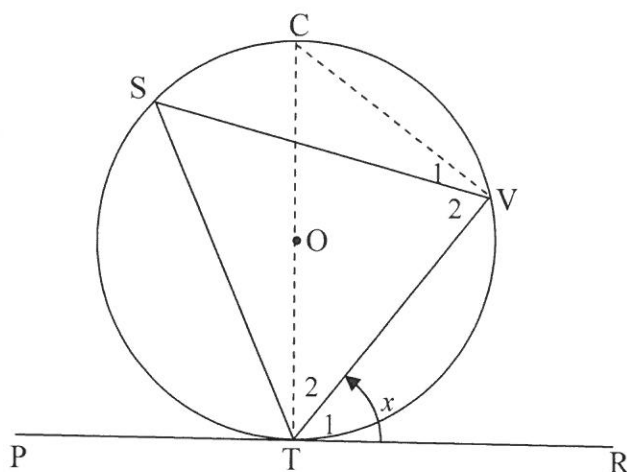## QUESTION 9

9.1 Complete the statement of the following theorem:

*The exterior angle of a cyclic quadrilateral is equal to ...* (1)

9.2 In the diagram below the circle with centre O passes through points S, T and V. PR is a tangent to the circle at T. VS, ST and VT are joined.



Given below is the partially completed proof of the theorem that states that $V\hat{T}R = \hat{S}$. **Using the above diagram, complete the proof of the theorem on DIAGRAM SHEET 3.**

Construction: Draw diameter TC and join CV.

| Statement | Reason |
|---|---|
| **Let:** $V\hat{T}R = \hat{T}_1 = x$ | |
| $\hat{V}_1 + \hat{V}_2 = .........$ | ..................................................................... |
| $\hat{T}_2 = 90° - x$ | ..................................................................... |
| $\therefore \hat{C} = .........$ | Sum of the angles of a triangle |
| $\therefore \hat{S} = x$ | ..................................................................... |
| $\therefore V\hat{T}R = \hat{S}$ | |

(5)

9.3     In the figure, TRSW is a cyclic quadrilateral with TW = WS. RT and RS are produced to meet tangent VWZ at V and Z respectively. PRQ is a tangent to the circle at R. RW is joined. $\hat{R}_2 = 30°$ and $\hat{R}_4 = 50°$.
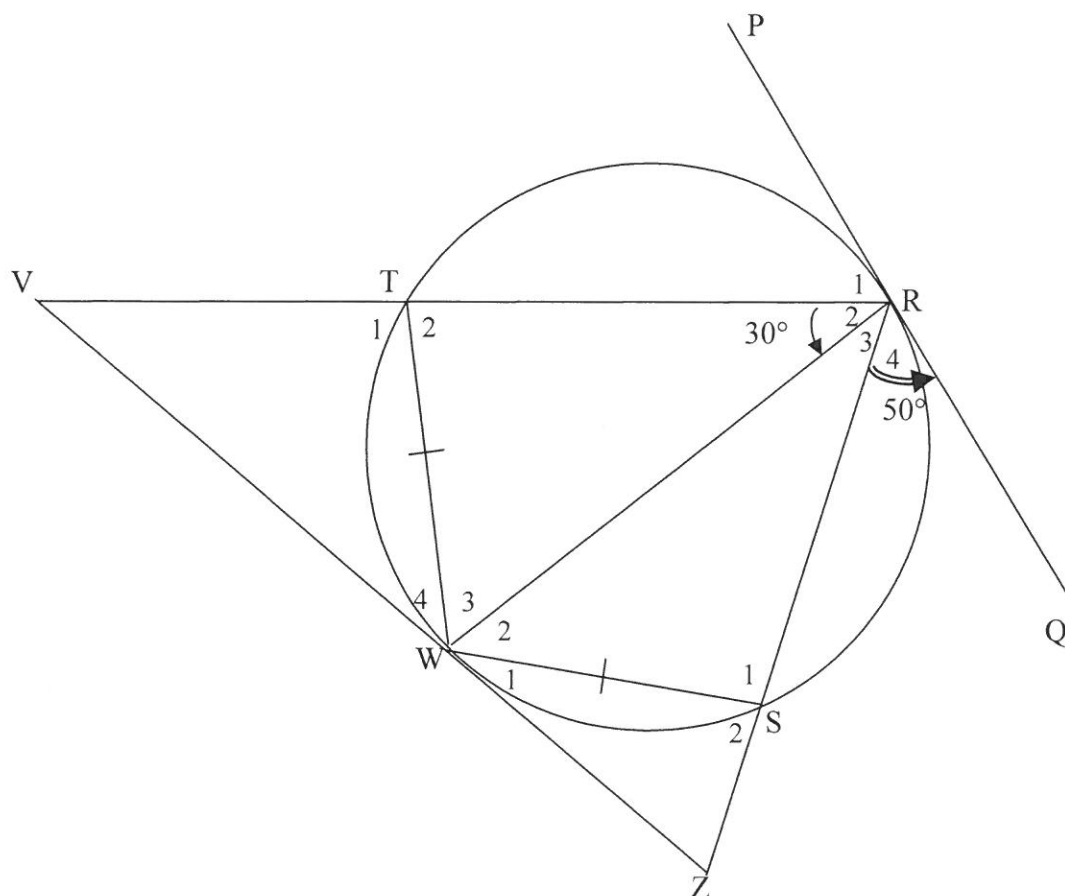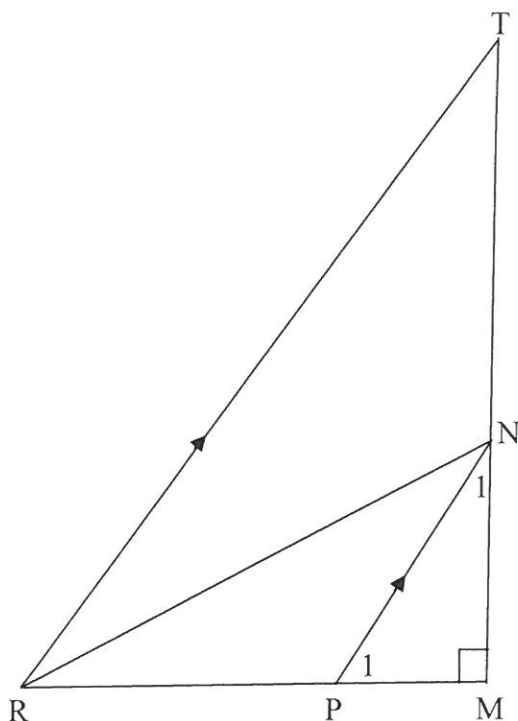


9.3.1       Give a reason why $\hat{R}_3 = 30°$.                                           (1)

9.3.2       State, with reasons, TWO other angles equal to 30°.               (3)

9.3.3       Determine, with reasons, the size of:

            (a)      $\hat{S}_2$                                                         (3)

            (b)      $\hat{V}$                                                          (4)

9.3.4       Prove that $WR^2 = RV \times RS$.                                    (5)

                                                                           **[22]**

**QUESTION 10**

In $\Delta TRM$, $\hat{M} = 90°$. NP is drawn parallel to TR with N on TM and P on RM. It is further given that RT = 3PN.



10.1    Give reasons for the statements below.
        **Use DIAGRAM SHEET 5.**

|       | Statement | Reason |
|-------|-----------|--------|
|       | In $\Delta PNM$ and $\Delta RTM$ : | |
| 10.1.1 | $\hat{N}_1 = \hat{T}$ | ..................................................... |
|       | $\hat{M}$ is common | |
| 10.1.2 | $\therefore \Delta PNM \ \| \| \ \Delta RTM$ | ..................................................... |

(2)

10.2    Prove that $\dfrac{PM}{RM} = \dfrac{1}{3}$.                                                                 (2)

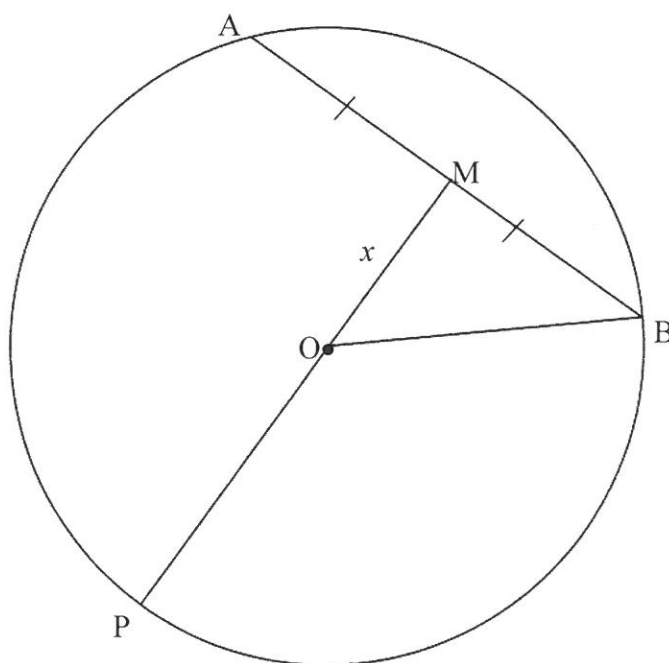10.3    Show that $RN^2 - PN^2 = 2RP^2$.                                                          (4)

**[8]**

**TOTAL:    150**

**CENTRE NUMBER:**

**EXAMINATION NUMBER:**

**DIAGRAM SHEET 1**

**QUESTION 1.3**

```
120      130      140      150      160      170      180      190      200
```

**QUESTION 7**

**DIAGRAM SHEET 2**

**QUESTION 8**
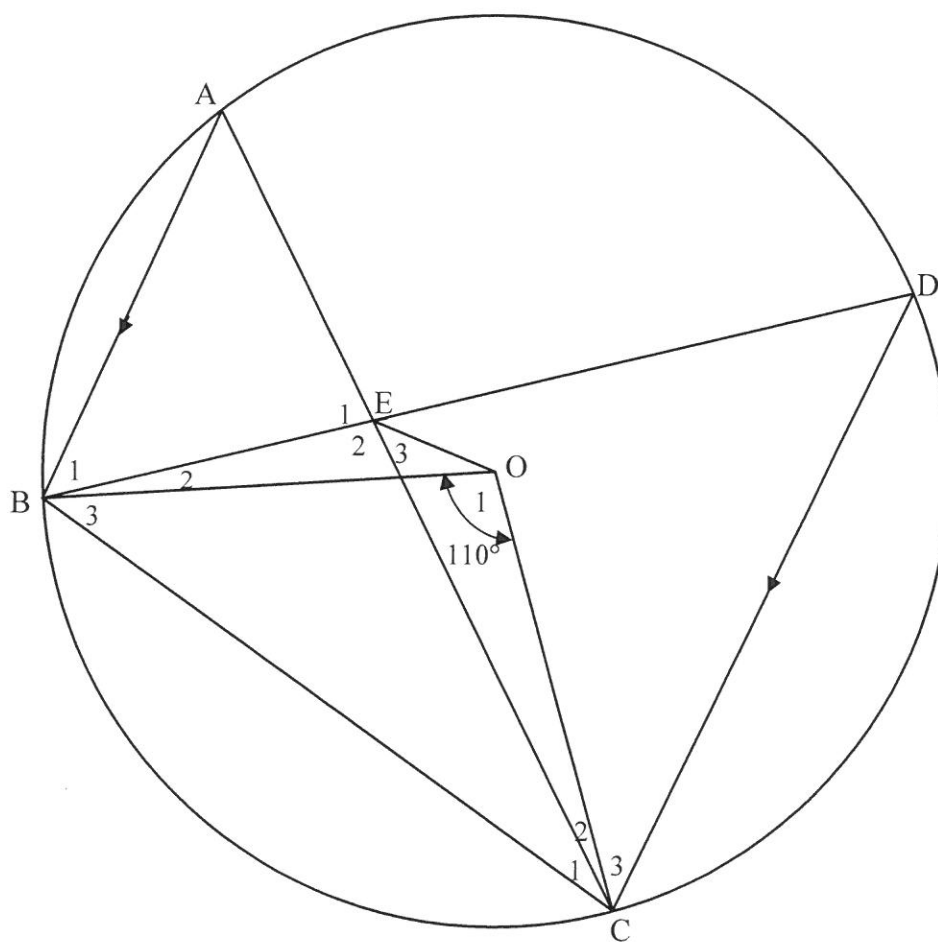
**CENTRE NUMBER:**

**EXAMINATION NUMBER:**

**DIAGRAM SHEET 3**

**QUESTION 9.2**
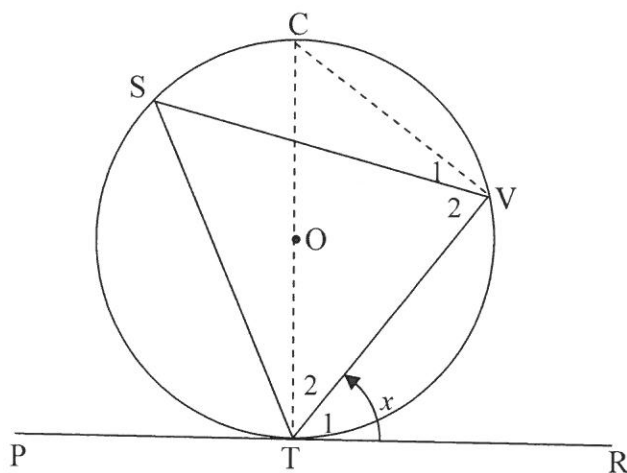


Construction: Draw diameter CT and join CV.

| Statement | Reason |
|---|---|
| **Let:** $V\hat{T}R = \hat{T}_1 = x$ | |
| $\hat{V}_1 + \hat{V}_2 = \ldots\ldots\ldots$ | ......................................................................................... |
| $\hat{T}_2 = 90° - x$ | ......................................................................................... |
| $\therefore \hat{C} = \ldots\ldots\ldots$ | Sum of the angles of a triangle |
| $\therefore \hat{S} = x$ | ......................................................................................... |
| $\therefore V\hat{T}R = \hat{S}$ | |

**CENTRE NUMBER:**

**EXAMINATION NUMBER:**

**DIAGRAM SHEET 4**

**QUESTION 9.3**
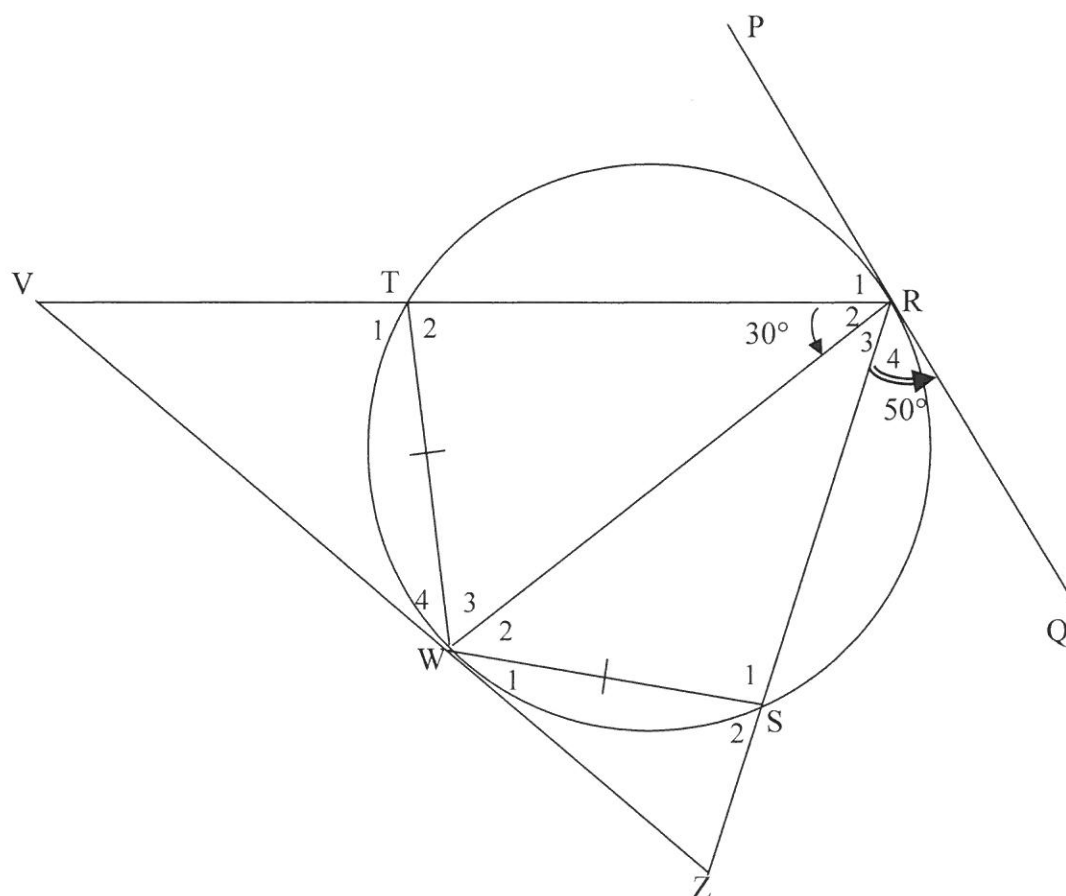
**CENTRE NUMBER:**

**EXAMINATION NUMBER:**

**DIAGRAM SHEET 5**

**QUESTION 10**
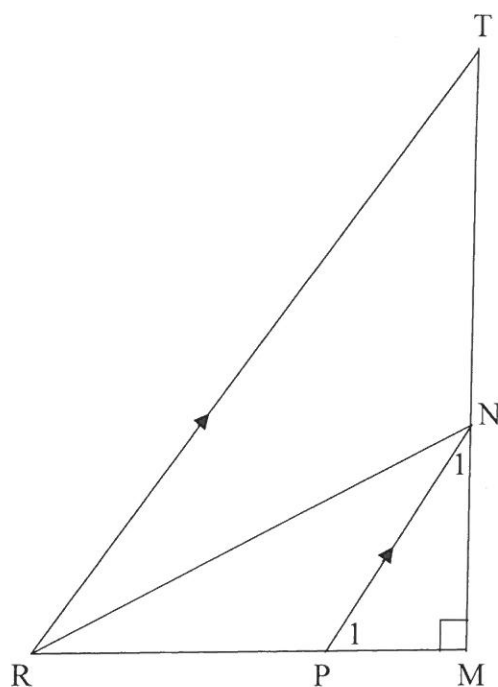


10.1

|  | **Statement** | **Reason** |
|---|---|---|
|  | In ΔPNM and ΔRTM : | |
| 10.1.1 | $\hat{N}_1 = \hat{T}$ | ................................................. |
|  | $\hat{M}$ is common | |
| 10.1.2 | ∴ ΔPNM ||| ΔRTM | ................................................. |

## INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad\qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad\qquad S_n = \frac{a(r^n - 1)}{r - 1} \quad ; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad\qquad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad\qquad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad\qquad y - y_1 = m(x - x_1) \qquad\qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad\qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad\qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad\qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad\qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad\qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad\qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad\qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$